CS6848 - Principles of Programming Languages

Principles of Programming Languages

V. Krishna Nandivada

IIT Madras

Last class

Interpreters

A Environment
B Cells
C Closures
D Recursive environments
E Interpreting OO (MicroJava) programs.

Outline

Introduction

- An interpreter executes a program as per the semantics.
- An interpreter can be viewed as an executable description of the semantics of a programming language.
- Program semantics is the field concerned with the rigorous mathematical study of the meaning of programming languages and models of computation.
- Formal ways of describing the programming semantics.
  - Operational semantics - execution of programs in the language is described directly (in the context of an abstract machine).
    - Big-step semantics (with environments) -is close in spirit to the interpreters we have seen earlier.
    - Small-step semantics (with syntactic substitution) - formalizes the inlining of a procedure call as an approach to computation.
  - Denotational Semantics - each phrase in the language is *translated* to a *denotation* - a phrase in some other language.
  - Axiomatic semantics - gives meaning to phrases by describing the logical axioms that apply to them.

Lambda Calculus

- The traditional syntax for procedures in the lambda-calculus uses the Greek letter λ (lambda), and the grammar for the lambda-calculus can be written as:

  $e \;::=\; x \mid \lambda x.e \mid e_1 e_2$

  $x \;\in\;$ Identifier (infinite set of variables)
- Brackets are only used for grouping of expressions. Convention for saving brackets:
  - that the body of a λ-abstraction extends "as far as possible."
  - For example, $\lambda x.xy$ is short for $\lambda x.(xy)$ and not $(\lambda x.x)y$.
  - Moreover, $e_1e_2e_3$ is short for $(e_1e_2)e_3$ and not $e_1(e_2e_3)$.

Extension of the Lambda-calculus

We will give the semantics for the following extension of the lambda-calculus:

$e \;::=\; x \mid \lambda x.e \mid e_1 e_2 \mid c \mid succ\ e$

$x \;\in\;$ Identifier (infinite set of variables)

$c \;\in\;$ Integer

Outline

Big step semantics

Here is a big-step semantics with environments for the lambda-calculus.

$$
\begin{array}{lcl}
w, v & \in & Value \\
v & ::= & c \mid (\lambda x.e, \rho) \\
\rho & \in & Environment \\
\rho & ::= & x_1 \mapsto v_1, \cdots x_n \mapsto v_n
\end{array}
$$

The semantics is given by the following five rules:

(1) $$\rho \vdash x \rhd v \quad (\rho(x) = v)$$

(2) $$\rho \vdash \lambda x.e \rhd (\lambda x.e, \rho)$$

(3) $$\frac{\rho \vdash e_1 \rhd (\lambda x.e, \rho') \quad \rho \vdash e_2 \rhd v \quad \rho', x \mapsto v \vdash e \rhd w}{\rho \vdash e_1 e_2 \rhd w}$$

(4) $$\rho \vdash c \rhd c$$

(5) $$\frac{\rho \vdash e \rhd c_1}{\rho \vdash \texttt{succ}\ e \rhd c_2} \quad \lceil c_2 \rceil = \lceil c_1 \rceil + 1$$

Outline

Small step semantics

- In small step semantics, one step of computation = either one primitive operation, or inline one procedure call.
- We can do steps of computation in different orders:

```
> (define foo
       (lambda (x y) (+ (* x 3) y)))
> (foo (+ 4 1) 7)
22
```

Let us calculate:

```
(foo (+ 4 1) 7)
=>   ((lambda (x y) (+ (* x 3) y))
       (+ 4 1) 7)
=>   (+ (* (+ 4 1) 3) 7)
=> 22
```


Small step semantics (contd.)

We can also calculate like this:

```
(foo
(+ 4 1) 7)

=> (foo 5 7)

=>   ((lambda (x y) (+ (* x 3) y))
       5 7)

=> (+ (* 5 3) 7)

=> 22
```


Free variables

A variable x occurs *free* in an expression E *iff* x is not bound in E.Examples:

- no variables occur free in the expression

  `(lambda (y) ((lambda (x) x) y))`

- the variable `y` occurs free in the expression

  `((lambda (x) x) y)`

An expression is *closed* if it does not contain free variables.
A program is a closed expression.

Methods of procedure application

Call by value

```
((lambda (x) x)
((lambda (y) (+ y 9)) 5))

=> ((lambda (x) x) (+ 5 9))

=> ((lambda (x) x) 14)

=> 14
```

Always evaluate the arguments first

- Example: Scheme, ML, C, C++, Java

Methods of procedure application

Call by name (or lazy-evaluation)

```
((lambda (x) x)
         ((lambda (y) (+ y 9) 5))

=> ((lambda (y) (+ y 9)) 5)

=> (+ 5 9)

=> 14
```

Avoid the work if you can

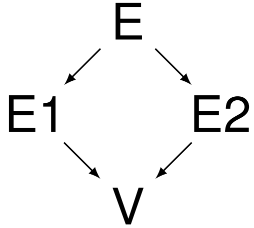
- Example: Miranda and Haskell

Lazy or eager: Is one more efficient? Are both the same?

Difference

- Q: If we run the same program using these two semantics, can we get different results?
- A:
  - If the run with call-by-value reduction terminates, then the run with call- by-name reduction terminates. (But the converse is in general false).
  - If both runs terminate, then they give the same result.

Church Rosser theorem



Call by value - too eager?

Sometimes call-by-value reduction fails to terminate, even though call-by- name reduction terminates.

```
  (define delta (lambda (x) (x x)))
    (delta delta)
=>  (delta delta)
=>  (delta delta)
=>  ...
```

Consider the program:

```
  (define const (lambda (y) 7))
  (const (delta delta))
```

- call by value reduction fails to terminate; cannot finish evaluating the operand.
- call by name reduction terminates.

Summary - calling convention

- call by value is more efficient but may not terminate
- call by name may evaluate the same expression multiple times.
- Lazy languages uses - call-by-need.
- Languages like Scala allow both call by value and name!

Beta reduction

- A procedure call which is ready to be “inlined” is called a *beta-redex*. Example `( (lambda (var) body ) rand )`
- In lambda-calculus call-by-value and call-by-name reduction allow the choosing of arbitrary beta-redex.
- The process of inlining a beta-redex for some reducible expression is called *beta-reduction*.

  `( (lambda (var) body ) rand )   body[var:=rand]`
- η conversion: A simple optimization:

$$(\lambda x\ (E\ x))\ =\ E$$

- A *conversion* when applied in the left-to-right direction is called a *reduction*.

Notes on reduction

- Applicative order reduction - A β reduction can be applied only if both the operator and the operand are already values. Else?
- Applicative order reduction (call by value), example: Scheme, C, Java.

Notes on reduction

- Is there a reduction strategy which is guaranteed to find the answer if it exists? – *leftmost* reduction (lazy evaluation).
- leftmost-reduction – reduce the β-redex whose left parenthesis comes first
- A lambda expression is in *normal* form if it contains no β-redexes.
- An expression in normal form – cannot be further reduced. e.g. constant or (lambda (x) x)
- Church-Rosser theorem $\rightarrow$ expression can have at most one normal form.
- leftmost reduction will find the normal form of an expression if one exists.

Name clashes

- Care must be taken to avoid name clashes. Example:

```
((lambda (x)
   (lambda (y) (y x)))
  (y 5))
```

should not be transformed into

```
(lambda (y) (y (y 5)))
```

- The reference to y in (y 5) should remain **free**!
- The solution is to change the name of the inner variable name `y` to some name, say `z`, that does not occur free in the argument `y 5`.

```
((lambda (x)
    (lambda (z) (z x)))
 (y 5))

=>  (lambda (z) (z (y x))) ;; y is free.
```


Substitution

- The notation $e[x := M]$ denotes e with M substituted for every free occurrence of x in such that a way that name clashes are avoided.
- We will define $e[x := M]$ inductively on e.

$$\begin{array}{lcl} x[x := M] & \equiv & M \\ y[x := M] & \equiv & y\ (x \neq y) \\ (\lambda x.e_1)[x := M] & \equiv & (\lambda x.e_1) \\ (\lambda y.e_1)[x := M] & \equiv & \lambda z.((e_1[y := z])[x := M]) \\ & & \text{(where } x \neq y \text{ and } z \text{ does not} \\ & & \text{occur free in } e_1 \text{ or } M). \\ (e_1 e_2)[x := M] & \equiv & (e_1[x := M])(e_2[x := M]) \\ c[x := M] & \equiv & c \\ (succ\ e_1)[x := M] & \equiv & succ\ (e_1[x := M]) \end{array}$$

- The renaming of a bound variable by a *fresh* variable is called *alpha-conversion*.
- Q: Can we avoid creating a new variable in the fourth rule ?

Small step semantics

Here is a small-step semantics with syntactic substitution for the lambda-calculus.

$$\begin{array}{lcl} v & \in & Value \\ v & ::= & c|\lambda x.e \end{array}$$

The semantics is given by the reflexive, transitive closure of the relation $\rightarrow_V$

$$\rightarrow_V \subseteq Expression \times Expression$$

(6) $$\lambda x.e\ v \rightarrow_V e[x := v]$$

(7) $$\frac{e_1 \rightarrow_V e_1'}{e_1 e_2 \rightarrow_V e_1' e_2}$$

(8) $$\frac{e_2 \rightarrow_V e_2'}{v e_2 \rightarrow_V v e_2'}$$

(9) $$\texttt{succ} c_1 \rightarrow_V c_2 (\lceil c_2 \rceil = \lceil c_1 \rceil + 1)$$

(10) $$\frac{e_1 \rightarrow_V e_2}{\texttt{succ}\ e_1 \rightarrow_V succ e_2}$$