IITM-CS2200 : Languages, Machines and Computation — Date : Jan 18, 2012

Tutorial #1

You can attempt the questions in order. Q1-Q3 are directly based on what we have already done in the class. The last two questions are set to let you think about some interesting challenges in design. These will motivate the ideas in the next 2-3 lectures.

1. Construct Finite Automata accepting each of the following languages over the alphabet $\{0, 1\}$. Write the transition diagram as well as transition table for the first part. Formally prove that your construction is correct.
   (a) The set of all strings ending with 00.
   (b) The set of all strings for which the number of zeros is even and the number of 1s is a multiple of 3.
   (c) The set of all strings not containing 110.

2. Fix $k \geq 1$ and $p \geq 2$,

$$A_{k,p} = \{x \in \{0, 1, \ldots, p-1\}^* : x \text{ is a } p\text{-ary representation of a multiple of } k.\}$$

   We designed an automaton for $A_{3,2}$ and proved it to be correct. Generalize the construction and the proof to arbitrary k and p (but fixed numbers).

3. If A and B are regular, is the concatenated language $AB = \{xy : x \in A, y \in B\}$ regular?

4. For a language $A \subseteq \Sigma^*$, let us define $rev(A)$ as follows :
   For strings the definition is inductive. $rev(\epsilon) = \epsilon$, and $\forall x \in \Sigma^*, a \in \Sigma$, define $rev(xa) = a.rev(x)$. Now for a language A, $rev(A) = \{rev(x) : x \in L\}$.

   If A is regular, attempt to construct an automaton for $rev(A)$.

5. Attempt to construct finite automata for the following languages over $\Sigma = \{0, 1\}$.
   (a) $\{x \in \Sigma^* : |x| \text{ is divisible by } 4\}$
   (b) $\{x \in \Sigma^* : |x| \text{ is a perfect square}\}$
   (c) $\{x \in \Sigma^* : |x| \text{ is a prime number}\}$