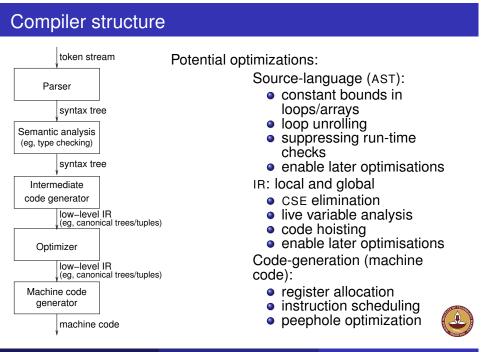
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## Optimization

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Goal: produce fast code

- What is optimality?
- Problems are often hard
- Many are intractable or even undecideable
- Many are NP-complete
- Which optimizations should be used?
- Many optimizations overlap or interact



## Optimization

Definition: An optimization is a transformation that is expected to:

- improve the running time of a program
- or decrease its space requirements

#### The point:

- "improved" code, not "optimal" code (forget "optimum")
- sometimes produces worse code
- range of speedup might be from 1.000001 to xxx

- applicable across broad range of machines
- remove redundant computations
- move evaluation to a less frequently executed place
- specialize some general-purpose code
- find useless code and remove it
- expose opportunities for other optimizations



## Machine-dependent transformations

## A classical distinction

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- capitalize on machine-specific properties
- improve mapping from IR onto machine
- replace a costly operation with a cheaper one
- hide latency
- replace sequence of instructions with more powerful one (use "exotic" instructions)

The distinction is not always clear: replace  ${\tt multiply}\ with\ {\tt shifts}\ and\ {\tt adds}$ 

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## Optimization

#### Desirable properties of an optimizing compiler

- code at least as good as an assembler programmer
- stable, robust performance
- architectural strengths fully exploited
- architectural weaknesses fully hidden
- broad, efficient support for language features
- instantaneous compiles

Unfortunately, modern compilers often drop the ball



## Scope of optimization

### Local

(single block)

(global)

(predictability)

- confined to straight-line code
- simplest to analyse
- time frame: '60s to present, particularly now

#### Intraprocedural

- consider the whole procedure
- What do we need to optimize an entire procedure?
- classical data-flow analysis, dependence analysis
- time frame: '70s to present

#### Interprocedural

(whole program)

- analyse whole programs
- What do we need to optimize and entire program?
- less information is discernible
- time frame: late '70s to present, particularly now

## Optimization

### Good compilers are crafted, not assembled

- consistent philosophy
- careful selection of transformations
- thorough application
- coordinate transformations and data structures
- attention to results

#### Compilers are engineered objects

- minimize running time of compiled code
- minimize compile time
- use reasonable compile-time space

Thus, results are sometimes unexpected



(code, time, space)

(serious problem)

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## Optimization

Three considerations arise in applying a transformation:

- safety
- profitability
- opportunity
- We need a clear understanding of these issues
  - the literature often hides them
  - every discussion should list them clearly



# Fundamental question Does the transformation change the **results** of executing the code?

yes  $\Rightarrow$  don't do it!

no  $\Rightarrow$  it is safe

#### Compile-time analysis

- may be safe in all cases
- analysis may be simple
- may require complex reasoning

(loop unrolling)
(DAGs and CSEs)
(data-flow analysis)

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## Opportunity

Fundamental question Can we efficiently locate sites for applying the transformation?

yes  $\Rightarrow$  compilation time won't suffer

no  $\Rightarrow$  better be highly profitable

#### Issues

- provides a framework for applying transformation
- systematically find all sites
- update safety information to reflect previous changes
- order of application

(hard)

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## Profitability

# Fundamental question <u>Is there a reasonable expectation that the</u> transformation will be an improvement?

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yes  $\Rightarrow$  do it! no  $\Rightarrow$  don't do it

#### Compile-time estimation

- always profitable
- heuristic rules
- compute benefit



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## Optimization

Successful optimization requires

- test for safety
- profit is *local improvement × executions* 
  - $\Rightarrow$  focus on loops:
    - loop unrolling
    - factoring loop invariants
    - strength reduction
- want to minimize side-effects like code growth



Idea: reduce loop overhead by creating multiple successive copies of the loop's body and increasing the increment appropriately

Safety: always safe

Profitability: reduces overhead

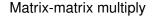
(instruction cache blowout) (subtle secondary effects)

**Opportunity:** loops

Unrolling is easy to understand and perform



## Example: loop unrolling



(assume 4-word cache line)

```
do i \leftarrow 1, n, 1

do j \leftarrow 1, n, 1

c(i,j) \leftarrow 0

do k \leftarrow 1, n, 4

c(i,j) \leftarrow c(i,j) + a(i,k) * b(k,j)

c(i,j) \leftarrow c(i,j) + a(i,k+1) * b(k+1,j)

c(i,j) \leftarrow c(i,j) + a(i,k+2) * b(k+2,j)

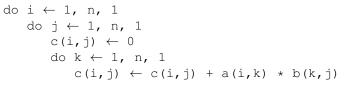
c(i,j) \leftarrow c(i,j) + a(i,k+3) * b(k+3,j)
```

- $2n^3$  flops,  $\frac{n^3}{4}$  loop increments and branches
- each iteration does 8 loads and 8 flops
- memory traffic is better
  - c(i,j) is reused
  - a(i,k) reference are from cache
  - b(k,j) is problematic



(put it in a register)

### Matrix-matrix multiply



•  $2n^3$  flops,  $n^3$  loop increments and branches

• each iteration does 2 loads and 2 flops

This is the most overstudied example in the literature



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## Example: loop unrolling

#### Matrix-matrix multiply

(to improve traffic on b)

```
do j \leftarrow 1, n, 1
   do i \leftarrow 1, n, 4
      c(i,j) \leftarrow 0
      do k \leftarrow 1, n, 4
          c(i,j) \leftarrow c(i,j) + a(i,k) * b(k,j)
             + a(i,k+1) * b(k+1,j) + a(i,k+2) * b(k+2,j)
             + a(i,k+3) * b(k+3,j)
          c(i+1,j) \leftarrow c(i+1,j) + a(i+1,k) * b(k,j)
             + a(i+1,k+1) * b(k+1,j)
             + a(i+1,k+2) * b(k+2,j)
             + a(i+1,k+3) * b(k+3,j)
          c(i+2,j) \leftarrow c(i+2,j) + a(i+2,k) * b(k,j)
             + a(i+2,k+1) * b(k+1,j)
             + a(i+2,k+2) * b(k+2,j)
             + a(i+2,k+3) * b(k+3,j)
          c(i+3,j) \leftarrow c(i+3,j) + a(i+3,k) * b(k,j)
             + a(i+3,k+1) * b(k+1,j)
             + a(i+3,k+2) * b(k+2,j)
             + a(i+3,k+3) * b(k+3,j)
```

## Example: loop unrolling

What happened?

- interchanged i and j loops
- ${\tilde{\circ}}$  unrolled  ${\tilde{1}}$  loop
- fused inner loops
- $2n^3$  flops,  $\frac{n^3}{16}$  loop increments and branches
- first assignment does 8 loads and 8 flops
- 2<sup>nd</sup> through 4<sup>th</sup> do 4 loads and 8 flops
- memory traffic is better
  - c(i,j) is reused
  - a(i,k) references are from cache
  - b(k,j) is reused

(register)

```
(register)
```

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## Loop optimizations: factoring loop-invariants

Loop invariants: expressions constant within loop body Relevant variables: those used to compute and expression

### Opportunity:

- identify variables defined in body of loop (LoopDef)
- Ioop invariants have no relevant variables in LoopDef
- assign each loop-invariant to temp. in loop header
- use temporary in loop body

Safety: loop-invariant expression may throw exception early Profitability:

- loop may execute 0 times
- loop-invariant may not be needed on every path through loop body



## Loop transformations

It is not as easy as it looks:

Safety : loop interchange? loop unrolling? loop fusion?

Opportunity : find memory-bound loop nests

Profitability : machine dependent

#### Summary

- chance for large improvement
- answering the fundamentals is tough
- resulting code is <u>ugly</u>

Matrix-matrix multiply is everyone's favorite example



(mostly)

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## Example: factoring loop invariants

```
foreach i=1 .. 100 do
    // LoopDef = {i,j,k, A}
    foreach j=1 .. 100 do
        // LoopDef = {j,k, A}
        foreach k=1 .. 100 do
        // LoopDef = {k, A}
        A[i,j,k] = i * j * k;
        end
        end
end
```

- 3 million index operations
- 2 million multiplications



### Example: factoring loop invariants (cont.)

Factoring the inner loop:	And the second loop:
foreach i=1 100 do	foreach $\underline{i=1}$ 100 do
$// LoopDef = \{i, j, k, A\}$	// LoopDef = $\{i, j, k, A\}$
foreach <u>j=1</u> 100 do	t3 = &A[i];
$// LoopDef = \{j, k, A\}$	foreach <u>j=1</u> 100 do
t1 = &A[i][j];	// LoopDef = $\{j, k, A\}$
t2 = i * j ;	t1 = &t3[j];
foreach k=1100 do	t2 = i * j ;
$// LoopDef = \{k, A\}$	foreach $k=1 \dots 100$ do
t1[k] = t * k;	$//$ LoopDef = {k,A}
end	t1[k] = t * k;
end	end
end	end
	end
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## Example: strength reduction in loops

```
From previous example:

foreach \underline{i=1} ... 100 do

t3 = \underline{\&A[i]};

t4 = i; // i * j0 = i

foreach \underline{j=1} ... 100 do

t1 = \underline{\&t3[j]};

t2 = t4; // t4 = i * j

t5 = t2; // t2 * k0 = t2

foreach \underline{k=1} ... 100 do

t1[k] = t5; // t5 = t2 * k

t5 = t5 + t2;

end

t4 = t4 + i;

end

end
```

## Strength reduction in loops

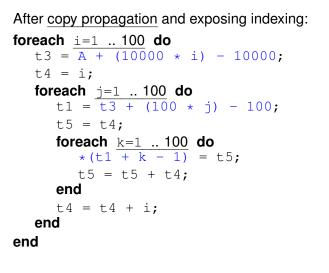
Loop induction variable: incremented on each iteration *i*<sub>0</sub>, *i*<sub>0</sub> + 1, *i*<sub>0</sub> + 2, ...
Induction expression: *ic*<sub>1</sub> + *c*<sub>2</sub>, where *c*<sub>1</sub>, *c*<sub>2</sub> are loop invariant *i*<sub>0</sub>*c*<sub>1</sub> + *c*<sub>2</sub>, (*i*<sub>0</sub> + 1)*c*<sub>1</sub> + *c*<sub>2</sub>, (*i*<sub>0</sub> + 2)*c*<sub>1</sub> + *c*<sub>2</sub>, ...
replace *ic*<sub>1</sub> + *c*<sub>2</sub> by *t* in body of loop
insert *t* := *i*<sub>0</sub>*c*<sub>1</sub> + *c*<sub>2</sub> before loop
insert *t* := *t* + *c*<sub>1</sub> at end of loop



```
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```

```
a second by
```

## Example: strength reduction in loops



## Example: strength reduction in loops

Applying strength reduction to exposed index expressions: t6 = A;foreach i=1 .. 100 do t3 = t6; t4 = i;t7 = t3;foreach j=1 .. 100 do t1 = t7; t5 = t4;t8 = t1;foreach  $k=1 \dots 100$  do \*t8 = t5;t5 = t5 + t4;t8 = t8 + 1;end t4 = t4 + i;t7 = t7 + 100;end t6 = t6 + 10000;end V.Krishna Nandivada (IIT Madras) CS6013 - Jan 2024 Again, copy propagation further improves the code.

## Loop optimizations

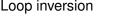
- Loop unswitching
- Loop tiling
- Loop unrolling
- Loop reversal
- Loop-invariant code motion
- Loop inversion

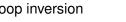
- Loop interchange

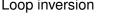
Loop distribution Strip mining

Vectorisation (brief)

Loop fusion











- semantic analysis and intermediate code generation:
  - loop unrolling
  - inline expansion
- Intermediate code generation:
  - build basic blocks with their Def and Kill sets
- Solution of the second seco
  - perform initial data flow analyses
  - assume worst case for calls if no interproc. analysis
- early data-flow optimizations: constant/copy propagation (may expose dead code, changing flow graph, so iterate)
- CSE and live/dead variable analyses
- translate basic blocks to target code: local optimizations (register allocation/assignment, code selection)
- peephole optimization



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