Counting

CS1200, CSE IIT Madras

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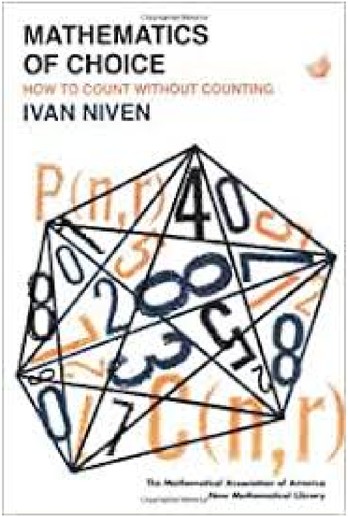
Counting (without counting)



- Basic Counting Techniques
- Pigeon Hole Principle (revisited)
- Permutations and Combinations
- Combinatorial Identities

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How to count is something that we are going to discuss extensively!

Basics of Counting: Product Rule

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Ex: What if we disallow roll numbers that contain the same first two alphabets? For example, CC1 is not a valid roll number.

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Assuming $m \leq n$, the number of one-to-one functions is $n \cdot (n-1) \cdots (n-m+1)$.

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Sol: This is a simple application of sum rule and the answer is $5 + 5 + 5 = 15$.

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Ex: List a way of selecting which is counted by the above, but is not possible.

Forming an office (of posts)

Qn: We have three posts in an office : president, treasurer, and a secretary and there are four people to fill up these posts: Arun, Babita, Charu and Dinesh. Arun does not want to be the president and one of Charu or Dinesh must be the secretary. In how many different ways can the office be constituted?

Calculate your answer and write down. Is it 18?

Lets see how to get 18. There are three choices for the post of president excluding Arun, three choices for the post of treasurer excluding one person who was selected for president, and two choices for the post of secretary.

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Consider selecting Charu for president, then we do not have three ways of selecting the treasurer.

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Can you enumerate the 18 ways? Try it! In fact the analysis has a flaw.

Ex: List a way of selecting which is counted by the above, but is not possible.

Consider selecting Charu for president, then we do not have three ways of selecting the treasurer. This is because Dinesh must be the secretary. Thus we have only two ways for selecting the treasurer and for each of these ways, we have exactly one way of selecting the secretary (which has to be Dinesh).

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Issue in the previous analysis: The number of ways in each case was not the same. Is there a way to count such that the number of ways in each case is the same for the remaining posts?

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Sol: Select the secretary, followed by the president, followed by the treasurer.

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Sol: Select the secretary, followed by the president, followed by the treasurer.

There are two ways to select the secretary, for each of these two ways there are two ways to select the president (excluding Arun as required)

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There are two ways to select the secretary, for each of these two ways there are two ways to select the president (excluding Arun as required) and for each of these ways there are two ways of selecting the treasurer.

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There are two ways to select the secretary, for each of these two ways there are two ways to select the president (excluding Arun as required) and for each of these ways there are two ways of selecting the treasurer.

This gives us a total of $2 \cdot 2 \cdot 2 = 8$ ways. Now list all of them and check that they are indeed valid.

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Sol: Select the secretary, followed by the president, followed by the treasurer.

There are two ways to select the secretary, for each of these two ways there are two ways to select the president (excluding Arun as required) and for each of these ways there are two ways of selecting the treasurer.

This gives us a total of $2 \cdot 2 \cdot 2 = 8$ ways. Now list all of them and check that they are indeed valid.

Ex: Does the order secretary followed by treasurer, followed by president work?

Two more simple (and useful) rules

Subtraction Rule: If a task can be done in either of n_1 ways or n_2 ways then the number of ways to do the task is $n_1 + n_2$ minus the number of ways that are common to the two different ways.

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Sol: Recall binary strings contain only 0s and 1s. For instance 10101111 is a valid string by the above, whereas 00001111 is an invalid string by the above.

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Sol: Recall binary strings contain only 0s and 1s. For instance 10101111 is a valid string by the above, whereas 00001111 is an invalid string by the above. The number of bit strings that start with 1 is 2^7, the number of strings that end with 00 are 2^6. Subtract the ones that start with 1 and end with 00, which is 2^5 to get $2^7 + 2^6 - 2^5$.

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Qn: Count the number of ways to seat 4 people around a round table where two seatings are the same if each person has the same left and right neighbour.

Sol: Lets number the seats as 1, 2, 3, 4 (say clockwise around the table). There are four ways to select a person for seat 1, three ways to select a person for seat 2, two ways to select a person for seat 3 and one way to select a person for seat 4. Thus we have 4! ways to select.

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However, note that this counts A, B, C, D and D, A, B, C as two different ways. These according to us are the same way since the neighbours of every person remains unchanged. Note that the seating A, B, C, D is counted 4 times in the above counting and this is true for every seating.

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Thus the total number of distinct seatings is $24/4 = 6$.

Summary

- Sum rule and product rules.
- Subtraction and division rules.
- Examples illustrating use if single and multiple rules.
- Pitfalls and common mistakes and possible ways to avoid them.
- Reference: Section 6.1 [KR].