

# CS6848 - Principles of Programming Languages

## Flow analysis

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## Flow analysis - What, Why and How

### What

- Tell what “flows” into a variable/expression. (instances - values, type of values, properties of values ...)
- One instance of flow analysis information - finite set of classes.
- Say the flow set of an expression  $e$  is  $\{A, B, C\}$ .
- $\implies e$  can evaluate to `null` or an instance of a class mentioned in  $\{A, B, C\}$ .

### Utility of such flow information:

- We can inline a message send (method call), if the flow set for the receiver is a singleton set.
- If the method of a class is not called at all, then we can discard thus “dead code”.

### How

- To compute flow sets for each expression, we will do flow analysis.



## Recap

- Idea of CPS
- Step by step approach to convert scheme to cps.
- Algorithm to convert Scheme programs to Tail form.
- Algorithm to convert programs in tail form to first-order form.
- Algorithm to convert programs in first-order form to imperative form.

### What you should be able to answer (necessary not sufficient)

- Given a scheme program convert it to imperative form.



## Our focus

- We will study flow analysis that will help in inlining.
- Our assumptions:
  - closed-world assumption:  
All parts of the program are known at the time of the analysis and will not change.
  - open-world assumption:  
Some parts of the program are not known or may change. Recall the principal type inference.



# Method inlining example

- Method inlining is a popular optimization in OO languages: Java, C++. (Why?)

```

class A{
    void m (Q arg) {
        arg.p();
    }
}
class B extends A{
    void m (Q arg)
    { ... }
}

class Q {
    void p()
    { ... }
}
class S extends Q {
    void p()
    { ... }
}
    
```

```

A x = new A();
B y = new B();

x.m(new Q());
y.m(new S());
    
```

- Flow sets for  $x$ : A
- Flow sets for  $y$ : B – can also be inlined
- What if there is some code in between?



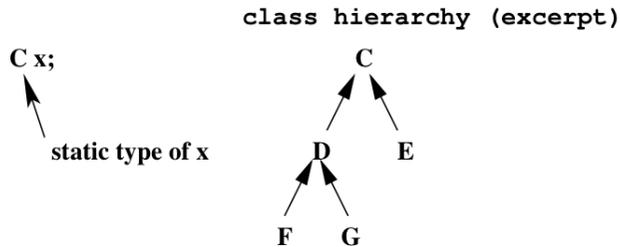
# Flow analysis

- Goal:** Find the call sites for each caller; unique callees are of interest.
- We will use a set based analysis.
  - The flow set for an expression is a set of class names.
  - Say the flow set of an expression  $e$  is  $\{A, B, C\}$ .
  - $\implies e$  can evaluate to `null` or an instance of a class mentioned in  $\{A, B, C\}$ .
- Note: Any flow analysis must be an approximation.
- Tradeoff - precision and speed.
- CHA  $\implies$  0-CFA  $\implies$  improved precision and cost
- CHA - Class hierarchy analysis, CFA - Control Flow Analysis



# CHA - Class hierarchy analysis

- Relies on the type system; and hence the type information.
- It is a type based analysis.
- Flow set of any expression  $e$ :
  - Say the static type of the expression  $e$  is A.
  - Flow set = all subtypes of the static type.
  - Example:



Flow set for  $x = \{ C, D, E, F, G \}$



# Revisit the example with CHA

```

class A{
    void m (Q arg) {
        arg.p();
    }
}
class B extends A{
    void m (Q arg)
    { ... }
}

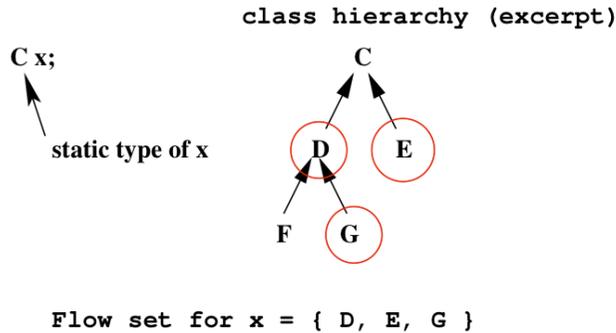
class Q {
    void p()
    { ... }
}
class S extends Q {
    void p()
    { ... }
}

A x = new A();
B y = new B();

//FS(x)={A, B}
x.m(new Q());
//FS(y)={B}
y.m(new S());
    
```



- Flow insensitive, context insensitive flow analysis.
- Can be done  $O(n^3)$  time.
- Does not rely on type system, but is type preserving.
- Example: (Say  $x$  gets values of type  $D$ ,  $E$ , and  $G$ .)



- 1 Generate constraints.
- 2 Solve constraints.
  - For each expression  $e$ , there is a flow variable  $\llbracket e \rrbracket$ .
  - Example:

Program	Constraints
<code>new C ();</code>	$C \in \llbracket \text{new C}() \rrbracket$
<code>x = e;</code>	$\llbracket x \rrbracket \supseteq \llbracket e \rrbracket$
<code>e1.m (e2);</code> and	$C \in \llbracket e1 \rrbracket \Rightarrow \llbracket e2 \rrbracket \subseteq \llbracket a \rrbracket$
<code>class C {</code> <code>B m(A a) {</code> <code>...</code> <code>return e; }</code> <code>}</code>	$C \in \llbracket e1 \rrbracket \Rightarrow \llbracket e \rrbracket \subseteq \llbracket e1.m(e2) \rrbracket$



## 0-CFA Constraint generation

- Assume that all program variable and argument names are distinct (rename otherwise).
- We will use the notation  $\llbracket \text{this} - C \rrbracket$  for the flow variable for the “this” in the class  $C$ .
- Generate constraints based on the syntax.
- We are looking at constraints in three forms:
  - $c \in X$  Beginning
  - $X \subseteq Y$  Propagation
  - $(c \in X) \Rightarrow (Y \subseteq Z)$  conditional
- A unique minimal solution is guaranteed.



## Constraint generation (contd.)

- “ $ID = EXP$ ” (assignment)  
 $\llbracket EXP \rrbracket \subseteq \llbracket ID \rrbracket$
- “this”. Say this occurs in a method in class  $C$ :  
 $C \in \llbracket \text{this} - C \rrbracket$
- “new  $C$ ” (object creation)  
 $C \in \llbracket \text{new C}() \rrbracket$
- “ $EXP.METH (EXP1, \dots, EXPn)$ ” (message send)  
Say  $C$  implements a method for the message  $METH$ :  
`retType METH (type1 ID1, ... type_n IDn) {`  
  `return EXP0 }`

Generate constraints:

$$C \in \llbracket EXP \rrbracket \Rightarrow \begin{cases} \llbracket EXP1 \rrbracket \subseteq \llbracket ID1 \rrbracket \\ \dots \\ \llbracket EXPn \rrbracket \subseteq \llbracket IDn \rrbracket \\ \llbracket EXP0 \rrbracket \subseteq \llbracket EXP.METH(EXP1, \dots, EXPn) \rrbracket \end{cases}$$



## Running on the example:

```

class A{
  void m (Q arg) {
    arg.p();
  }
}
class B extends A{
  void m (Q arg)
  { ... }
}

class Q {
  void p()
  { ... }
}

class S extends Q {
  void p()
  { ... }
}

```

### Generate constraints

Starting	Propagation	Conditional
$A \in \llbracket \text{new A}() \rrbracket$	$\llbracket \text{new A}() \rrbracket \subseteq \llbracket x \rrbracket$	$A \in \llbracket x \rrbracket \Rightarrow \llbracket \text{new Q}() \rrbracket \subseteq \llbracket A.\text{arg} \rrbracket$
$B \in \llbracket \text{new B}() \rrbracket$	$\llbracket \text{new B}() \rrbracket \subseteq \llbracket y \rrbracket$	$B \in \llbracket x \rrbracket \Rightarrow \llbracket \text{new Q}() \rrbracket \subseteq \llbracket B.\text{arg} \rrbracket$
$Q \in \llbracket \text{new Q}() \rrbracket$		$A \in \llbracket y \rrbracket \Rightarrow \llbracket \text{new S}() \rrbracket \subseteq \llbracket A.\text{arg} \rrbracket$
$S \in \llbracket \text{new S}() \rrbracket$		$B \in \llbracket y \rrbracket \Rightarrow \llbracket \text{new S}() \rrbracket \subseteq \llbracket B.\text{arg} \rrbracket$



## Recap

- Introduction to flow analysis.
- CHA.
- Constraint generation for CFA.



## Constraint generation. Example 2

```

class A implements I {
  I x = new D();
  public I m(I f) {
    return f.m(x);
  }
}

class B implements I {
  public I m(I g) {
    return this;
  }
}

... new A().m(new B()).m(new C()) ...

```



## Constraints for example 2

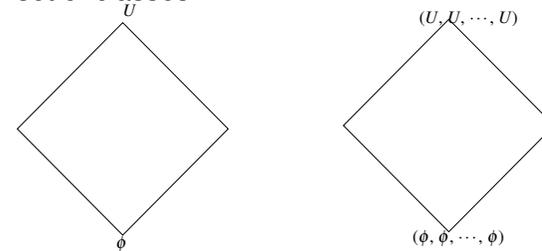
$$\begin{aligned}
 &D \in \llbracket x \rrbracket \\
 &B \in \llbracket \text{this-B} \rrbracket \\
 &A \in \llbracket \text{new A}() \rrbracket \quad B \in \llbracket \text{new B}() \rrbracket \\
 &C \in \llbracket \text{new C}() \rrbracket \quad D \in \llbracket \text{new D}() \rrbracket \\
 &A \in \llbracket f \rrbracket \Rightarrow \begin{cases} \llbracket x \rrbracket \subseteq \llbracket f \rrbracket \\ \llbracket f.m(x) \rrbracket \subseteq \llbracket f.m(x) \rrbracket \end{cases} \\
 &B \in \llbracket f \rrbracket \Rightarrow \begin{cases} \llbracket x \rrbracket \subseteq \llbracket g \rrbracket \\ \llbracket \text{this-B} \rrbracket \subseteq \llbracket f.m(x) \rrbracket \end{cases} \\
 &A \in \llbracket \text{new A}() \rrbracket \Rightarrow \begin{cases} \llbracket \text{new B}() \rrbracket \subseteq \llbracket f \rrbracket \\ \llbracket f.m(x) \rrbracket \subseteq \llbracket \text{new A}().m(\text{new B}()) \rrbracket \end{cases} \\
 &B \in \llbracket \text{new A}() \rrbracket \Rightarrow \begin{cases} \llbracket \text{new B}() \rrbracket \subseteq \llbracket g \rrbracket \\ \llbracket \text{this-B} \rrbracket \subseteq \llbracket \text{new A}().m(\text{new B}()) \rrbracket \end{cases} \\
 &A \in \llbracket \text{new A}().m(\text{new B}()) \rrbracket \Rightarrow \begin{cases} \llbracket \text{new C}() \rrbracket \subseteq \llbracket f \rrbracket \\ \llbracket f.m(x) \rrbracket \subseteq \llbracket \text{new A}().m(\text{new B}()).m(\text{new C}()) \rrbracket \end{cases} \\
 &B \in \llbracket \text{new A}().m(\text{new B}()) \rrbracket \Rightarrow \begin{cases} \llbracket \text{new C}() \rrbracket \subseteq \llbracket g \rrbracket \\ \llbracket \text{this-B} \rrbracket \subseteq \llbracket \text{new A}().m(\text{new B}()).m(\text{new C}()) \rrbracket \end{cases}
 \end{aligned}$$


## What you should be able to answer? (necessary not sufficient)

- Given a Java/C++ program inline methods using CHA.
- Given a Java/C++ program generate flow constraints for 0-CFA.



- For each flow variable, we want to compute the flow set.
- We go with the closed world assumption.  $\Rightarrow$  maximal set of classes present in flow set is finite (the total number of classes).
- We use  $U$  to denote the maximal set of classes.
- The flow set for any expression  $\in P(U)$ .
- The set of flow sets for all the expressions  $\subseteq$  powerset of a finite set of classes.

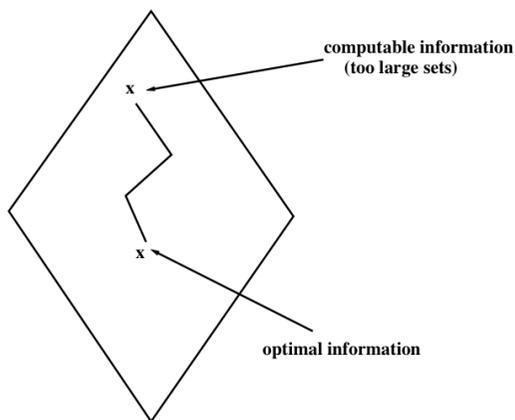


- Power set is a lattice (read: properties of lattices).
- The top of the lattice corresponds to trivial flow information.



# Property of conservative flow analysis

- The minimal solution is above the optimal information.



- We will start with the most trivial solution.
- Iteratively improve the solution.



# Constraint solver

- Takes one constraint at a time.
- At any point of time it maintains the minimal solution.
- Internally constraints are represented as a graph  $(N, E)$ .
  - $N$ : set of flow variables.
  - $E: (v \rightarrow w \in E) \Rightarrow v \subseteq w$   
(Why is it one way?)
- The value of a flow variable  $X$  is stored in a bit vector  $B(X)$ 
  - Initialized to all 0s.
- Each bit  $i$  (which corresponds to a class), has an associated set of pending constraints (may be empty) corresponding to the conditional constraints; given by  $K(X, i)$ 
  - For example:  $C \in X \Rightarrow Y \subseteq Z: Y \subseteq Z \in K(X, i)$ , where  $i$  is the bit corresponding to class  $C$ .
  - Note:  $B(i)$  will be 0.



## Solver details

```

Function Insert( $i \in X$ )
begin
  | Propagate( $X, i$ );
end
Function Insert( $X \subseteq Y$ )
begin
  | Add an edge  $X \rightarrow Y$ ;
  foreach  $i \in B(X)$  do
  | | Propagate( $Y, i$ );
  end
end
Function Insert( $c \in X \Rightarrow Y \subseteq Z$ )
begin
  | if  $B(X, c)$  then
  | | Insert ( $Y \subseteq Z$ );
  | end
  | else
  | |  $K(X, c) = K(X, c) \cup \{(Y \subseteq Z)\}$ 
  | end
end

```

```

Function Propagate( $v, i$ )
begin
  | if  $\neg B(v, i)$  then
  | |  $B(v, i) = true$ ;
  | | foreach  $(v \rightarrow w) \in Edges$  do
  | | | Propagate ( $w, i$ );
  | | end
  | | foreach  $k \in K(v, i)$  do
  | | | Insert ( $k$ );
  | | end
  | |  $K(v, i) = \{\}$ 
  | end
end

```



## Running on example

### Generate constraints

Starting	Propagation	Conditional
$A \in \llbracket new\ A() \rrbracket$	$\llbracket new\ A() \rrbracket \subseteq \llbracket x \rrbracket$	$A \in \llbracket x \rrbracket \Rightarrow \llbracket new\ Q() \rrbracket \subseteq \llbracket A.arg \rrbracket$
$B \in \llbracket new\ B() \rrbracket$	$\llbracket new\ B() \rrbracket \subseteq \llbracket y \rrbracket$	$B \in \llbracket x \rrbracket \Rightarrow \llbracket new\ Q() \rrbracket \subseteq \llbracket B.arg \rrbracket$
$Q \in \llbracket new\ Q() \rrbracket$		$A \in \llbracket y \rrbracket \Rightarrow \llbracket new\ S() \rrbracket \subseteq \llbracket A.arg \rrbracket$
$S \in \llbracket new\ S() \rrbracket$		$B \in \llbracket y \rrbracket \Rightarrow \llbracket new\ S() \rrbracket \subseteq \llbracket B.arg \rrbracket$



## Useless assignment

Run the OCFA algorithm on the constraints generated in Example 2.



## Complexity analysis of the algorithm

- Say the size of the program is  $n$ .
- Number of classes:  $O(n)$ .
- Number of nodes (flow variables):  $O(n)$
- Number of edges:  $O(n^2)$
- Number of constraints added:  
At each call site ( $O(n)$ ), for each class  $O(n)$ , add  $O(n)$  constraints.  
 $O(n^3)$
- Max size of  $K(v, i)$ , for any given  $v$ , and  $i$ :  $O(n)$ .
- Work done:
  - Each bit (class) is propagated along a specific edge at most once –  $O(n^2)$ . And each propagate may process  $O(n)$  Insert functions. =  $O(n^3)$
  - Each of the constraint may
    - be inserted into and deleted from a list once
    - cause the creation of a single edge.
- Cost =  $O(n^3)$ .
- In practise – mostly linear.



- Flow analysis using 0-CFA and some simple improvements.

### What you should be able to answer? (necessary not sufficient)

- Given a set of flow constraints solve them to get the flow sets.

### Reminder

- Assignment due in 5 days.



## Identify dead code

- Idea: Don't generate constraints for parts of program that is unreachable.
- Take the example of library code - most of the code in the libraries is "dead code" for any program.

### Modified solver

```

L = φ;
foreach k ∈ constraints (main) do
  | Insert(k);
end
// Updates the reachable methods of main in Live
while Live is not empty do
  | m = Get a method from Live;
  | foreach k ∈ constraints (m) do
    | Insert(k);
  | end
  // Updates the reachable methods of m in Live
end

```

- Complexity? The algorithm is still  $O(n^3)$ .
- Will be efficient in practise.



- The algorithm is not very precise.
- Several challenges:
  - huge class libraries.
  - polymorphic methods.
  - polymorphic container classes.
- Can improve by
  - dead code detection.
  - code duplication.
- We will study couple of ways.



## Method duplication

- Say A and B implement the interface I.

```

class C {
  I id (I x) { // A polymorphic identity function
    return x; // flow set for x = {A, B}
  }
}
new C().id(new A()).m(5);
new C().id(new B()).m(5);

```

- Is there a way to get the flow set of x to singleton sets?
- Create a copy of each method implementation for each syntactic invocation.

```

class C2 {
  I id1 (I x) { // Convert to a monomorphic identity function?
    return x; // flow set for x = {A}
  }
  I id2 (I x) { // Convert to a monomorphic identity function?
    return x; // flow set for x = {B}
  }
}
new C2().id1(new A()).m(5);
new C2().id2(new B()).m(5);

```



## Class duplication

```
class C{
  I x;
  C put (I v) { x = v; return this; }
  I get() {return x; } // flow set for x = {A, B} }
new C().put(new A()).get().m(5);
new C().put(new B()).get().m(5);
```

- Is there a way to get the flow set of x to singleton sets?
- Create a copy of each class for each syntactic object creation (via new).

```
class C1{
  I x;
  C1 put (I v) { x = v; return this; }
  I get() {return x; } // flow set for x = {A} }
class C2{
  I x;
  C1 put (I v) { x = v; return this; }
  I get() {return x; } // flow set for x = {B} }
new C1().put(new A()).get().m(5);
new C2().put(new B()).get().m(5);
```



## Closure conversion - a quick revisit

- Closure conversion - converting higher order functions to first order (has an environment that maps variables to values).

### Translating Closures to C.

- (define f (lambda (x) (let (g (lambda () x)) g)))

```
(set! a (f 10))
(a) ?
```

- Value should be 10.
- How to translate it to C?



## Closure conversion to C

- Naive translation is problematic.

```
typedef int (*fp)() ; // function pointer
int g () {
  return x ; // Oops: which x is this?
}
fp f(int x) {
  return g ;
}
```



## Closure conversion to C

- No nested functions in C. So use globals.

```
typedef int (*fp)() ; // function pointer
int globalX;
int g () {
  return globalX;
}
fp f(int x) {
  globalX = x ;
  return g ;
}
```

- Any problem? – a = f(10); b = f(20); a(); b();



## Closure conversion - revisited

- Create an environment and pass free variables.
- At each application site - remember that it is a closure.

```
typedef int (*fp) () ; // function pointer
int g(e) {
    return e["x"];
}
(fp, env) f(int x) {
    return (g, {"x" = x}) ;
}
```

- $(t1Fp, t2Env) = f(10);$
- $a = f(10) \rightarrow$   
     $a[0] = t1Fp;$   
     $a[1] = t2Env;$
- $(t3Fp, t4Env) = f(20);$
- $b = f(20) \rightarrow$   
     $b[0] = t3Fp;$   
     $b[1] = t4Env;$
- $a() \rightarrow a[0](a[1])$
- $b() \rightarrow b[0](b[1])$



## Recap

- Flow analysis using 0-CFA and some simple improvements.
- Closure conversion.

### What you should be able to answer? (necessary not sufficient)

- Given a set of flow constraints solve them to get the flow sets.
- Translate closures in Scheme to C.

### Reminder

- Assignment due in 4 days.

