Cryptographic Hash Functions

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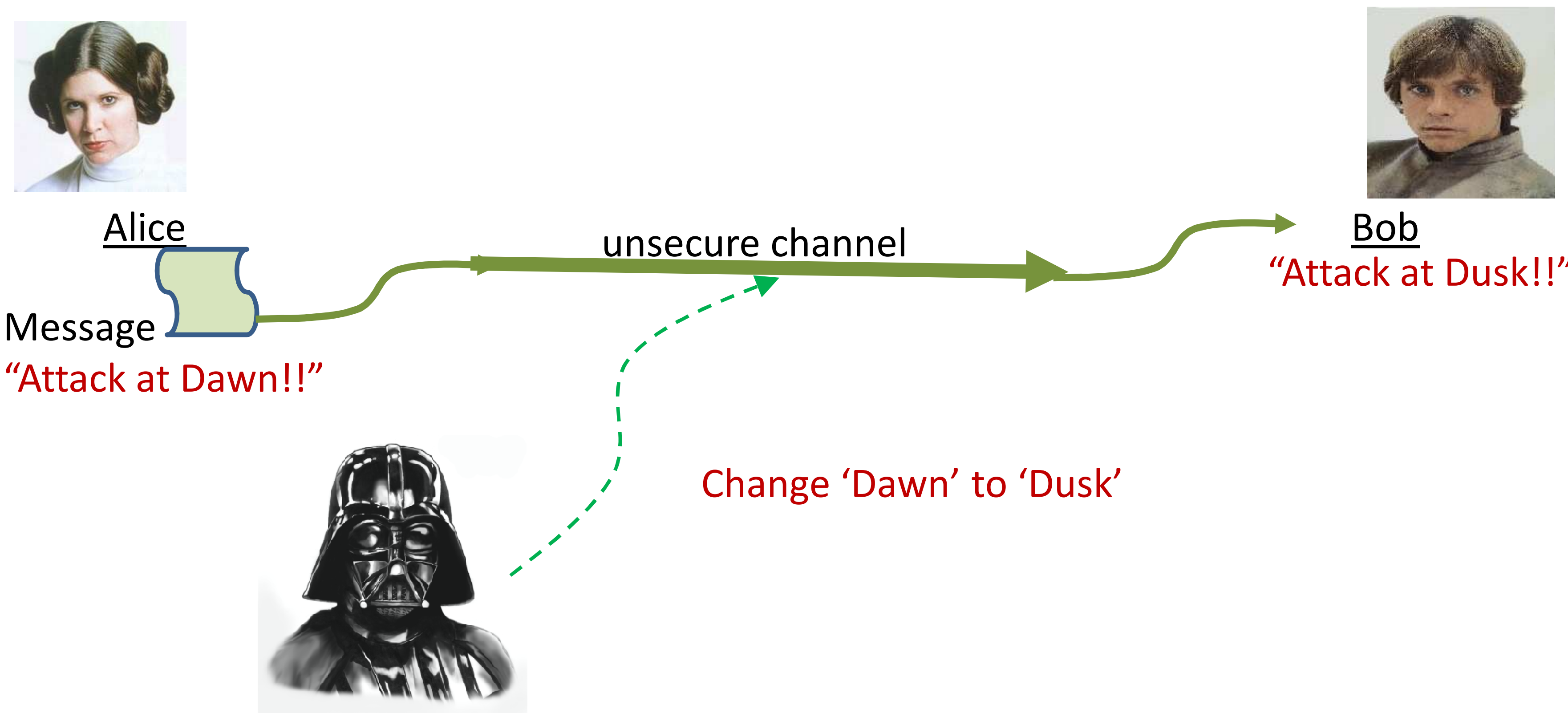
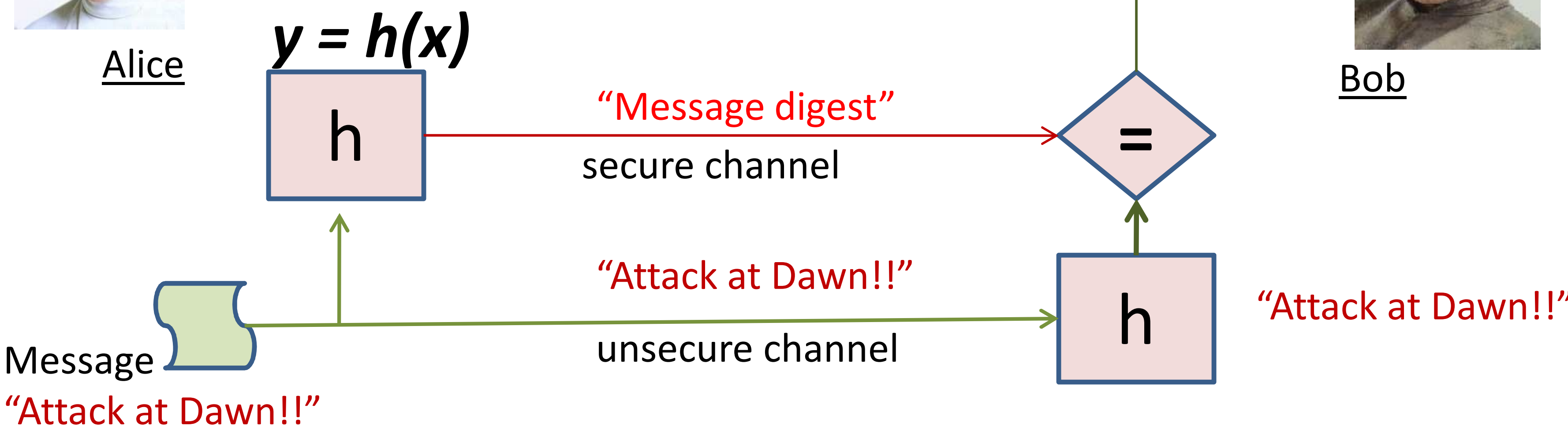
Issues with Integrity



How can Bob ensure that Alice’s message has not been modified?

Note…. We are not concerned with confidentiality here

Hashes

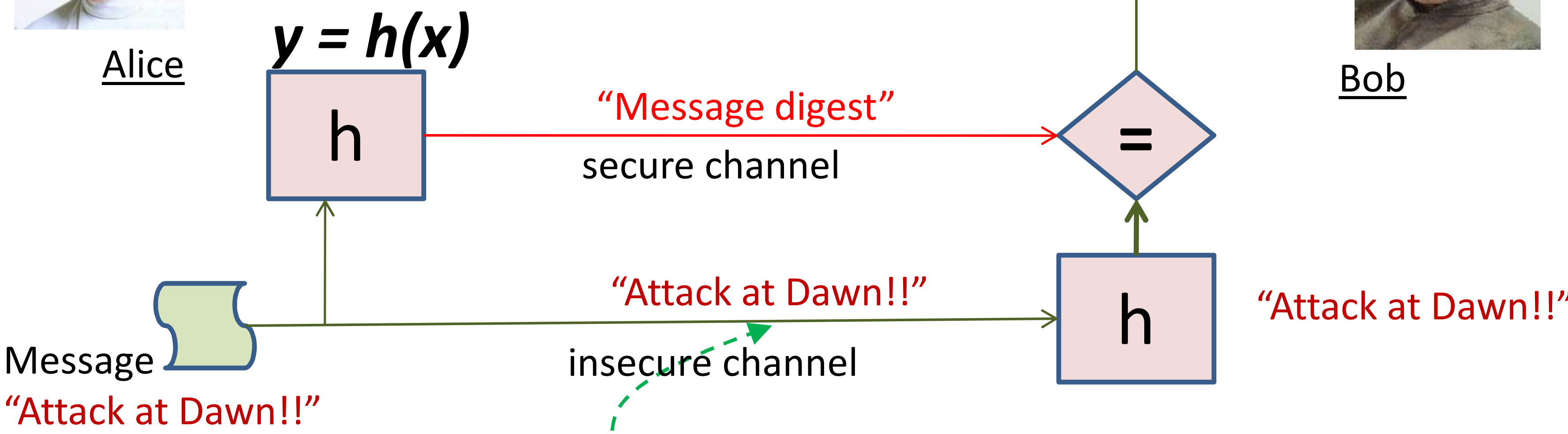


Alice passes the message through a hash function, which produces a fixed length message digest.

- **The message digest is representative of Alice's message.**
- **Even a small change in the message will result in a completely new message digest**
- **Typically of 160 bits, irrespective of the message size.**

Bob re-computes a message hash and verifies the digest with Alice's message digest.

Integrity with Hashes



$y = h(x)$

$y = h(x')$

Mallory does not have access to the digest y. Her task (to modify Alice's message) is much more difficult.

If she modifies x to x', the modification can be detected unless h(x) = h(x')

Hash functions are specially designed to resist such collisions

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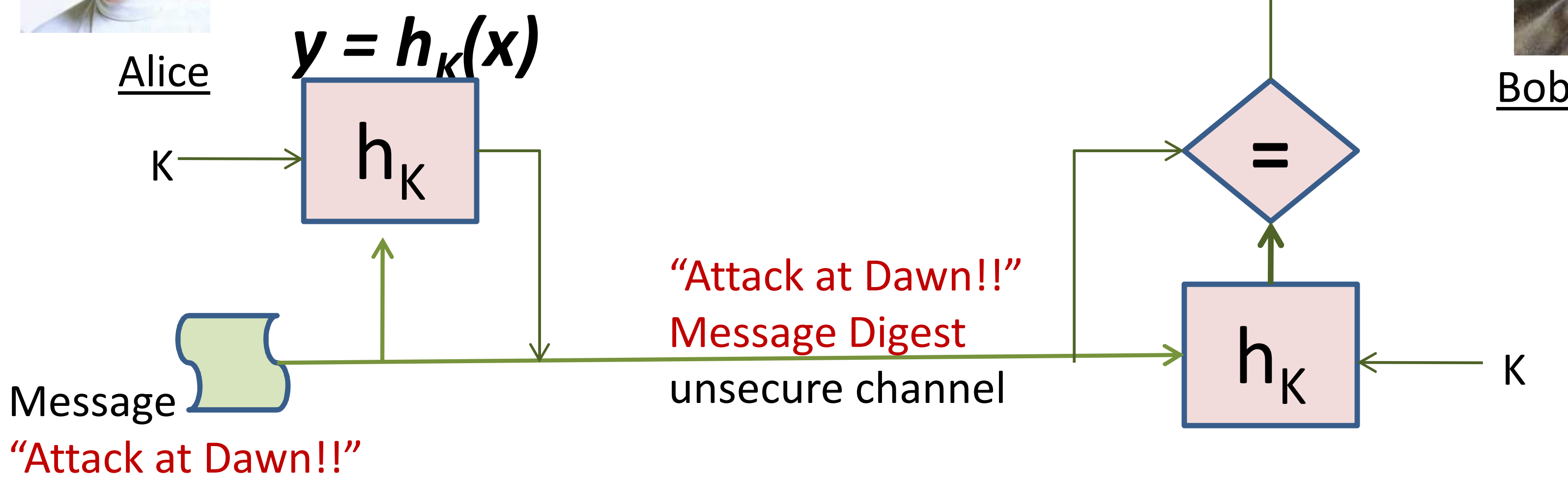
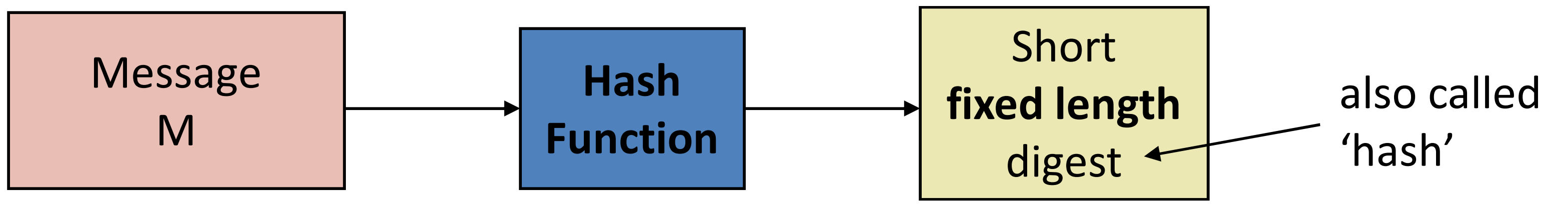
Message Authentication Codes (MAC)



MACs allow the message and the digest to be sent over an insecure channel

However, it requires Alice and Bob to share a common key

Avalanche Effect



Hash functions provide unique digests with high probability.

Even a small change in **M** will result in a new digest

SHA256("short sentence")

0x 0acdf28f4e8b00b399d89ca51f07fef34708e729ae15e85429c5b0f403295cc9

SHA256("The quick brown fox jumps over the lazy **dog**")

0x d7a8fbb307d7809469ca9abcb0082e4f8d5651e46d3cdb762d02d0bf37c9e592

SHA256("The quick brown fox jumps over the lazy **dog.**")

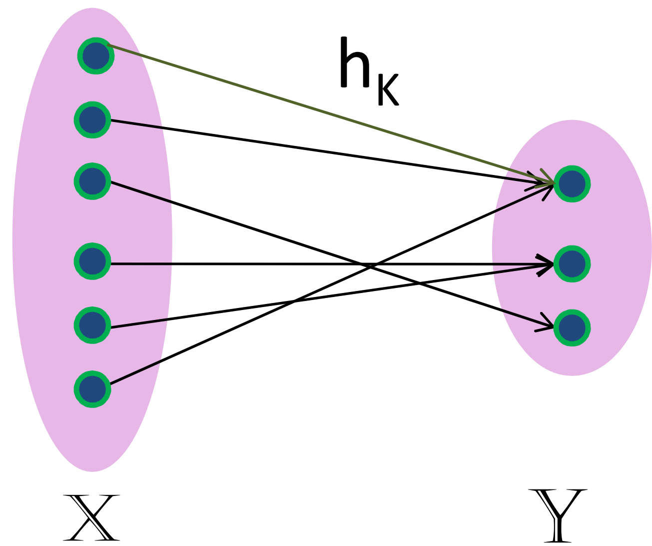
(extra period added)

0x ef537f25c895bfa782526529a9b63d97aa631564d5d789c2b765448c8635fb6c

Hash functions in Security

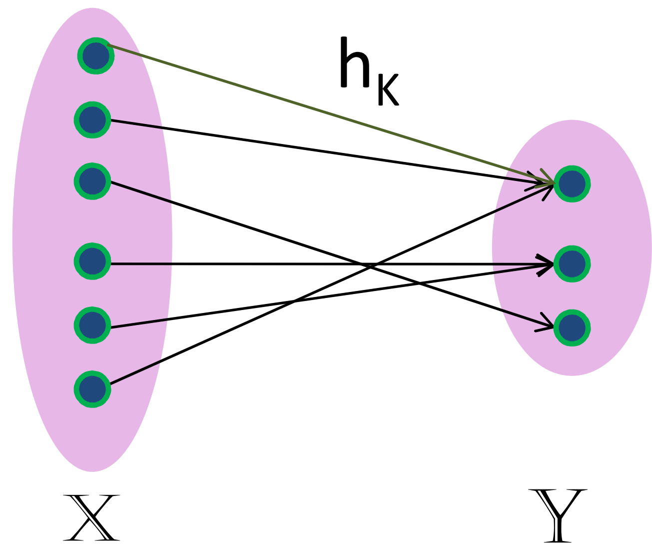
- Digital signatures
- Random number generation
- Key updates and derivations
- One way functions
- MAC
- Detect malware in code
- User authentication (storing passwords)

Hash Family



- The hash family is a 4-tuple defined by ($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{K}$,$\mathbb{H}$)
- $\mathbb{X}$ is a set of messages
  (may be infinite, we assume the minimum size is at least 2|$\mathbb{Y}$|)
- $\mathbb{Y}$ is a finite set of message digests (aka authentication tags)
- $\mathbb{K}$ is a finite set of keys
- Each K Ɛ $\mathbb{K}$, defines a keyed hash function h_K Ɛ $\mathbb{H}$

Hash Family : some definitions



- Valid pair under K : (x,y) Ɛ $\mathbb{X}$x$\mathbb{Y}$ such that, x = h_K(y)
- Size of the hash family:

  is the number of functions possible from set $\mathbb{X}$ to set $\mathbb{Y}$

  $|\mathbb{Y}| =$ M and $|\mathbb{X}| =$ N

  then the number of mappings possible is M^N

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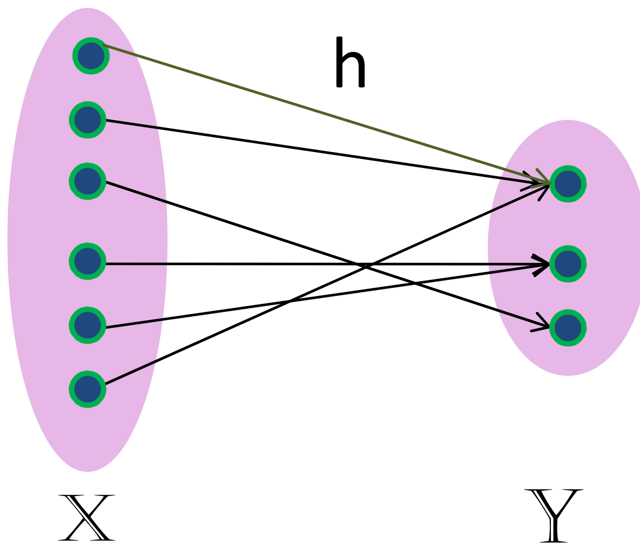
Unkeyed Hash Function



- The hash family is a 4-tuple defined by ($\mathbb{X}$,$\mathbb{Y}$,$\mathbb{K}$,$\mathbb{H}$)
- $\mathbb{X}$ is a set of messages
  (may be infinite, we assume the minimum size is at least 2|$\mathbb{Y}$|)
- $\mathbb{Y}$ is a finite set of message digests
- In an unkeyed hash function : |$\mathbb{K}$ | = 1
- We thus have only one mapping function in the family

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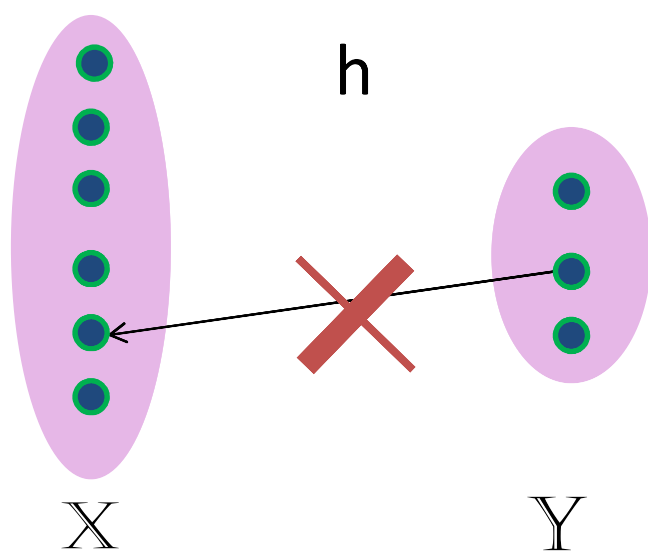
Hash function Requirement
Preimage Resistant

- Also know as **one-wayness problem**
- If Mallory happens to know the message digest, she should not be able to determine the message
- Given a hash function h : $\mathbb{X} \rightarrow \mathbb{Y}$ and an element y Ɛ $\mathbb{Y}$. Find any x Ɛ $\mathbb{X}$ such that, h(x) = y



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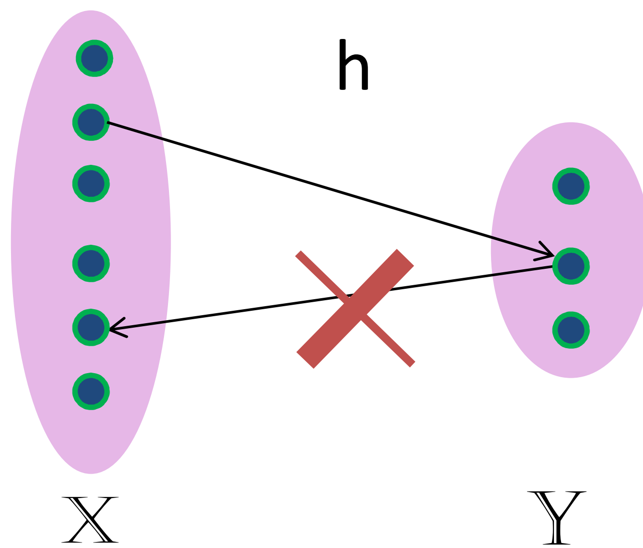
Hash function Requirement (Second Preimage)

- Mallory has x and can compute h(x), she should not be able to find another message x' which produces the same hash.
  - It would be easy to forge new digital signatures from old signatures if the hash function used weren't second preimage resistant
- Given a hash function h : $\mathbb{X} \rightarrow \mathbb{Y}$ and an element x Ɛ $\mathbb{X}$,Find, x' Ɛ $\mathbb{X}$ such that, h(x) = h(x')



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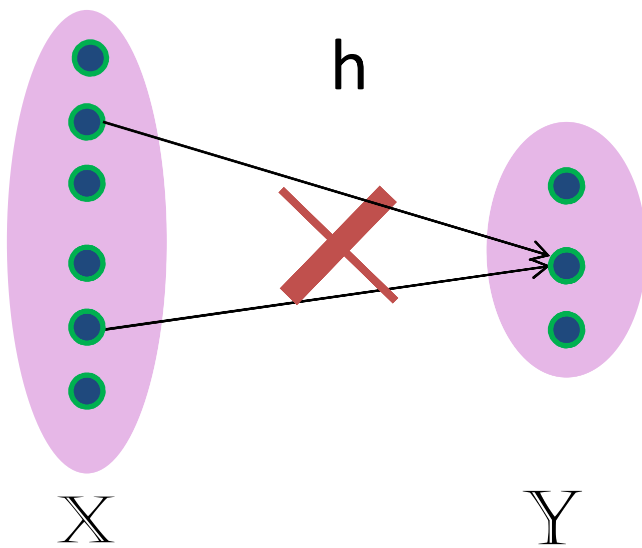
Hash Function Requirement (Collision Resistant)

- Mallory should not be able to find two messages x and x’ which produce the same hash
- Given a hash function $h : \mathbb{X} \rightarrow \mathbb{Y}$ and an element $x \in \mathbb{X}$, find, $x, x' \in \mathbb{X}$ and $x \neq x'$ such that, $h(x) = h(x')$



There is no collision Free hash Function

Hash Function Requirement (No shortcuts)

- For a message m, the only way to compute its hash is to evaluate the function h(m)
- This should remain to irrespective of how many hashes we compute
  - Even if we have computed $h(m_1)$, $h(m_2)$, $h(m_3)$, ……., $h(m_{1000})$ There should not be a shortcut to compute $h(m_{1001})$
  - An example where this is not true :
    eg. Consider h(x) = ax mod n
    If $h(x_1)$ and $h(x_2)$ are known, then $h(x_1+x_2)$ can be calculated

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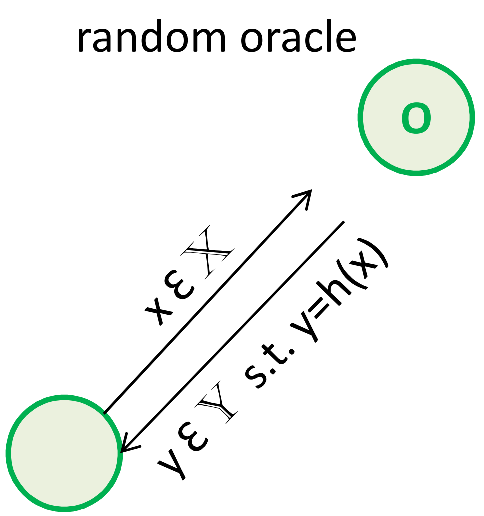
The Random Oracle Model

- The ideal hash function should be executed by applying h on the message x.
- The RO model was developed by Bellare and Rogaway for analysis of ideal hash functions



- Let $\mathbb{F}^{(\mathbb{X},\mathbb{Y})}$ be the set of all functions mapping $\mathbb{X}$ to $\mathbb{Y}$.
- The oracle picks a random function h from $\mathbb{F}^{(\mathbb{X},\mathbb{Y})}$. only the Oracle has the capability of executing the hash function.
- All other entities, can invoke the oracle with a message x Ɛ $\mathbb{X}$. The oracle will return y = h(x).

We do not know h. Thus the only way to compute h(x) is to query the oracle.

Independence Property

- Let h be a randomly chosen hash function from the set $\mathbb{F}^{(\mathbb{X},\mathbb{Y})}$
- If x_1 Ɛ $\mathbb{X}$ and a different x_2 Ɛ $\mathbb{X}$ then

$$Pr[h(x_1) = h(x_2)] = 1/M$$

where M = | $\mathbb{Y}$ |

this means, the hash digests occur with uniform probability

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Complexity of Problems in the RO model

- 3 problems : First pre-image, Second pre-image, Collision resistance
- We study the complexity of breaking these problems
  - Use **Las Vegas randomized algorithms**
    - A Las-Vegas algorithm may succeed or fail
    - If it succeeds, the answer returned is always correct
  - Worst case success probability
  - Average case success probability (e)
    - Probability that the algorithm returns success, averaged over all problem instances is at least e
  - **(e, Q) Las Vegas algorithm:**
    - Is an algorithm which can make Q queries and have an average success probability of e

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Las Vegas Algorithm Example

- Find a person who has a birthday today in at-most Q queries

```
BirthdayToday(){
        X = set of Q randomly chosen people
        for x in X{
            if (birthday(x) == today) return x
        }
        return FAILURE;
}
```

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Las Vegas Algorithm Example

- Find a person who has a birthday today in at-most Q queries

```
BirthdayToday(){
        X = set of Q randomly chosen people
        for x in X{
            if (birthday(x) == today) return x
        }
        return FAILURE;
}
```

- Let E be the event that a person has a birthday today

$$\Pr \textit{ that a person does not have a birthday today is } \left(1 - \frac{1}{365}\right)$$

$$\Pr[\textit{Success in } Q \textit{ trials}] = 1 - \Pr[\textit{Failure in } Q \textit{ tries}] = 1 - \left(1 - \frac{1}{365}\right)^Q$$

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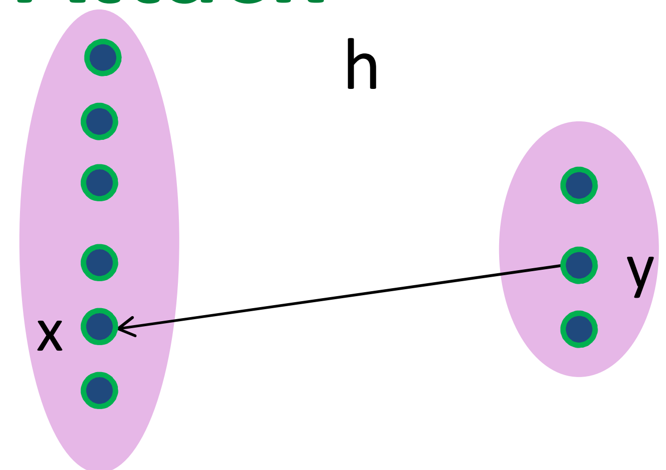
First Preimage Attack

Problem : Given a hash y, find an x such that h(x) = y

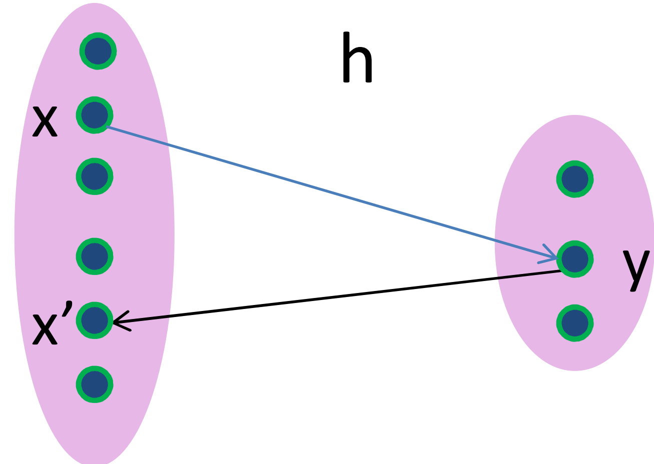


Ideal hash function queried using the RO access

```
First_PreImage_Attack(h, y, Q){
    choose Q distinct values from X (say x_1, x_2, …., x_Q)
    for(i=1; i<=Q; ++i){
        if (h(x_i) == y) return x_i
    }
    return FAIL
}
```

|Y| = M $\Pr[\, Success \;\; in\; Q\; trials \;\; on\; average \,] = 1 - \left(1 - \frac{1}{M}\right)^{Q}$

Second Preimage Attack



Problem : Given an x, find an x' ($\neq$x) such that h(x') = h(x)

Extra Oracle query

```
Second_PreImage_Attack(h, x, Q){
    choose Q-1 distinct values from X (say x_1, x_2, ...., x_{Q-1})
    y = h(x)
    for(i=1; i<=Q-1; ++i){
        if (h(x_i) == y) return x_i
    }
    return FAIL
}
```

$$\Pr[\,Success \;\; in\; Q\; trials \;\; on\; average\;\,] = 1 - \left(1 - \frac{1}{M}\right)^{Q-1}$$

Finding Collisions

```
Find_Collisions(h, Q){
    choose Q distinct values from 𝒳 (say x_1, x_2, ...., x_Q)
    for(i=1; i<=Q; ++i) y_i = h(x_i)
    if there exists (y_j == y_k) for j ≠k then return (x_j, x_k)
    return FAIL
}
```

$$Success\,\Pr obability\,(\varepsilon)\,is\,\varepsilon = 1 - \prod_{i=1}^{Q-1}\left(1 - \frac{i}{M}\right)$$

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Birthday Paradox

- Find the probability that at-least two people in a room have the same birthday

$$\textit{Event } A : \textit{atleast two people in the room have the same birthday}$$

$$\textit{Event } A' : \textit{no two people in the room have the same birthday}$$

$$\Pr[A] = 1 - \Pr[A']$$

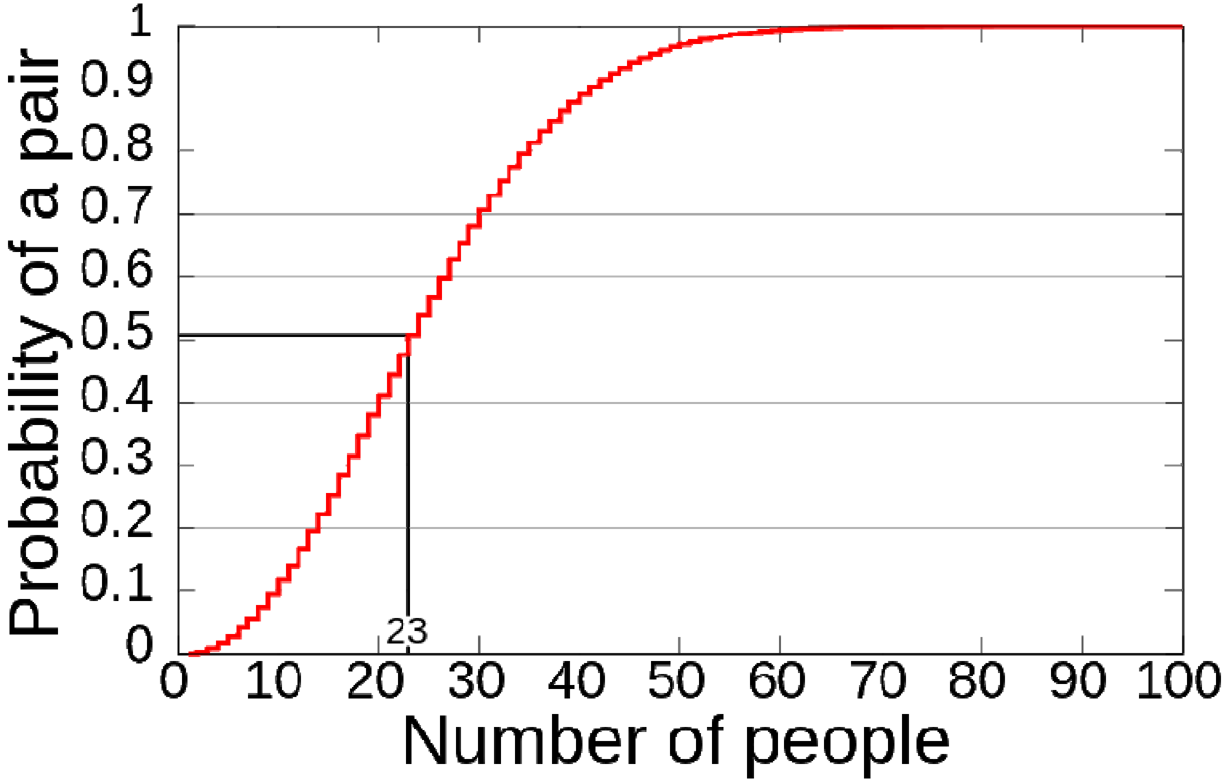
$$\Pr[A'] = 1 \times \left(1 - \frac{1}{365}\right) \times \left(1 - \frac{2}{365}\right) \times \left(1 - \frac{3}{365}\right) \cdots\cdots \left(1 - \frac{Q-1}{365}\right)$$

$$= \prod_{i=1}^{Q-1} \left(1 - \frac{i}{365}\right)$$

$$\Pr[A] = 1 - \prod_{i=1}^{Q-1} \left(1 - \frac{i}{365}\right)$$

Birthday Paradox

- If there are 23 people in a room, then the probability that two birthdays collide is 1/2



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Collisions in Birthdays to Collisions in Hash Functions

```
Find_Collisions(h, Q){
    choose Q distinct values from X (say x_1, x_2, …., x_Q)
    for(i=1; i<=Q; ++i) y_i = h(x_i)
    if there exists (y_j == y_k) for j ≠k then return (x_j, x_k)
    return FAIL
}
```

$$Success\,\Pr obability\,(\varepsilon)\,is\,\varepsilon = 1 - \prod_{i=1}^{Q-1}\left(1 - \frac{i}{M}\right)$$

|Y| = M

Relationship between Q, M, and success

$$Q \approx \sqrt{2M \ln \frac{1}{1-\varepsilon}}$$

$$If\ \varepsilon = 0.5\ then\ Q \approx 1.17\sqrt{M}$$

Q always proportional to square root of M.
Ɛ only affects the constant factor

Birthday Attacks and Message Digests

$$Q \approx 1.17\sqrt{M}$$

- If the size of a message digest is 40 bits
- $M = 2^{40}$
- A birthday attack would require 2^{20} queries

- Thus to achieve 128 bit security against collision attacks, hashes of length at-least 256 is required

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Comparing Security Criteria

- Finding collisions is easier than solving pre-image or second preimage
- Do reductions exist between the three problems?

collision resistance →second preimage

- We can reduce collision resistance to second preimage problem

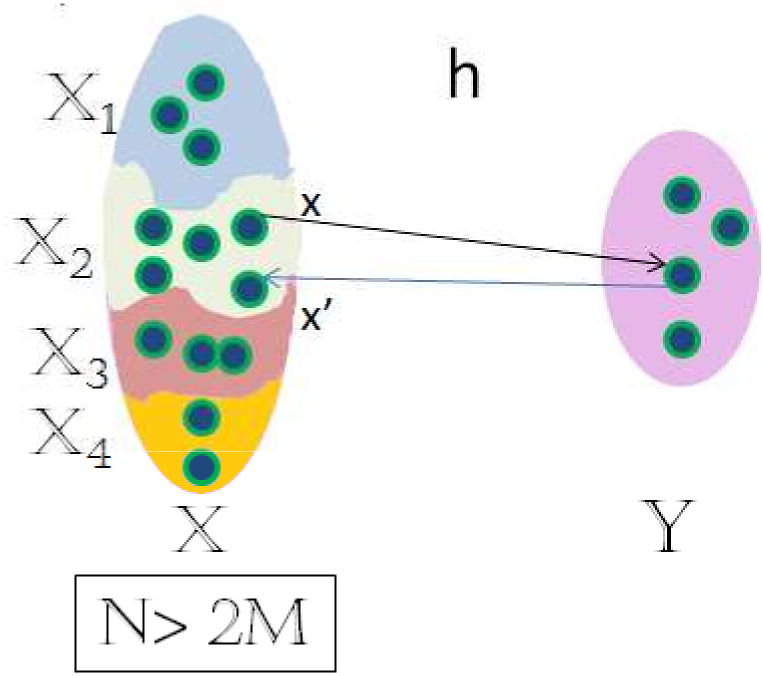
  collision resitance →2nd preimage

  – i.e. If we have an algorithm to attack the 2nd preimage problem, then we can solve the collision problem

```
findCollisions1(h,  Q){
    choose x randomly from X
    if(Second_PreImage_Attack(h, x, Q)  == x')
        return (x,x')
    else
        return FAIL
}
```

collision resistance → preimage

```
Find_Collisions2(h,  Q){
    choose x randomly from X
    y = h(x)
    x' = PreImage_Attack(h, y, Q-1)
    if (x ≠ x')
        return (x,x')
    else
        return FAIL
}
```



$$X = X_1 \cup X_2 \cup X_3 \cup X_4$$

X_i is an equivalence class. The number of such X_i formed is $|Y|$

Assume Preimage_Attack always finds the pre-image of y in Q-1 queries to the Oracle, **then, Find_Collisions2 is a (1/2, Q) Las Vegas algorithm**

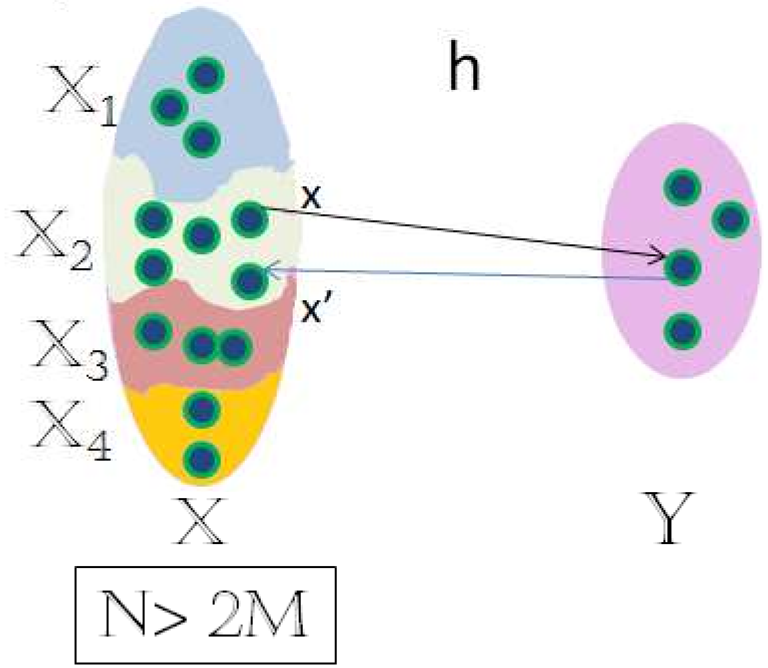
Proof

$y \in Y$ *partitions* X *as follows.*

$X_y = \{x \in X \mid s.t.\, h(x) = y\}$

Number of partitions of X *is* $|Y| = M$

$(assume\ X \le \dfrac{M}{2})$



$$\Pr[success] = \Pr[x \neq x'] = \frac{1}{N}\sum_{y}\sum_{X_y}\left(1 - \frac{1}{|X_y|}\right)$$

$$= \frac{1}{N}\sum_{y}|X_y|\left(1 - \frac{1}{|X_y|}\right)$$

$$= \frac{1}{N}\sum_{y}(|X_y| - 1) \quad = \frac{1}{N}(N - M)$$

$$\ge \left(\frac{N - N/2}{N}\right) \qquad (use\ N \ge 2M)$$

$$= \frac{1}{2}$$

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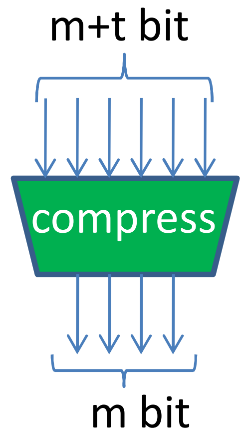
Iterated Hash Functions

- So far, we've looked at hash functions where the message was picked from a finite set $\mathbb{X}$
- What if the message is of an infinite size?
  - We use an iterated hash function
- The core in an iterated hash function is a function called compress
  - Compress, hashes from m+t bit to m bit

$$compress : \{0,1\}^{m+t} \rightarrow \{0,1\}^{m}$$
$$t \geq 1$$



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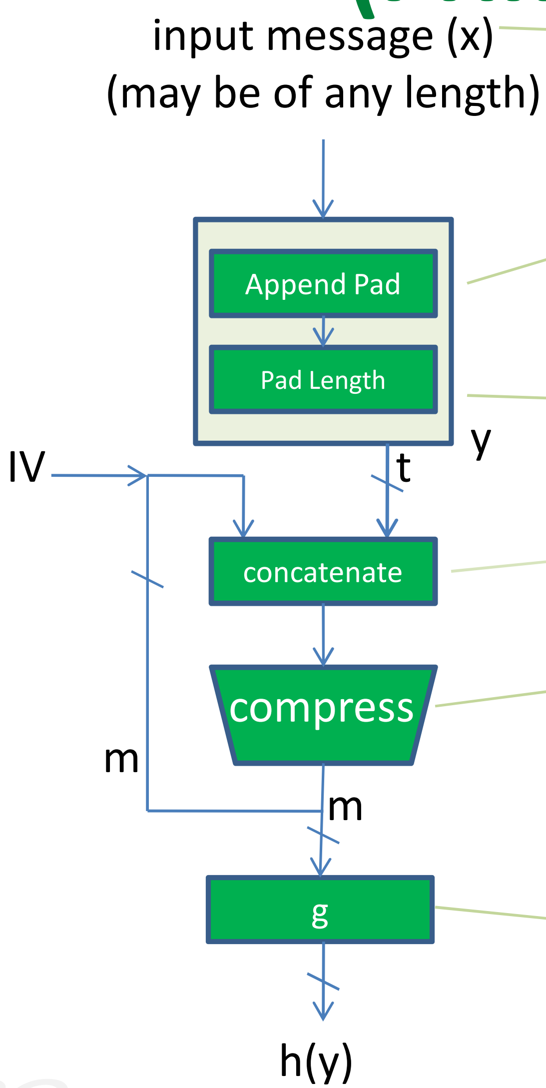
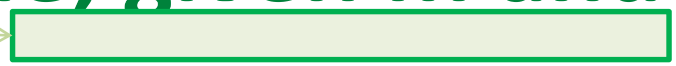
Iterated Hash Function (Principle, given m and t)





- must be at-least m+t+1 in length

- Input message is padded so that its length is a multiple of t

- Number of bits in the pad appended
- Concatinate previous m bit output with next t bit block (IV used only during initialization)
- The compress function is invoked iteratively for each t bit block in the message. For the first operation, an initialization vector is used
- After all t bit blocks are processed, there is a post processing step, and finally the hash is obtained. This step is optional.

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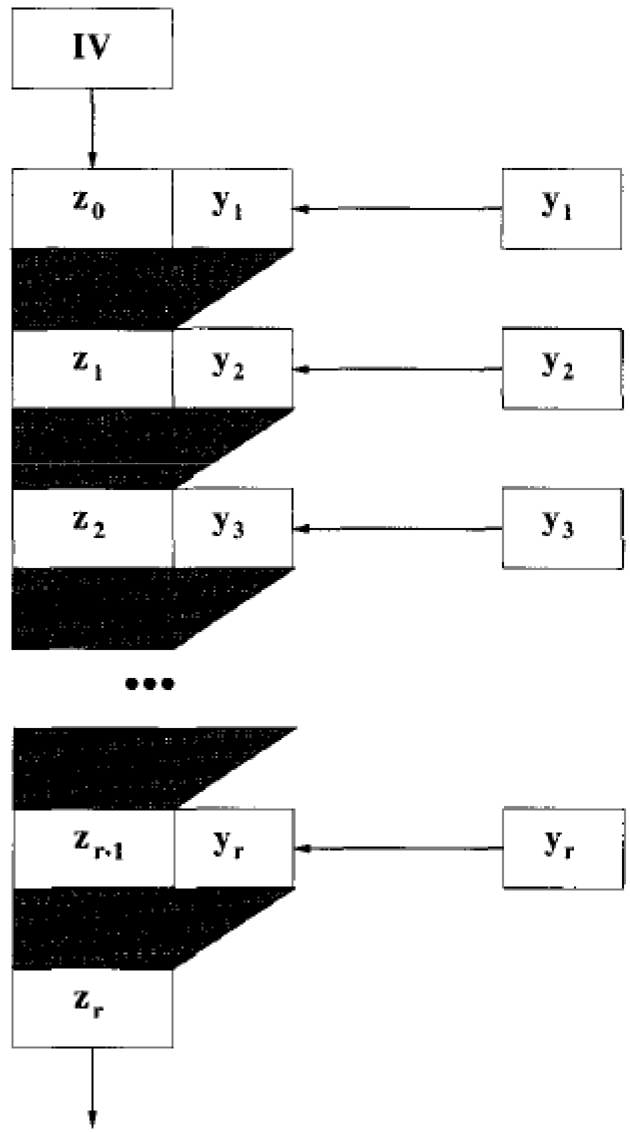
Iterated Hash Function (Principle)

- Another perspective



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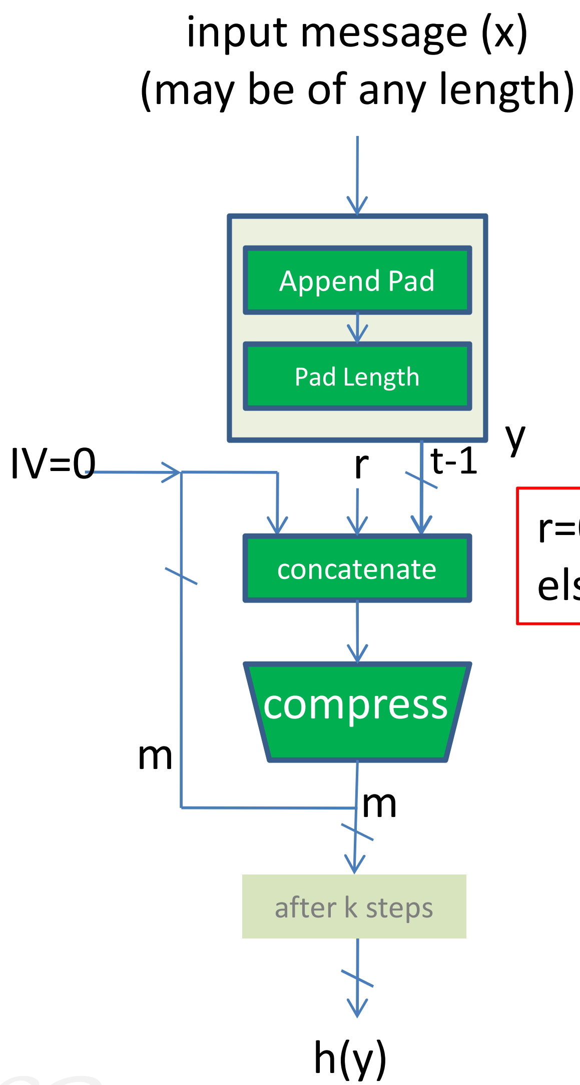
Merkle-Damgard Iterated Hash Function



$$h : \{0,1\}^{m+t} \rightarrow \{0,1\}^{m}$$

$$X = \bigcup_{i=m+t+1}^{\infty} \{0,1\}^{i}$$

r=0 for the first iteration
else r=1

Itrated hash function construction
That uses a compress function h

If h is collision resistant then the Merkle Damgard construction is collision resistant

Merkle-Damgard Iterated Hash Function

Algorithm : $\textsc{Merkle-Damgård}(x)$

external compress

comment: compress : $\{0,1\}^{m+t} \to \{0,1\}^m$, where $t \geq 2$

$n \leftarrow |x|$

$k \leftarrow \lceil n/(t-1) \rceil$

$d \leftarrow k(t-1) - n$

for $i \leftarrow 1$ **to** $k-1$

  do $y_i \leftarrow x_i$

$y_k \leftarrow x_k \,||\, 0^d$

$y_{k+1} \leftarrow$ the binary representation of d

$z_1 \leftarrow 0^{m+1} \,||\, y_1$

$g_1 \leftarrow \textbf{compress}(z_1)$

for $i \leftarrow 1$ **to** k

$$\textbf{do} \begin{cases} z_{i+1} \leftarrow g_i \,||\, 1 \,||\, y_{i+1} \\ g_{i+1} \leftarrow \textbf{compress}(z_{i+1}) \end{cases}$$

$h(x) \leftarrow g_{k+1}$

return $(h(x))$

Message length

k :Num of blocks of in x. Each block has length t-1
Note that t cannot be = 1

Amount of padding required to make message a multiple of t-1

Apply padding

Append d

IV is 0^m

On Merkle-Damgard Construction

Theorem: If the compress function is collision resistant then the Merkle-Damgard construction is collision resistant

Proof: We show the contra-positive…

If the Merkle-Damgard construction results in a collision then the compress function is NOT collision resistant

Merkle-Damgard Construction is Collision Resistant (Proof)

- Assume we have two message x and x’ which result in the same hash.
- Proof proceeds by considering 2 cases:

(1) $\boxed{|x| \neq |x'| \bmod (t-1)}$

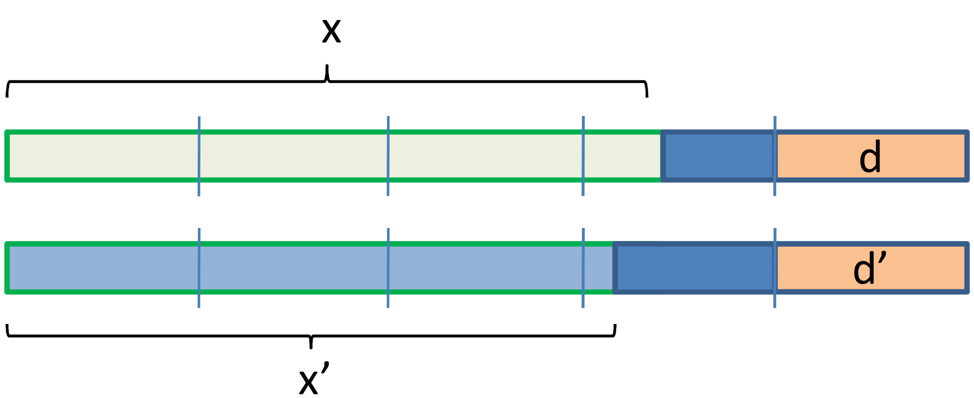
(2) $\boxed{|x| = |x'| \bmod (t-1)}$

(2a) $\boxed{|x| = |x'|}$

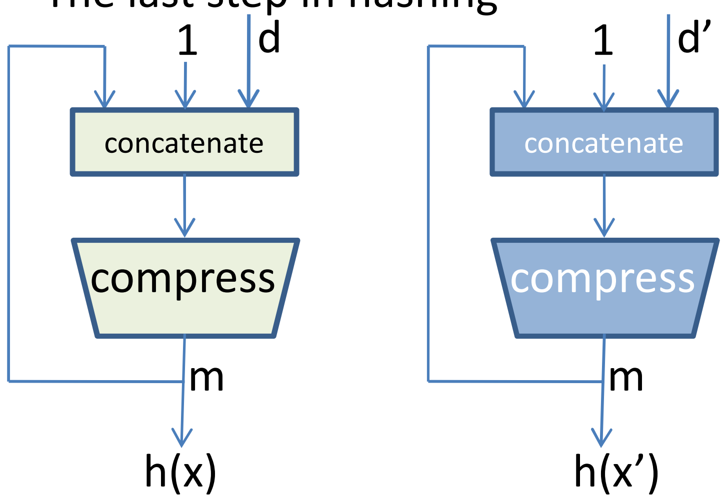
(2b) $\boxed{|x| \neq |x'|}$

Case 1 $|x| \neq |x'| \bmod (t-1)$

- This means that the padding (resp. d and d') applied to x and x' is different (i.e. d ≠ d')



The last step in hashing



If h(x) = h(x') then
compress(xx||1||d) = compress(xx||1||d')

Since d≠ d', we have a collision in compress.

Case 1 formally : $|x| \neq |x'| \bmod (t-1)$

case 1: $|x| \not\equiv |x'| \pmod{t-1}$.

Here $d \neq d'$ and $y_{k+1} \neq y'_{\ell+1}$. We have

$$\begin{aligned}
\mathbf{compress}(g_k \parallel 1 \parallel y_{k+1}) &= g_{k+1} \\
&= h(x) \\
&= h(x') \\
&= g'_{\ell+1} \\
&= \mathbf{compress}(g'_\ell \parallel 1 \parallel y'_{\ell+1}),
\end{aligned}$$

which is a collision for **compress** because $y_{k+1} \neq y'_{\ell+1}$.

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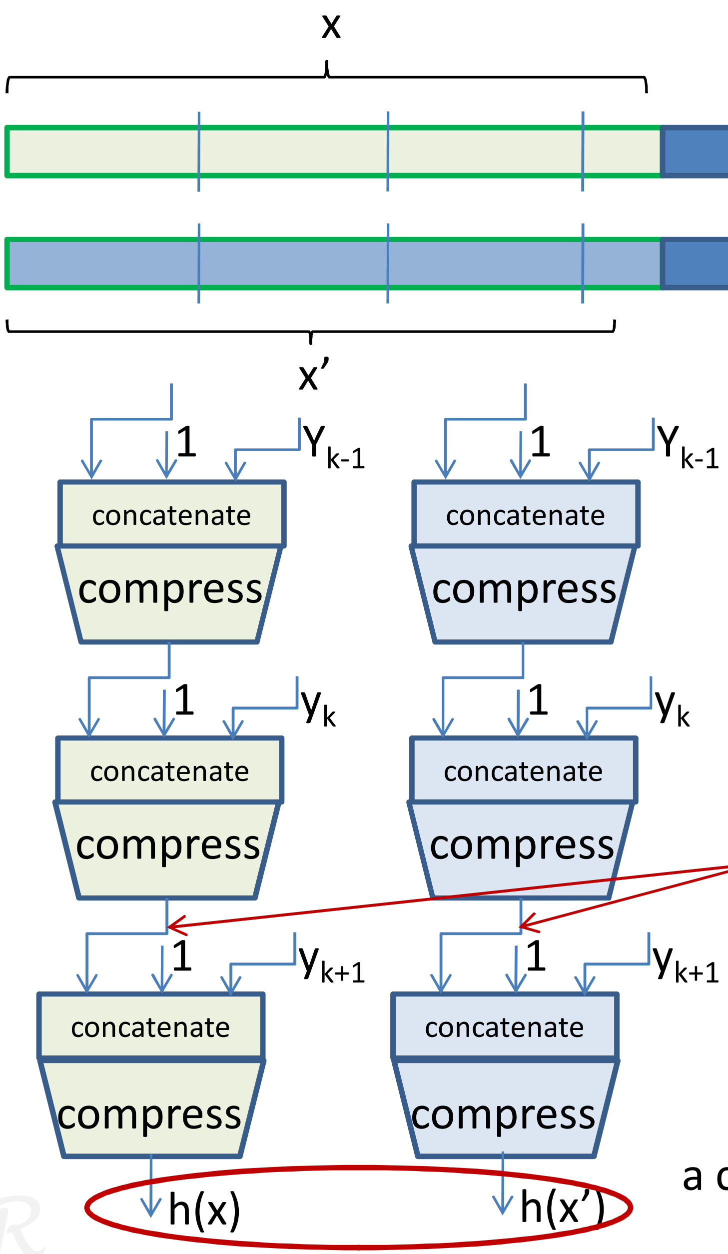
Case 2a : $|x| = |x'| \bmod (t-1)$ *and* $|x| = |x'|$

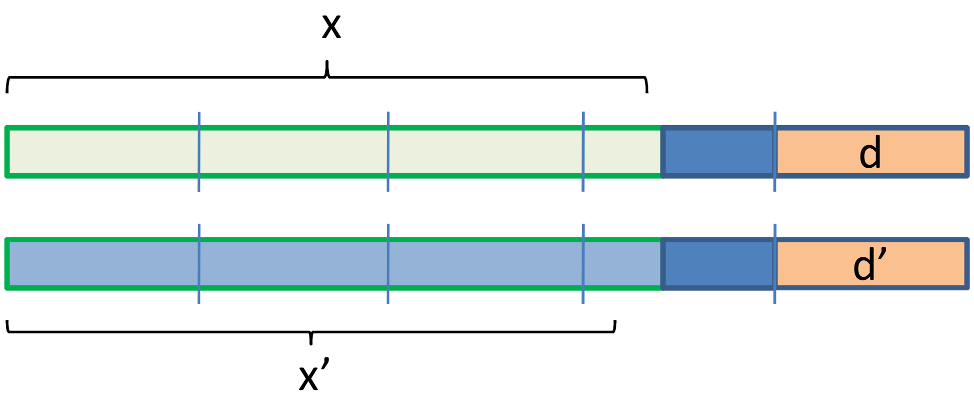


In this case, padding in x and x’ are the same. Hence d = d’.
… can’t use the old trick ☹

These may or may not collide.
If they collide, we are done : we have shown a collision in compress. If they don’t collide we look at the previous iteration

a collision here

Case 2a : $|x| = |x'| \bmod (t-1)$ and $|x| = |x'|$



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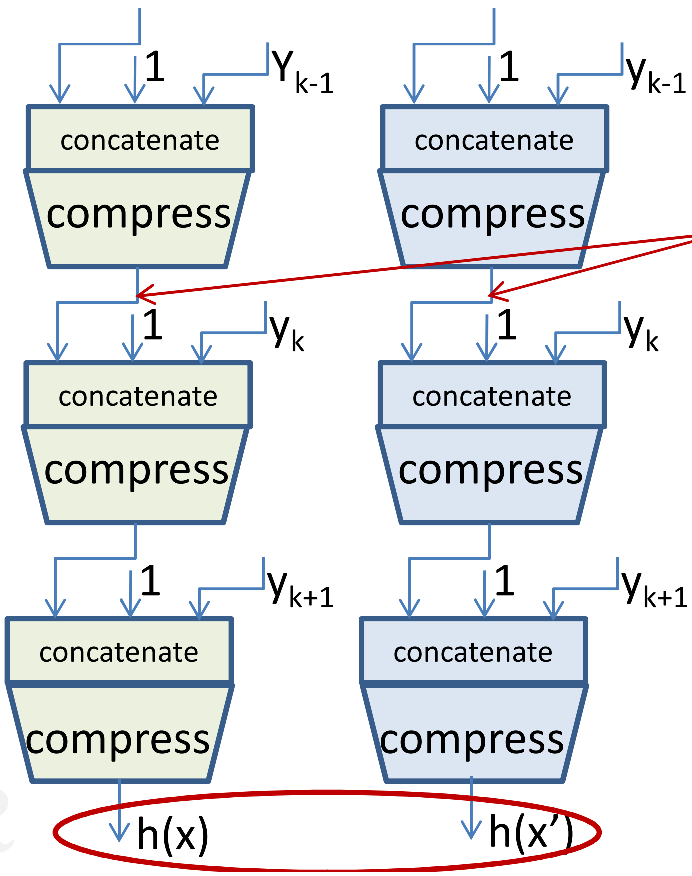

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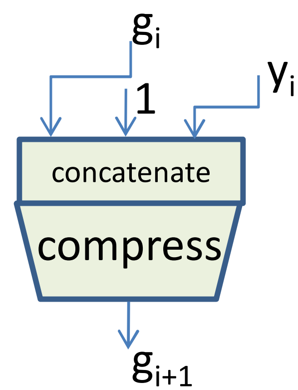
If they collide, we are done :
We have shown a collision in compress.
If they don’t collide we look at the previous iteration

We continue this back tracking, until we find a collision. We will definitely find a collision at some point because x ≠ x’.

Case 2a formally : $|x| = |x'| \bmod (t-1)$ *and* $|x| \neq |x'|$

Here we have $k = \ell$ and $y_{k+1} = y'_{k+1}$. We begin as in case 1:



$$\begin{aligned}
\mathbf{compress}(g_k \parallel 1 \parallel y_{k+1}) &= g_{k+1} \\
&= h(x) \\
&= h(x') \\
&= g'_{k+1} \\
&= \mathbf{compress}(g'_k \parallel 1 \parallel y'_{k+1}).
\end{aligned}$$

If $g_k \neq g'_k$, then we find a collision for **compress**, so assume $g_k = g'_k$. Then we have

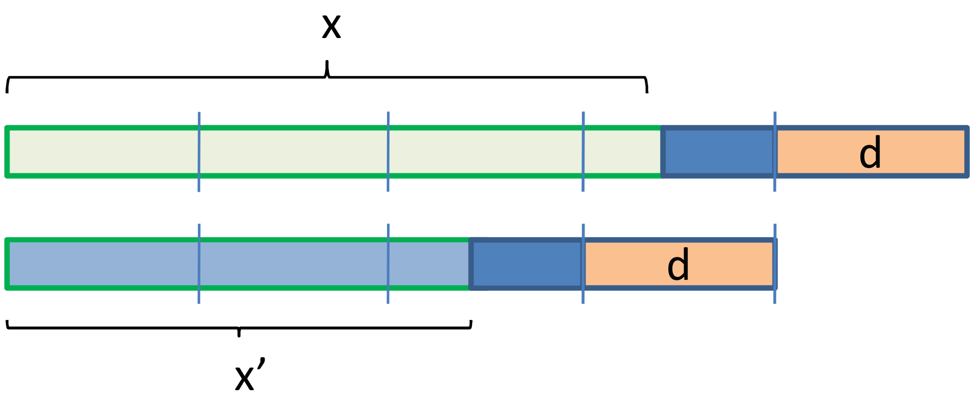
$$\begin{aligned}
\mathbf{compress}(g_{k-1} \parallel 1 \parallel y_k) &= g_k \\
&= g'_k \\
&= \mathbf{compress}(g'_{k-1} \parallel 1 \parallel y'_k).
\end{aligned}$$

Either we find a collision for **compress**, or $g_{k-1} = g'_{k-1}$ and $y_k = y'_k$. Assuming we do not find a collision, we continue working backwards, until finally we obtain

$$\begin{aligned}
\mathbf{compress}(0^{m+1} \parallel y_1) &= g_1 \\
&= g'_1 \\
&= \mathbf{compress}(0^{m+1} \parallel y'_1).
\end{aligned}$$

but $y_1 = y_1'$ implies x=x'.
which is a contradiction.

Case 2b : $|x| = |x'| \bmod (t-1) \quad and \quad |x| \neq |x'|$

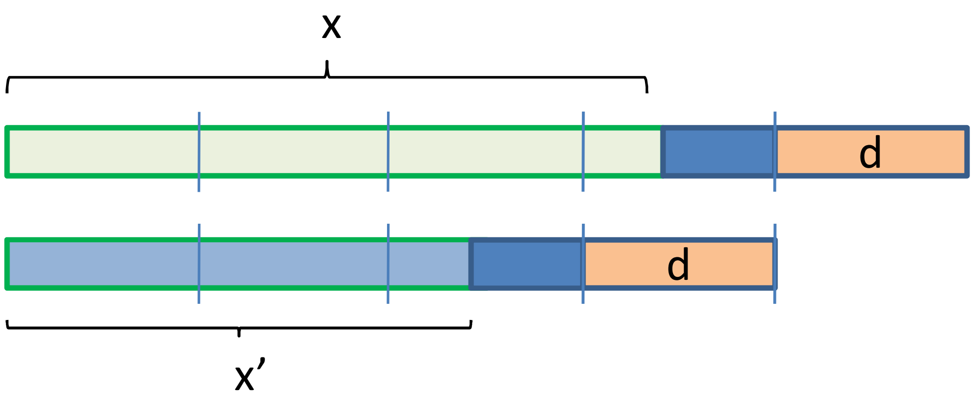


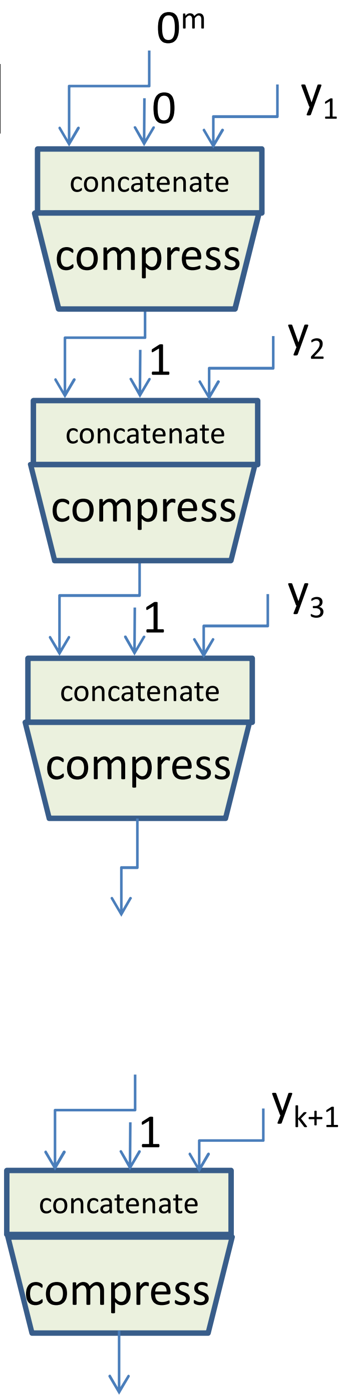
Note here that d=d' even though lengths of the messages are not the same.

In most cases, the proof would proceed similar to case 2a.

But there is a cornercase.

Case 2b : $|x| = |x'| \bmod (t-1) \quad and \quad |x| \neq |x'|$





- The corner case: x = (x''|x')

  back tracking in such as case will not help find a collision

- Handling this case:

  the inserted bit r

  (r=0 for the 1^{st} round, else r=1)

Case 2b formally : $|x| = |x'| \bmod (t-1)$ *and* $|x| \neq |x'|$

case 2b: $|x| \neq |x'|$.

Without loss of generality, assume $|x'| > |x|$, so $\ell > k$. This case proceeds in a similar fashion as case 2a. Assuming we find no collisions for **compress**, we eventually reach the situation where

$$\begin{aligned}\mathbf{compress}(0^{m+1} \parallel y_1) &= g_1 \\ &= g'_{\ell-k+1} \\ &= \mathbf{compress}(g'_{\ell-k} \parallel 1 \parallel y'_{\ell-k+1}).\end{aligned}$$

But the $(m+1)$st bit of

$$0^{m+1} \parallel y_1$$

is a 0 and the $(m+1)$st bit of

$$g'_{\ell-k} \parallel 1 \parallel y'_{\ell-k+1}$$

is a 1. So we find a collision for **compress**.

Merkle-Damgard-2 (for the case when t=1)

Algorithm : MERKLE-DAMGÅRD2(x)

external compress
comment: compress : $\{0,1\}^{m+1} \to \{0,1\}^m$

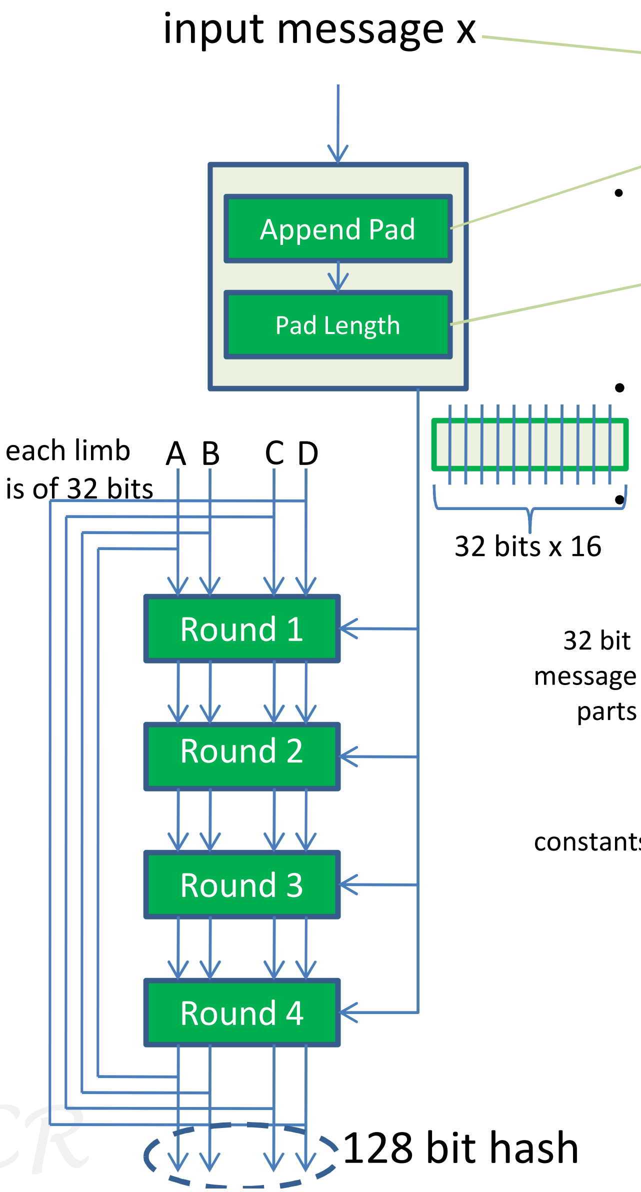
$n \leftarrow |x|$
$y \leftarrow 11 \,\|\, f(x_1) \,\|\, f(x_2) \,\|\, \cdots \,\|\, f(x_n)$
denote $y = y_1 \,\|\, y_2 \,\|\, \cdots \,\|\, y_k$, where $y_i \in \{0,1\}, 1 \leq i \leq k$
$g_1 \leftarrow$ **compress**$(0^m \,\|\, y_1)$
for $i \leftarrow 1$ **to** $k-1$
  do $g_{i+1} \leftarrow$ **compress**$(g_i \,\|\, y_{i+1})$
return (g_k)

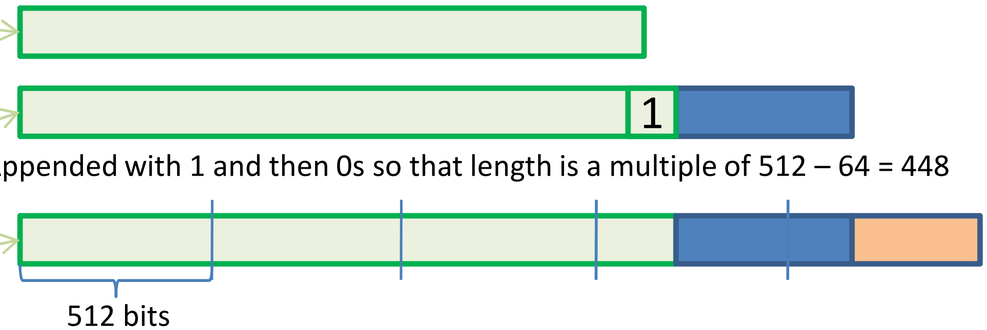
CR

Hash Functions in Practice

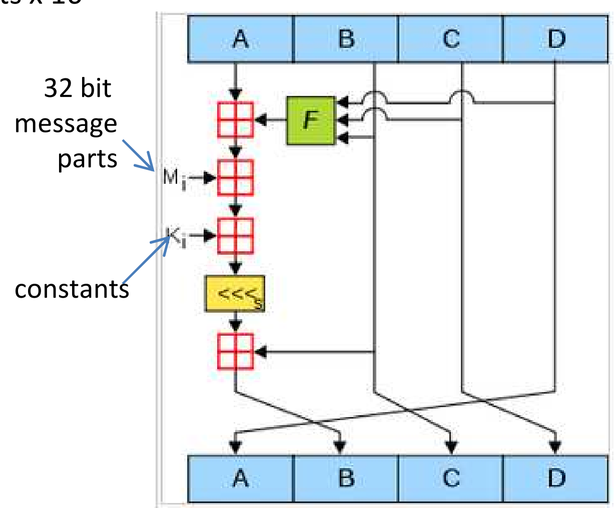
- MD5
- NIST specified "secure hash algorithm"
  - SHA0 : published in 1993. 160 bit hash.
    - There were unpublished weaknesses in this algorithm
    - The first published weakness was in 1998, where a collision attack was discovered with complexity 2^{61}
  - SHA1 : published in 1995. 160 bit hash.
    - SHA0 replaced with SHA1 which resolved several of the weaknesses
    - SHA1 used in several applications until 2005, when an algorithm to find collisions with a complexity of 2^{69} was developed
    - In 2010, SHA1 was no longer supported. All applications that used SHA1 needed to be migrated to SHA2
  - SHA2 : published in 2001. Supports 6 functions: 224, 256, 384, 512, and two truncated versions of 512 bit hashes
    - No collision attacks on SHA2 as yet. The best attack so far assumes reduced rounds of the algorithm (46 rounds)
  - SHA3 : published in 2015. Also known as Kecchak

MD5





- Appended with 1 and then 0s so that length is a multiple of 512 – 64 = 448
- Message length appended (in 64 bits) and split into blocks of 512 bits
- Each round has 16 similar operations of this **modified Feistel form**



round operations

round 1 $F(B,C,D) = (B \wedge C) \vee (\neg B \wedge D)$

round 2 $G(B,C,D) = (B \wedge D) \vee (C \wedge \neg D)$

round 3 $H(B,C,D) = B \oplus C \oplus D$

round 4 $I(B,C,D) = C \oplus (B \vee \neg D)$

Collisions in MD5 (Timeline)

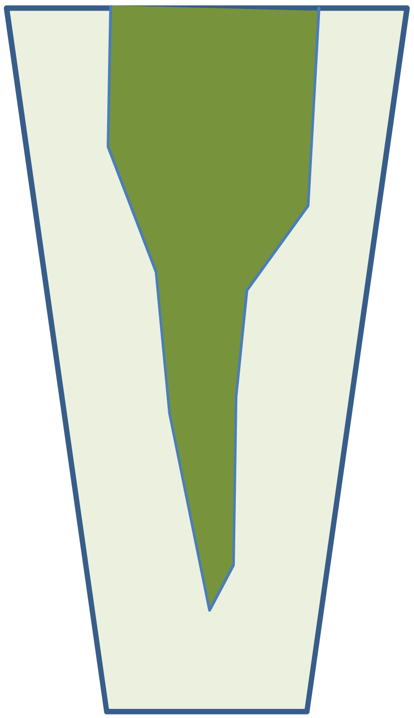
- A birthday attack on MD5 has complexity of 2^{64}
- Small enough to brute force collision search
- 1996, collisions on the inner functions of MD5 found
- 2004, collisions demonstrated practically
- 2007, chosen-prefix collisions demonstrated

> Given two different prefixes p1, p2 find two appendages m1 and m2 such that hash(p1 || m1) = hash(p2 || m2)

- 2008, rogue SSL certificates generated
- 2012, MD5 collisions used in cyberwarfare
  - **Flame malware** uses an MD5 prefix collision to fake a Microsoft digital code signature

MD5 Collisions demos : http://www.mscs.dal.ca/~selinger/md5collision/

Collision attack on MD5 like hash functions



- Analyze differential trails
- A bit different from block ciphers
  - No secret key involved
  - We can choose M and M* as we want
- We have a valid attack if probability of trail is $P > 2^{-N/2}$

SHA1

input message (x)
(may be of any length less than 2^{64})

Algorithm : SHA-1-PAD(x)

comment: $|x| \leq 2^{64} - 1$

$d \leftarrow (447 - |x|) \bmod 512$
$\ell \leftarrow$ the binary representation of $|x|$, where $|\ell| = 64$
$y \leftarrow x \,||\, 1 \,||\, 0^d \,||\, \ell$

global $K_0, \ldots, K_{79}$
$y \leftarrow$ SHA-1-PAD(x)
denote $y = M_1 \,||\, M_2 \,||\, \cdots \,||\, M_n$, where each M_i is a 512-bit block

IV
$H_0 \leftarrow$ 67452301
$H_1 \leftarrow$ EFCDAB89
$H_2 \leftarrow$ 98BADCFE
$H_3 \leftarrow$ 10325476
$H_4 \leftarrow$ C3D2E1F0

for $i \leftarrow 1$ **to** n
do
- denote $M_i = W_0 \,||\, W_1 \,||\, \cdots \,||\, W_{15}$, where each W_i is a word
- **for** $t \leftarrow 16$ **to** 79
  - **do** $W_t \leftarrow \mathbf{ROTL}^1(W_{t-3} \oplus W_{t-8} \oplus W_{t-14} \oplus W_{t-16})$
- $A \leftarrow H_0$
- $B \leftarrow H_1$
- $C \leftarrow H_2$
- $D \leftarrow H_3$
- $E \leftarrow H_4$
- **for** $t \leftarrow 0$ **to** 79
  do
  - $temp \leftarrow \mathbf{ROTL}^5(A) + \mathrm{f}_t(B, C, D) + E + W_t + K_t$
  - $E \leftarrow D$
  - $D \leftarrow C$
  - $C \leftarrow \mathbf{ROTL}^{30}(B)$
  - $B \leftarrow A$
  - $A \leftarrow temp$
- $H_0 \leftarrow H_0 + A$
- $H_1 \leftarrow H_1 + B$
- $H_2 \leftarrow H_2 + C$
- $H_3 \leftarrow H_3 + D$
- $H_4 \leftarrow H_4 + E$

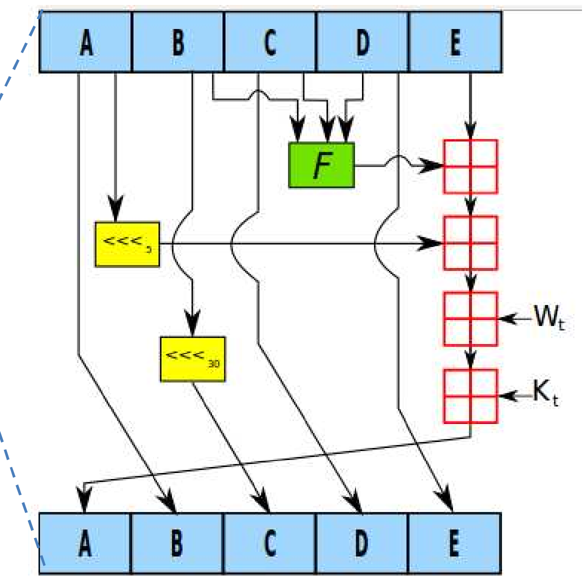
return $(H_0 \,||\, H_1 \,||\, H_2 \,||\, H_3 \,||\, H_4)$

32*5=160 bit hash output

each word is 32 bits (512/16=32)

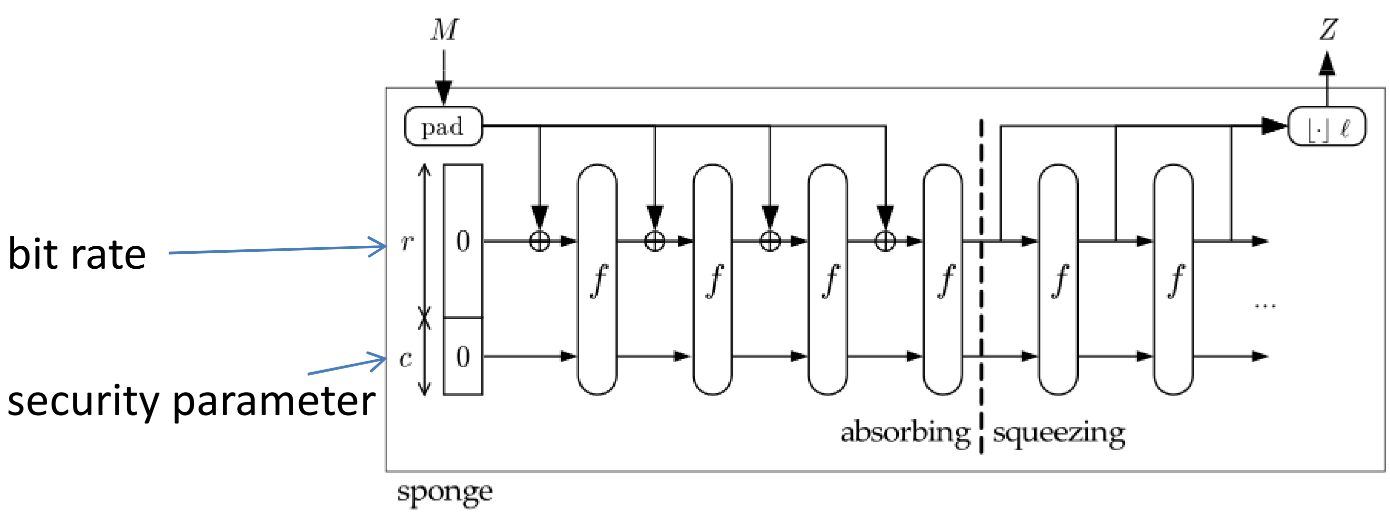
expand to 79 words

$$\mathrm{f}_t(B, C, D) = \begin{cases} (B \wedge C) \vee ((\neg B) \wedge D) & \text{if } 0 \leq t \leq 19 \\ B \oplus C \oplus D & \text{if } 20 \leq t \leq 39 \\ (B \wedge C) \vee (B \wedge D) \vee (C \wedge D) & \text{if } 40 \leq t \leq 59 \\ B \oplus C \oplus D & \text{if } 60 \leq t \leq 79. \end{cases}$$



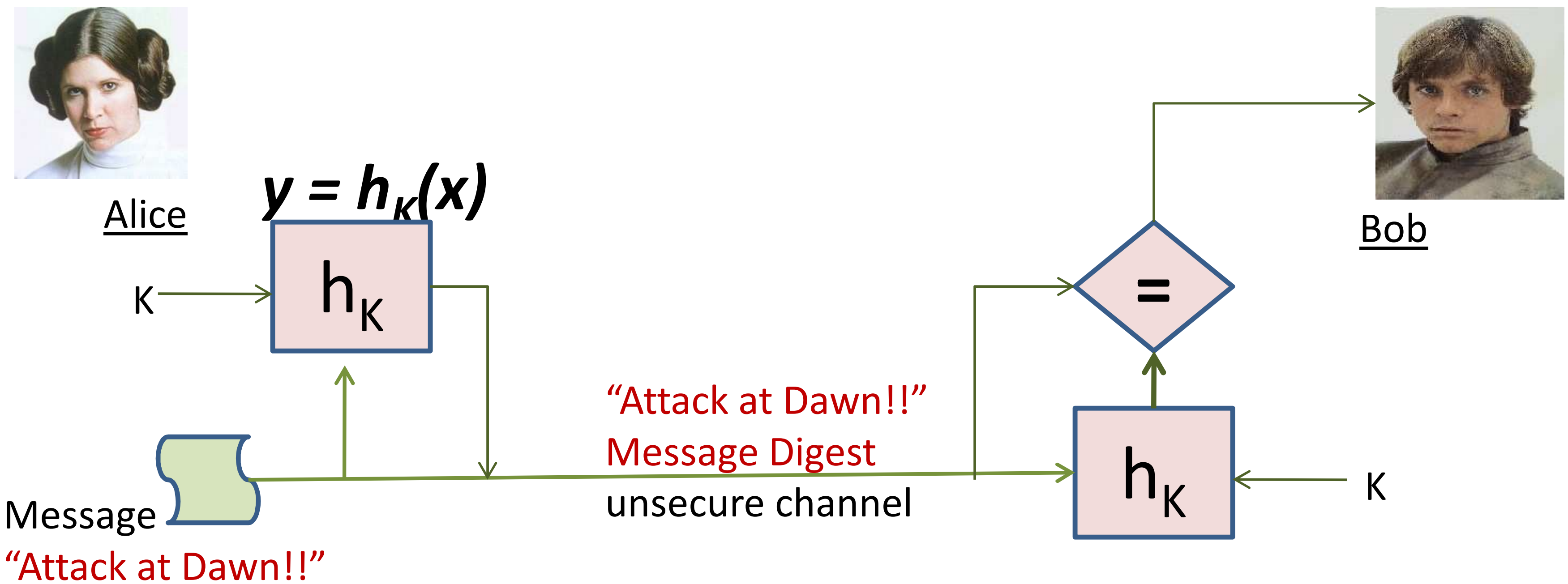
Kacchak and the SHA3

- Uses a sponge construction
  - Achieves variable length hash functions



Success of an attack against Kecchak < $N^2/2^{c+1}$
where N is number of calls to f

Message Authentication Codes (Keyed Hash Functions)



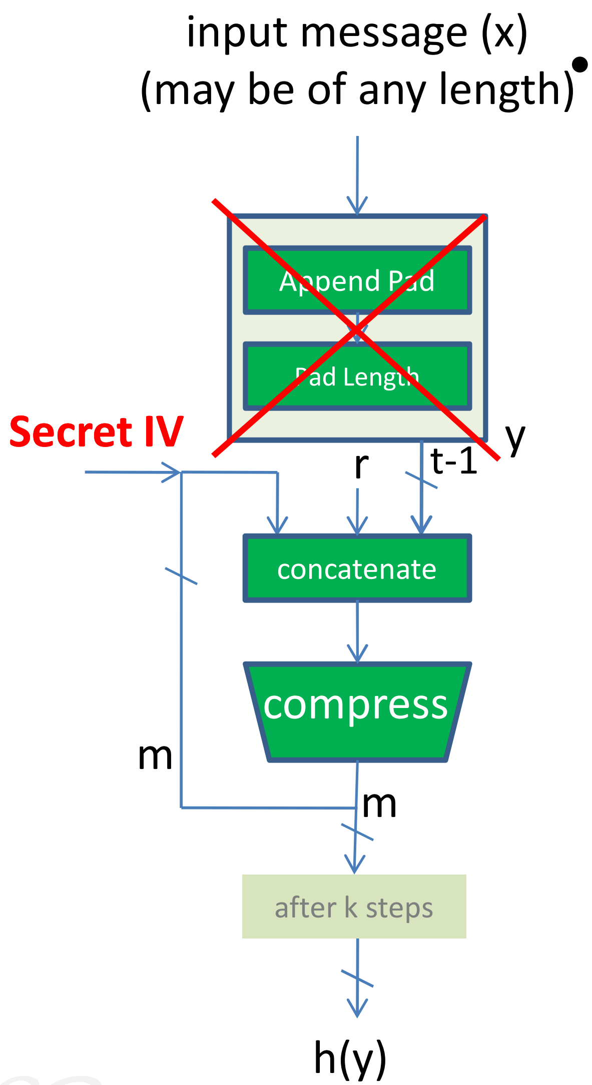
Provides Integrity and Authenticity
Integrity : Messages are not tampered
Authenticity : Bob can verify that the message came from Alice
(Does not provide non-repudiation)

How to construct MACs? recall … shortcuts

- For a message m, the only way to compute its hash is to evaluate the function $h_K(m)$
- This should remain to irrespective of how many hashes we compute
  - Even if we have computed $h_K(m_1)$, $h_K(m_2)$, $h_K(m_3)$, ……., $h_K(m_{1000})$
    It should be difficult to compute $h_K(x)$ without knowing the value of K

CR

Constructing a MAC
(First Attempt)

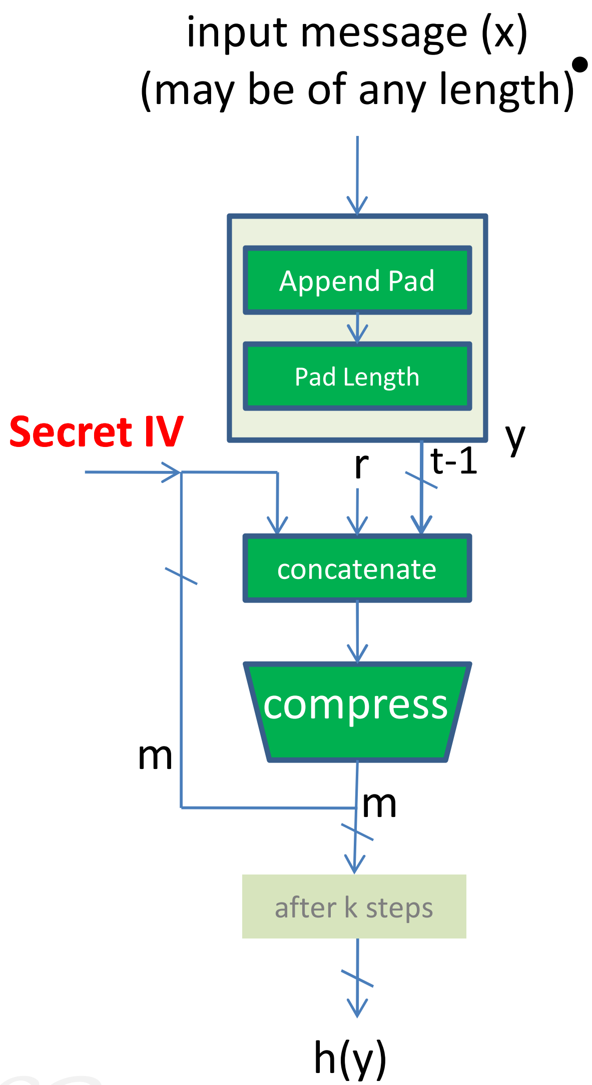


- Won’t work if no preprocessing step
  - attackers could append messages and get the same hash

    $x \rightarrow h_K(x)$,

    $x \,||\, x' \rightarrow \text{compress}(h_K(x) \,||\, x')$

Constructing a MAC (First Attempt)



- Won't work if preprocessing step present

$$\text{suppose } y = x \,\|\, pad(x) \qquad \text{where} \; |y| = rt$$

$$\text{consider } x' = x \,\|\, pad(x) \,\|\, w \qquad \text{where} \; |w| = t$$

$$y' = x' \,\|\, pad(x') = x \,\|\, pad(x) \,\|\, w \,\|\, pad(x')$$

$$\text{where} \; |y'| = r't \quad \text{for some integer } r' > r$$

$$\text{Let } z_r = h_K(x)$$

$$z_{r+1} \leftarrow compress(h_K(x) \,\|\, y_{r+1})$$

$$z_{r+2} \leftarrow compress(z_{r+1} \,\|\, y_{r+2})$$

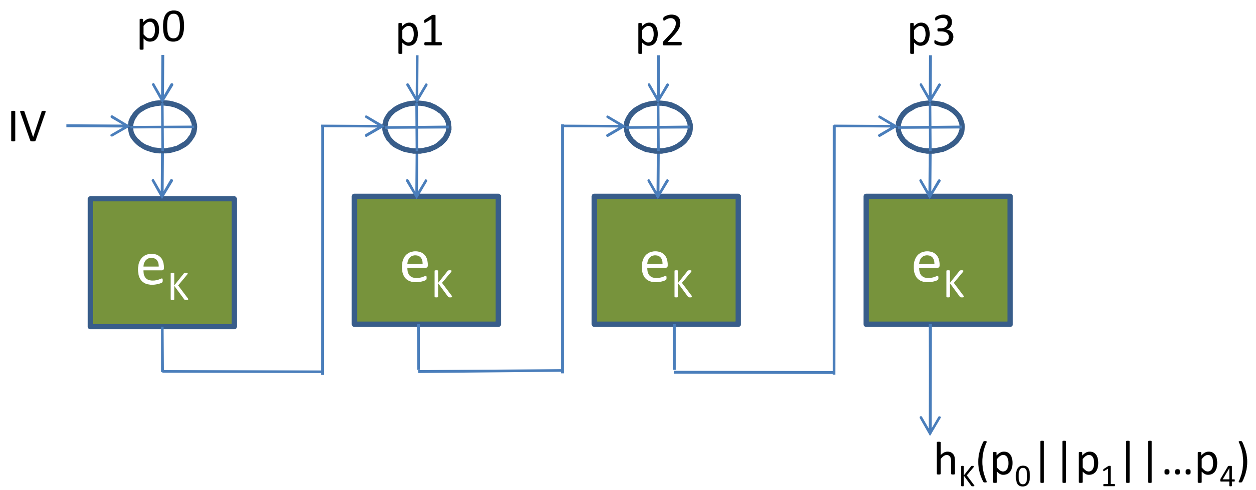
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$z_{r'} \leftarrow compress(z_{r'-1} \,\|\, y_{r'})$$

$$thus \;\; h_K(x') = z_{r'}$$

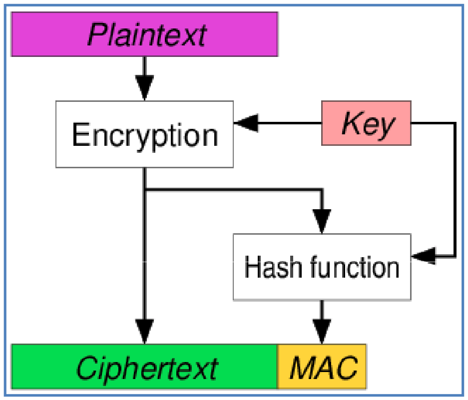
CR

CBC-MAC

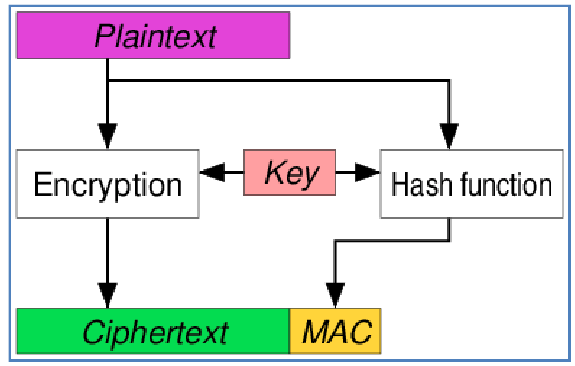


Authenticated Encryption

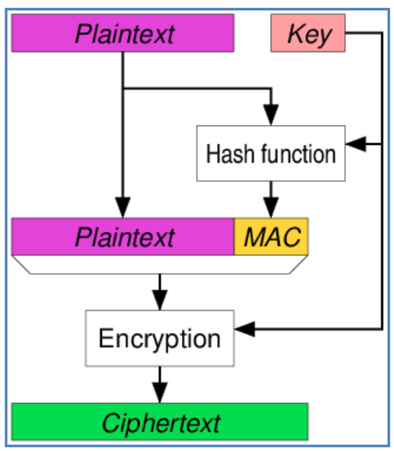
- Achieves Confidentiality, Integrity, and Authentication



EtM
(encrypt then MAC)



E&M



MtE
(MAC then Encrypt)

CR

Using CBC-MAC for Authenticated Encryption

1. Consider $p = (p_0, p_1, p_2, p_3)$ is a message Alice sends to Bob
   1. She encrypts it with CBC as follows

      $c_0 = E_k(p_0)$; $c_1 = E_k(p_1 + c_0)$; $c_2 = E_k(p_2 + c_1)$; $c_3 = E_k(p_3 + c_2)$
   2. She computes **mac** = CBC-MAC$_k$(p)

      She transmits (**c**, **mac**) to Bob : where **c** = (c_0, c_1, c_2, c_3)
2. Mallory modifies one or more of the ciphertexts (c_0, c_1, c_2) to (c_0', c_1', c_2')
3. Bob will
   1. Decrypt (c_0', c_1', c_2') to (p_0', p_1', p_2')
   2. And use it compute the MAC **mac'**

We show that **mac' = c_3** irrespective of how Mallory modifies the ciphertext

CR

Using CBC-MAC for Authenticated Encryption

Alice's side (encryption)	Bob's side (decryption)	
$c_0 = E_k(p_0)$	$p'_0 = D_k(c'_0)$	$(assume\ IV = 0)$
$c_1 = E_k(p_1 \oplus c_0)$	$p'_1 = D_k(c'_1) \oplus c'_0$	
$c_2 = E_k(p_2 \oplus c_1)$	$p'_2 = D_k(c'_2) \oplus c'_1$	
$c_3 = E_k(p_3 \oplus c_2)$	$p'_3 = D_k(c_3) \oplus c'_2$	

$$
\begin{aligned}
mac' &= CBCMAC(p') \\
&= E_k(p'_3 \oplus E_k(p'_2 \oplus E_k(p'_1 \oplus E_k(p'_0)))) \\
&= E_k(p_3 \oplus c'_2) \\
&= E_k(D_k(c_3) \oplus c'_2 \oplus c'_2) \\
&= E_k(D_k(c_3)) \\
&= c_3
\end{aligned}
$$

Without modifying the final ciphertext, Mallory can change any other ciphertext as she pleases. The CBC-MAC will not be altered.

Moral of the story: Never use CBC-MAC with CBC encryption!!

Counter Mode + CBC-MAC for Authenticated Encryption

Consider $p = (p_0, p_1, p_2, p_3)$ is a message Alice sends to Bob

1. She encrypts p with counter mode as follows
   $c_0 = p_0 + E_k(ctr)$; $c_1 = p_1 + E_k(ctr + 1)$;
   $c_2 = p_2 + E_k(ctr + 2)$; $c_3 = p_3 + E_k(ctr + 3)$

2. She computes **mac** = $\text{CBC-MAC}_k(p)$
   She transmits (**c**, **mac**) to Bob : where **c** = (c_0, c_1, c_2, c_3)