Amplifying lower bounds by means of self-reducibility

Eric Allender, Michal Koucky

Presented by,
Balagopal

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Contents

- Lower bounds problem
- Amplifying lower bounds
- Self-reducibility
- Some simple examples
- Self-reducubility for BFE
- Self-reducibility for MAJ

Lower bounds problem

- To separate circuit complexity classes, we need to prove *superpolynomial* size lower bounds.
- Best known lower bounds are linear.
- Can we prove super-linear lower bounds?

Amplifying lower bounds

- Certain problems have a structure which allows *amplifying* super-linear lower bounds to prove super-polynomial lower bounds.
- Note that the amplification depends on the problem.

Self-reducibility

- Solve L_n using constant depth circuits using AND_2, OR_2, NOT and oracle gates for L_{n^ϵ}
- These reductions are called pure $\leq_T^{NC^0}$ downward self-reductions
- The work is done mainly by the oracle gates

Some simple examples

- AND_n using $AND_{\sqrt{n}}$
- The product over a finite monoid, $(W_M)_n$ using $(W_M)_{\sqrt{n}}$
- How can we use this to amplify lower bounds?

Self-reducibility of BFE

- *BFE* - Given a boolean formula and the values of variables, check whether it evaluates to 1.
- The formula is well-balanced. That is, it can be represented as a complete binary tree.
- *BFE* is NC^1-complete.

Self-reducibility of MAJ

- MAJ_n - TC^0 complete
- Can be computed by NC^0 circuits with $MAJ_{\sqrt{n}}$ oracle gates
- **Idea:** Compute the sum of n 1-bit integers and compare with $n/2$
- **Key lemma:** Given m, ℓ-bit integers transform to ℓ, $\ell + \log(m+1)$ bit integers such that $y_1 + \ldots y_m = z_1 + \ldots z_\ell$ using $O(\ell m)$ MAJ_{2m} gates.

Known results

- BFE requires $n^{1+\epsilon_d}$ size on depth d TC^0 circuits
- If the dependence on depth is eliminated, we have shown $TC^0 \neq NC^1$

Thank you