Advanced Counting Techniques

CS1200, CSE IIT Madras

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Advanced Counting Techniques

- Principle of Inclusion-Exclusion ✓
- Recurrences and its applications ✓
- Solving Recurrences

Recurrences

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Today's class: (Some) Techniques to solve recurrences.

There is no single recipe to solve all recurrences. However, we will show techniques that apply to a wide variety.

Some familiar recurrences

Fibonacci Sequence Recurrence

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\begin{aligned}
f(n) &= n & \text{if } n = 0 \text{ or } n = 1 \\
&= f(n-1) + f(n-2) & \textit{otherwise}
\end{aligned}
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Towers of Hanoi Recurrence

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n-1) + 1 & \textit{otherwise} \end{cases}$$

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Binary Search Recurrence

assume $n = 2^k$ for $k \geq 0$

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Repeated Substitution Method : Example 1

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At this step we can guess that $\boxed{T(n) = \log n + 1}$. Recall $n = 2^k$ for $k \geq 0$.

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$$\begin{aligned} T(n) &= 1 & \text{if } n = 1 \\ &= 2T(n-1) + 1 & \textit{otherwise} \end{aligned}$$

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Another recurrence

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\begin{aligned}
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$$\begin{aligned} &= 3^k T(n-k) + 2\sum_{i=0}^{k-1} 3^i (n-i) \\ &= 3^n T(0) + 2\sum_{i=0}^{n-1} 3^i (n-i) \\ &= 2n\left(\sum_{i=0}^{n-1} 3^i\right) - 2\left(\sum_{i=0}^{n-1} 3^i i\right) \end{aligned}$$

we need to solve $\sum_{i=0}^{n-1} 3^i i$

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this needs a derivation and proof!

In general: good to know sum of commonly occurring sums.

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Ex: Solve by repeated substitution

$$T(n) = \begin{cases} 2 & \text{if } n = 0 \\ 2\sqrt{T(n-1)} & \textit{otherwise} \end{cases}$$

$$T(n) = \begin{cases} 12 & \text{if } n = 0 \\ 20 & \text{if } n = 1 \\ 2T(n-1) - T(n-2) & \textit{otherwise} \end{cases}$$

Linear recurrences

Linear Homogeneous Recurrences with constant coefficients

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$$

$1 \leq i \leq k$, c_i is a real number and $c_k \neq 0$

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Examples:

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Non Examples:

- $T(n) = nT(n-1)$ and $T(0) = 1$ does not have constant coefficients
- $T(n) = T(n-1) \cdot T(n-2)$ and $T(0) = 0, T(1) = 1$. not linear
- $T(n) = n \cdot T(n-1)$ and $T(1) = 1$. does not have constant coefficients

Linear Homogeneous Recurrences with constant coefficients

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \ldots + c_k a_{n-k}$$

$1 \leq i \leq k$, c_i is a real number and $c_k \neq 0$

Our goal: To derive a systematic way of solving such recurrences.

Upcoming next week

- A technique to solve linear homogeneous equations.
- A technique to solve linear non-homogeneous equations.
- References: Section 8.2[KR]