CS6013 - Modern Compilers: Theory and Practise

Register Allocation

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Opening remarks

What have we done so far?

- Compiler overview.
- Scanning and parsing.
- JavaCC, visitors and JTB
- Semantic Analysis - specification, execution, attribute grammars.
- Type checking, Intermediate Representation, Intermediate code generation.
- Control flow analysis, interval analysis, structural analysis
- Data flow analaysis, intra-procedural and inter-procedural constant propagation.
- Points-to analysis

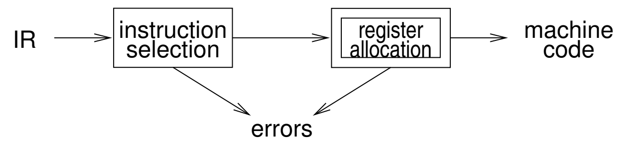
Announcement:

- Assignment 5 is out. Due in three weeks.

Today: Liveness analysis and register allocation.

Register allocation

Register allocation



Register allocation:

- have value in a register when used
- limited resources
- can effect the instruction choices
- can move loads and stores
- optimal allocation is difficult
  $\Rightarrow$ NP-complete for $k \geq 1$ registers

Liveness analysis

Problem:

- IR contains an unbounded number of temporaries
- machine has bounded number of registers

Approach:

- temporaries with disjoint live ranges can map to same register
- if not enough registers then spill some temporaries (i.e., keep them in memory)

The compiler must perform liveness analysis for each temporary:

It is live if it holds a value that may be needed in future

Example

$$
\begin{array}{rl}
 & a \leftarrow 0 \\
L_1: & b \leftarrow a + 1 \\
 & c \leftarrow c + b \\
 & a \leftarrow b \times 2 \\
 & \text{if } a < N \text{ goto } L_1 \\
 & \text{return } c
\end{array}
$$

Liveness analysis

Gathering liveness information is a form of data flow analysis operating over the CFG:

- We will treat each statement as a different basic block.
- liveness of variables "flows" around the edges of the graph
- assignments define a variable, v:
  - *def*(v) = set of graph nodes that define v
  - *def*$[n]$ = set of variables defined by n
- occurrences of v in expressions use it:
  - *use*(v) = set of nodes that use v
  - *use*$[n]$ = set of variables used in n

Definitions

- v is live on edge e if there is a directed path from SRC(e) to a use of v that does not pass through any *def*(v)
- v is live-in at node n if live on all of n's in-edges
- v is live-out at n if live on any of n's out-edges
- $v \in$ *use*$[n] \Rightarrow v$ live-in at n
- v live-in at $n \Rightarrow v$ live-out at all $m \in$ *pred*$[n]$
- v live-out at $n, v \notin$ *def*$[n] \Rightarrow v$ live-in at n

Liveness analysis

Define:

$$in[n] = \text{variables live-in at } n$$
$$out[n] = \text{variables live-out at } n$$

Then:

$$out[n] = \bigcup_{s \in succ(n)} in[s]$$
$$succ[n] = \phi \Rightarrow out[n] = \phi$$

Note:

$$in[n] \supseteq use[n]$$
$$in[n] \supseteq out[n] - def[n]$$

$use[n]$ and $def[n]$ are constant (independent of control flow)
Now, $v \in in[n]$ iff. $v \in use[n]$ or $v \in out[n] - def[n]$
Thus, $in[n] = use[n] \cup (out[n] - def[n])$

Iterative solution for liveness

N : Set of nodes of CFG;
foreach $n \in N$ **do**
  $in[n] \leftarrow \phi$;
  $out[n] \leftarrow \phi$;
end
repeat
  foreach n ∈ Nodes **do**
    $in'[n] \leftarrow in[n]$;
    $out'[n] \leftarrow out[n]$;
    $in[n] \leftarrow use[n] \cup (out[n] - def[n])$;
    $out[n] \leftarrow \bigcup_{s \in succ[n]} in[s]$;
  end
until $\forall n, in'[n] = in[n] \vee out'[n] = out[n]$;

Notes

- should order computation of inner loop to follow the "flow"
- liveness flows backward along control-flow arcs, from out to in
- nodes can just as easily be basic blocks to reduce CFG size
- could do one variable at a time, from uses back to defs, noting liveness along the way

Iterative solution for liveness

Complexity: for input program of size N

- $\leq N$ nodes in CFG
  $\Rightarrow \leq N$ variables
  $\Rightarrow N$ elements per *in/out*
  $\Rightarrow O(N)$ time per set-union
- **for** loop performs constant number of set operations per node
  $\Rightarrow O(N^2)$ time for **for** loop
- each iteration of **repeat** loop can only add to each set
  sets can contain at most every variable
  $\Rightarrow$ sizes of all in and out sets sum to $2N^2$,
  bounding the number of iterations of the **repeat** loop

$\Rightarrow$ worst-case complexity of $O(N^4)$

- ordering can cut **repeat** loop down to 2-3 iterations
  $\Rightarrow O(N)$ or $O(N^2)$ in practice

Least fixed points

There is often more than one solution for a given dataflow problem (see example).

Any solution to dataflow equations is a conservative approximation:

- v has some later use downstream from n
  $\Rightarrow v \in out(n)$
- but not the converse

Conservatively assuming a variable is live does not break the program; just means more registers may be needed.

Assuming a variable is dead when really live will break things.

Many possible solutions but we want the "smallest": the least fixpoint.

The iterative algorithm computes this least fixpoint.

Register allocation - by Graph coloring

- Step 1:
  - Select target machine instructions assuming infinite registers (temps).
  - If a instruction requires a special register – replace that temp with that register.
- Step 2:
  - Construct an interference graph.
  - Solve the register allocation problem by coloring the graph.
  - A graph is said to be colored if each each pair of neighboring nodes have different colors.

Parts of slides: sources - Andrew Myers

Graph coloring - a simplistic approach

Input: G - the interference graph, K - number of colors

repeat

    `// Simplify`

    repeat

        Remove a node n and all its edges from G, such that degree of n is less than K;

        Push n onto a stack;

    until G has no node with degree less than K ;

    `//` G `is either empty or all of its nodes have degree` $\geq$ `K`

    `// Spill`

    if G is not empty **then**

        Take one node m out of G, and mark it for spilling;

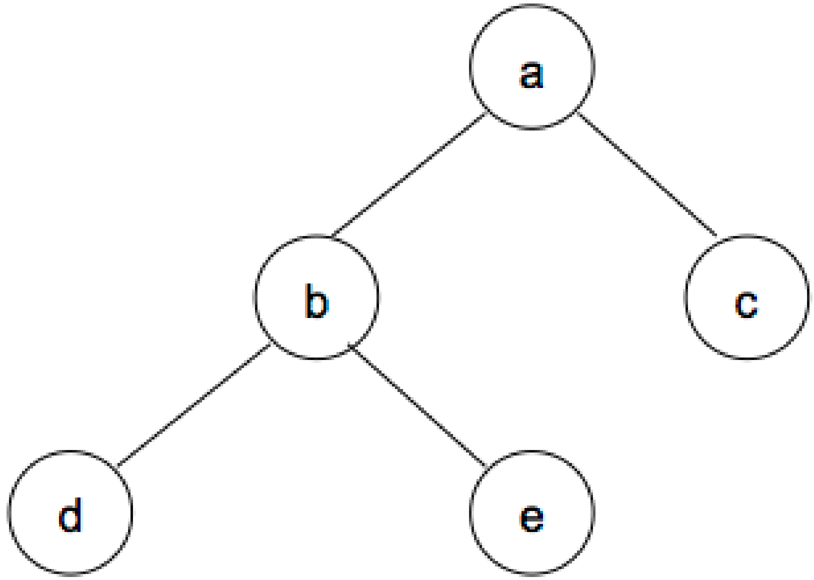
        Remove all the edges of m from G;

    end

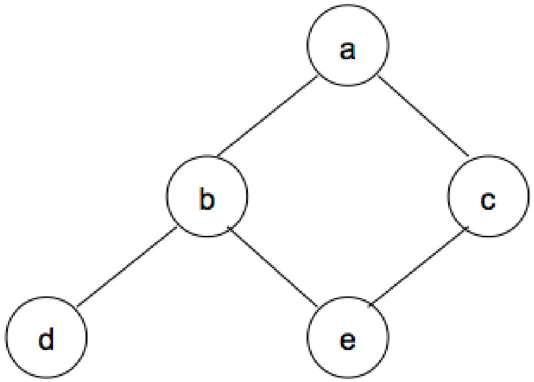
until G is empty ;

Take one node at a time from the stack and assign a non conflicting color.

Example 1, available colors = 2



Example 2



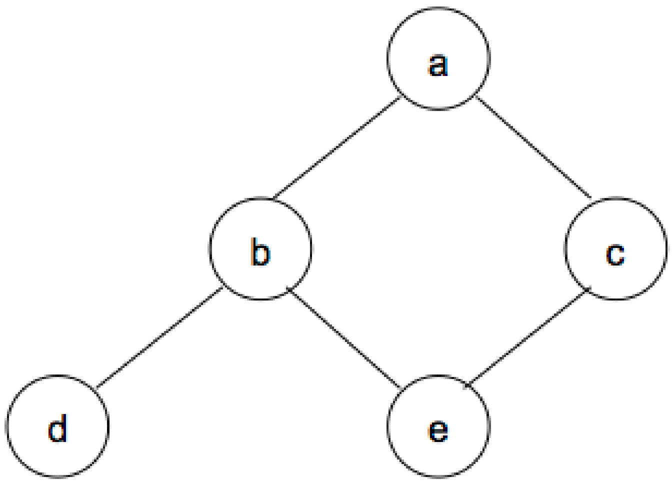
We have to spill.

Graph coloring - Kempe's heuristic

- Algorithm dating back to 1879.

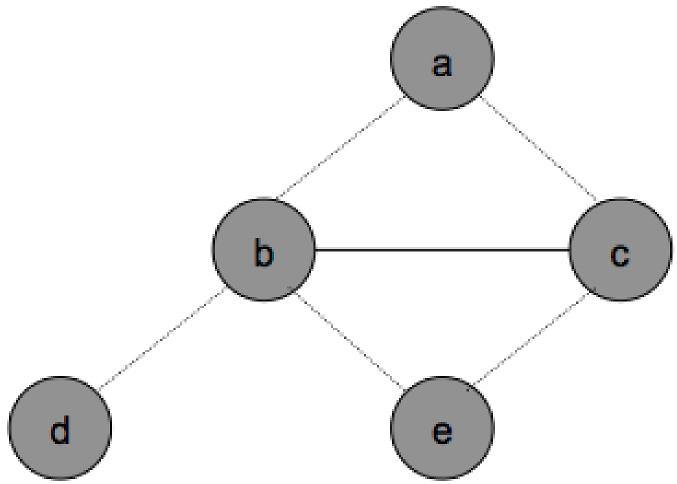
Input: G - the interference graph, K - number of colors
repeat
  repeat
    Remove a node n and all its edges from G, such that degree of n is less than K;
    Push n onto a stack;
  until G has no node with degree less than K ;
  `//` G `is either empty or all of its nodes have degree` $\geq$ `K`
  if G is not empty **then**
    Take one node m out of G.;
    push m onto the stack;
  end
until G is empty ;
Take one node at a time from the stack and assign a non conflicting color (if possible, else spill).

Example 2 (revisited)



We don't have to spill.

Example 3



Don't have a choice. Have to spill.

Spilling

- We need to generate extra instructions to load variables from the stack and store them back.
- The load and store may require registers again:
  - Naive approach: Keep a separate register (wasteful).
  - Rewrite the code - by introducing a temporary; rerun the liveness + ra.
    (Note: the new temp has much smaller live range).

Example: rewrite code

Consider: `add t1 t2`

- Suppose `t2` has to be spilled, say to `[sp-4]`.
- Invent a new temp `t35`, and rewrite:

  `mov t35 [sp-4] add t1 t35`
- `t35` has a very short live range and less likely to interfere.
- Now rerun the algo.

Register allocation - Linear scan

Register allocation is **expensive**.

- Many algorithms use heuristics for graph coloring.
- Allocation may take time quadratic in the number of live intervals.

Not suitable

- Online compilers – need to generate code quickly. e.g. JIT compilers.
- Sacrifice efficient register allocation for compilation speed.

Linear scan register allocation - Massimiliano Poletto and Vivek Sarkar, ACM TOPLAS 1999

- Complexity linear in the number of variables (assuming the number of register is not too large).

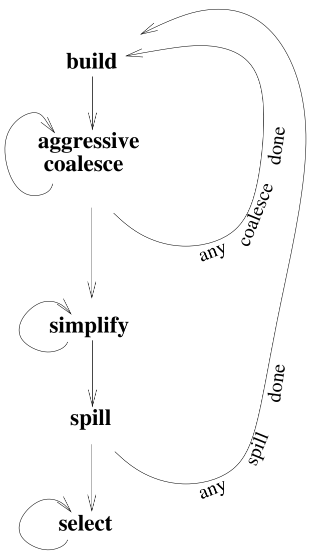
Register allocation - Chaitins

1. Simplify
2. Spill
3. Select: assign colors to nodes
   1. start with empty graph and keep adding nodes:
   2. if adding a non-spill node – will have a color (basis for removal)
   3. if adding spill node and no color available (neighbors already K-colored) then mark as an actual spill; break;
   4. continue to select nodes.
4. Start over: if select has no actual spills then finished, otherwise
   1. rewrite code: fetch spills at use, store at definition
   2. recalculate liveness and repeat

Coalescing

- Can delete a move instruction when source s and destination d do not interfere:
  - coalesce them into a new node whose edges are the union of those of s and d
- In principle, any pair of non-interfering nodes can be coalesced
  - unfortunately, the union is more constrained and new graph may no longer be K-colorable
  - overly aggressive

Simplification with aggressive coalescing



Conservative coalescing

Apply tests for coalescing that preserve colorability.
Suppose a and b are candidates for coalescing into node ab.
Briggs: coalesce only if ab has $< K$ neighbors of significant degree $\geq K$

- simplify first removes all insignificant-degree neighbors
- ab will then be adjacent to $< K$ neighbors
- simplify can then remove ab

George: coalesce only if all significant-degree neighbors of a already interfere with b

- simplify removes all insignificant-degree neighbors of a
- remaining significant-degree neighbors of a already interfere with b; coalescing does not increase degree of any node

Iterated register coalescing

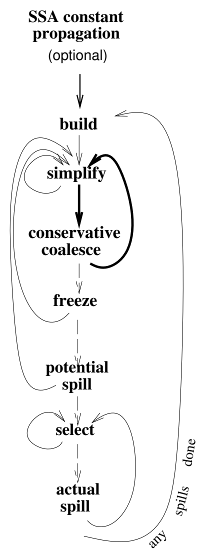
Interleave simplification with coalescing to eliminate most moves while guaranteeing not to introduce spills:

1. Build interference graph G and distinguish move-related from non-move-related nodes. A move-related node is one that is either the source or destination of a move instruction.
2. Simplify: remove non-move-related nodes of low degree one at a time
3. Coalesce: conservatively coalesce move-related nodes
   - remove associated move instruction
   - if resulting node is non-move-related it can now be simplified
   - repeat simplify and coalesce until only significant-degree or uncoalesced moves

Iterated register coalescing (cont.)

4. Freeze: if unable to simplify or coalesce
   1. look for move-related node of low-degree
   2. freeze its associated moves (give up on coalescing)
   3. now treat as non-move-related; resume iteration of simplify and coalesce
5. Spill: if no low-degree nodes
   1. select candidate for spilling
   2. remove to stack and continue simplifying
6. Select: pop stack assigning colors (with actual spills)
7. Start over: if select has no actual spills then finished, otherwise
   1. rewrite code: fetch spills before use, store after def
   2. recalculate liveness and repeat

Iterated register coalescing



Precolored nodes

Precolored nodes correspond to machine registers (e.g., stack pointer, arguments, return address, return value)

- select and coalesce can give an ordinary temporary the same color as a precolored register, if they don't interfere
- e.g., argument registers can be reused inside procedures for a temporary
- simplify, freeze and spill cannot be performed on them
- also, precolored nodes interfere with other precolored nodes

So, treat precolored nodes as having infinite degree
This also avoids needing to store large adjacency lists for precolored nodes; coalescing can use the George criterion

Temporary copies of machine registers

Since precolored nodes don't spill, their live ranges must be kept short:

1. use move instructions
2. move callee-save registers to fresh temporaries on procedure entry, and back on exit, spilling between as necessary
3. register pressure will spill the fresh temporaries as necessary, otherwise they can be coalesced with their precolored counterpart and the moves deleted

Criteria for spilling

During register allocation, we identify that one of the live ranges from a given set, has to be spilled. Criteria?

- Random! Adv? Disadv?
- One with maximum degree
- One that has the longest life
- One with the shortest life (take advantage of the cache).
- One with least cost.
  - Cost = Dynamic (load cost + store cost)
  - How to handle loops, conditionals?
  - Cost of load, store

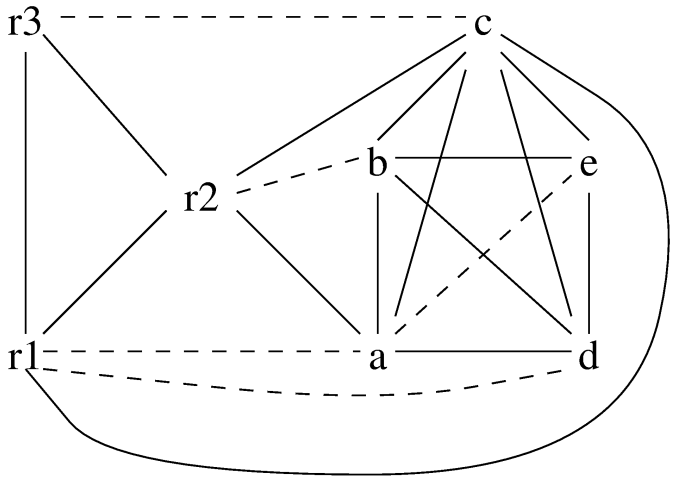
Example

```
enter:
  c := r3
  a := r1
  b := r2
  d := 0
  e := a
loop:
  d := d + b
  e := e - 1
  if e > 0 goto loop
  r1 := d
  r3 := c
  return [ r1, r3 live out ]
```

- Temporaries are a, b, c, d, e
- Assume target machine with $K = 3$ registers: r1, r2 (caller-save/argument/result), r3 (callee-save)
- The code generator has already made arrangements to save r3 explicitly by copying into temporary a and back

Example (cont.)

Interference graph:



Example (cont.)

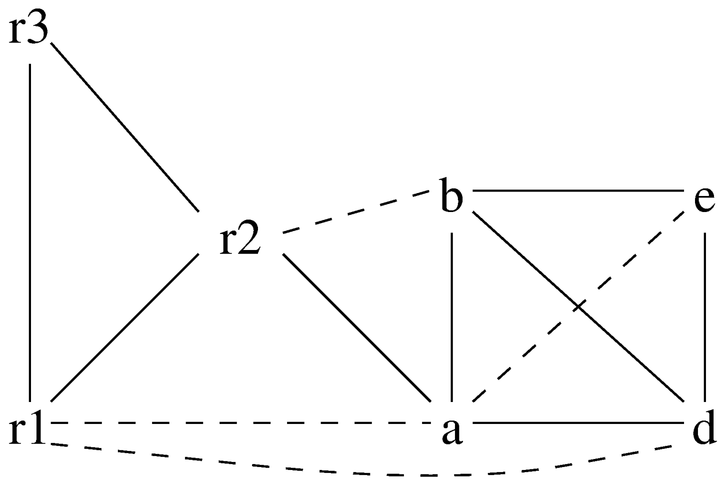
- No opportunity for simplify or freeze (all non-precolored nodes have significant degree $\geq K$)
- Any coalesce will produce a new node adjacent to $\geq K$ significant-degree nodes
- Must spill based on priorities:

Node	uses + defs outside loop		uses + defs inside loop		degree		priority
a	(2	$+10\times$	0	)/	4	=	0.50
b	(1	$+10\times$	1	)/	4	=	2.75
c	(2	$+10\times$	0	)/	6	=	0.33
d	(2	$+10\times$	2	)/	4	=	5.50
e	(1	$+10\times$	3	)/	3	=	10.30

- Node c has lowest priority so spill it

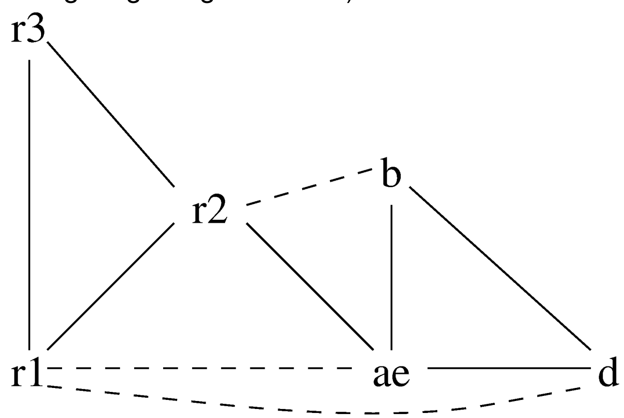
Example (cont.)

Interference graph with `c` removed:



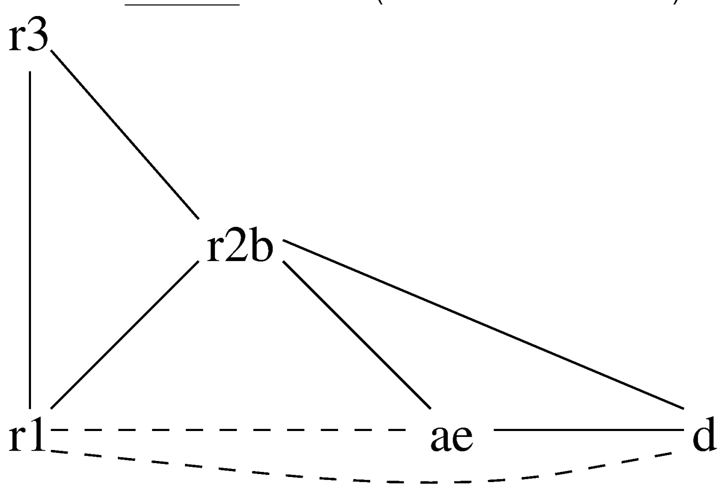
Example (cont.)

Only possibility is to coalesce `a` and `e`: `ae` will have $< K$ significant-degree neighbors (after coalescing `d` will be low-degree, though high-degree before)



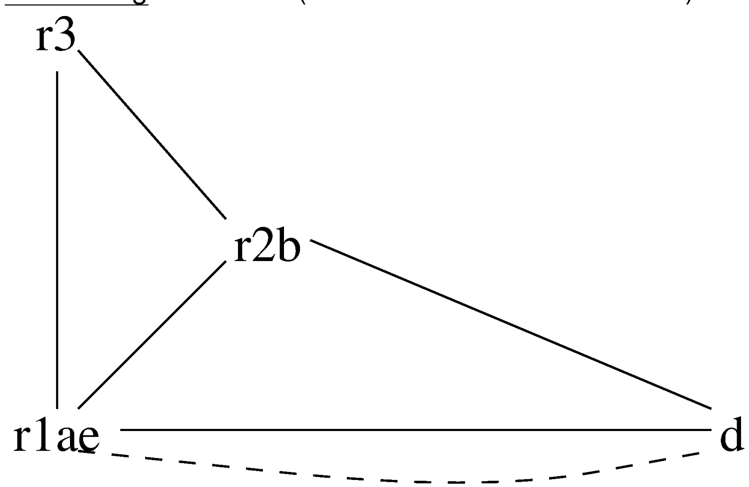
Example (cont.)

Can now coalesce `b` with `r2` (or coalesce `ae` and `r1`):



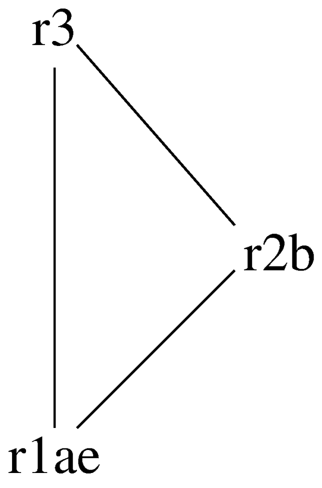
Example (cont.)

Coalescing `ae` and `r1` (could also coalesce `d` with `r1`):



Example (cont.)

Cannot coalesce `r1ae` with `d` because the move is constrained: the nodes interfere. Must simplify `d`:



Example (cont.)

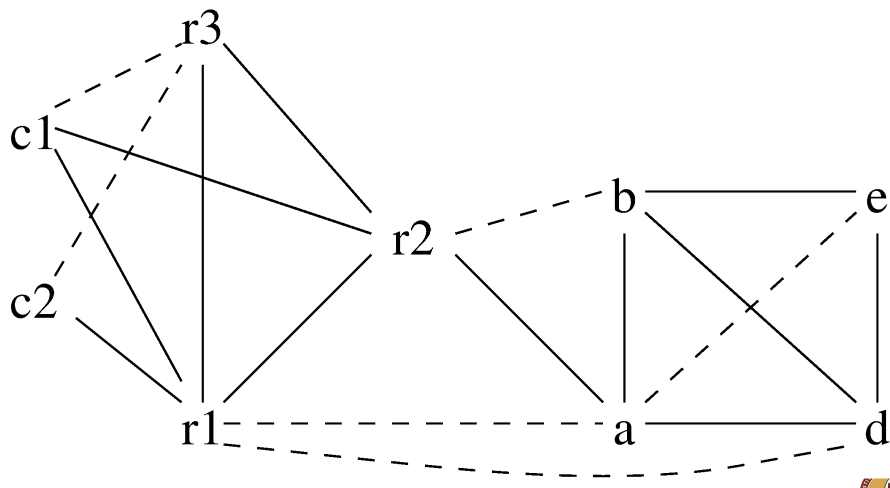
- Graph now has only precolored nodes, so pop nodes from stack coloring along the way
  - `d` ≡ `r3`
  - `a`, `b`, `e` have colors by coalescing
  - `c` must spill since no color can be found for it
- Introduce new temporaries `c1` and `c2` for each use/def, add loads before each use and stores after each def

Example (cont.)

```
enter:
  c1 := r3
  M[c_loc] := c1
  a := r1
  b := r2
  d := 0
  e := a
loop:
  d := d + b
  e := e - 1
  if e > 0 goto loop
  r1 := d
  c2 := M[c_loc]
  r3 := c2
  return [ r1, r3 live out ]
```

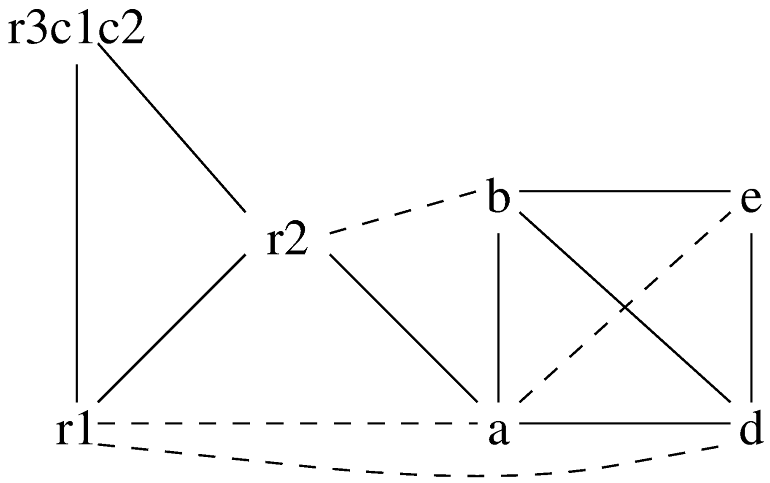

Example (cont.)

New interference graph:



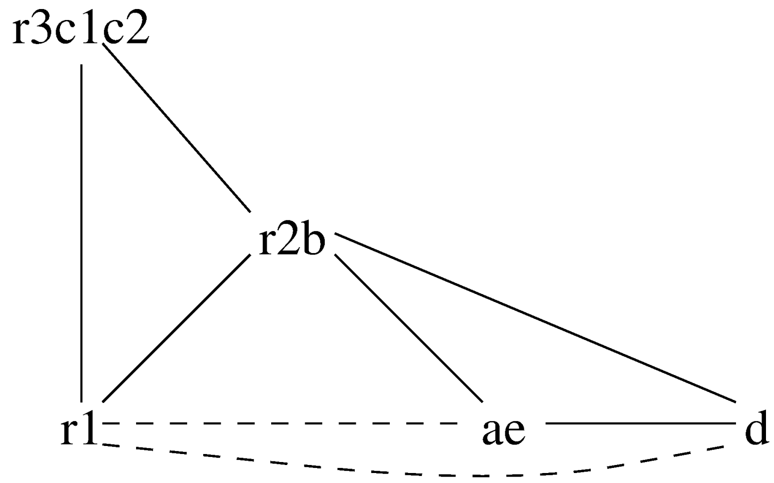
Example (cont.)

Coalesce `c1` with `r3`, then `c2` with `r3`:



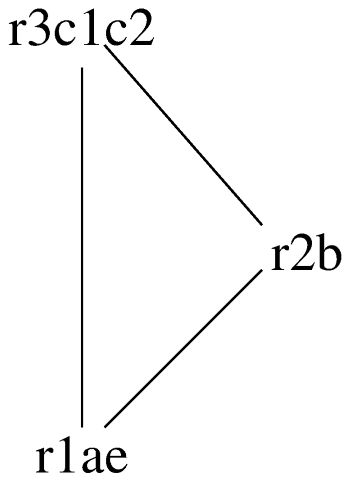
Example (cont.)

As before, coalesce `a` with `e`, then `b` with `r2`:



Example (cont.)

As before, coalesce `ae` with `r1` and simplify `d`:



Example (cont.)

Pop `d` from stack: select `r3`. All other nodes were coalesced or precolored. So, the coloring is:

- `a` ≡ `r1`
- `b` ≡ `r2`
- `c` ≡ `r3`
- `d` ≡ `r3`
- `e` ≡ `r1`

Example (cont.)

Rewrite the program with this assignment:

```
enter:
  r3 := r3
  M[c_loc] := r3
  r1 := r1
  r2 := r2
  r3 := 0
  r1 := r1
loop:
  r3 := r3 + r2
  r1 := r1 - 1
  if r1 > 0 goto loop
  r1 := r3
  r3 := M[c_loc]
  r3 := r3
  return [ r1, r3 live out ]
```


Example (cont.)

- Delete moves with source and destination the same (coalesced):

```
enter:
  M[c_loc] := r3
  r3 := 0
loop:
  r2 := r3 + r2
  r1 := r1 - 1
  if r1 > 0 goto loop
  r1 := r3
  r3 := M[c_loc]
  return [ r1, r3 live out ]
```

- One uncoalesced move remains