Counting

CS1200, CSE IIT Madras

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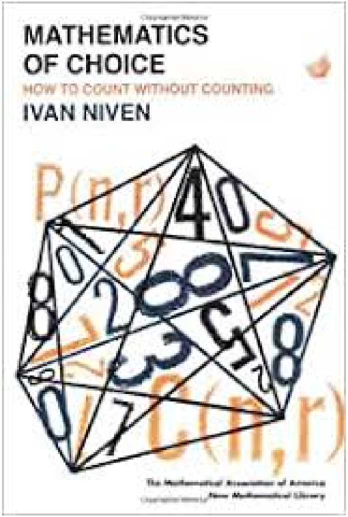
Counting (without counting)



- Basic Counting Techniques ✓
- Pigeon Hole Principle (revisited) ✓
- Permutations and Combinations
- Combinatorial Identities

Permutation

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Example: Let $X = \{p_1, p_2, \ldots, p_{10}\}$ be a set of ten people. How many ways can we arrange three of them in a queue?

Sol: We can use product rule here. We have 10 choices for the first person (in the queue), 9 choices for the second and 8 choices for the third. Thus, we have $10 \cdot 9 \cdot 8$ ways of arranging three people.

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$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

This is immediately proved by application of product rule.

Note that $P(n, 0) = 1$, since there is exactly one way to order zero elements.

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Sol: Note that there are 5! permutations of the given 5 letters. To count the number of permutations containing ABC as a string, we consider replacing the three letters by a single letter X and now count the permutations of X, D, E. Clearly there are $3! = 6$. For every permutation of X, D, E we replace the occurrence of X by A, B, C and hence achieve the desired permutation.

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Thus the total number of ways is selecting a permutation of men (5! ways) times selecting a permutation of women (5! ways) and for each of these selections, there are two possibilities of combining them. Hence the final answer is $\boxed{2 \cdot 5! \cdot 5!}$.

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An r-permutation of n objects can be obtained by an r-combination of that set and then ordering the elements in each r-combination. Thus

$$P(n,r) = C(n,r) \cdot P(r,r) = C(n,r) \cdot r!$$

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2. Amongst these 12 people, 2 people will either be in the team or both wont be in the team. How many teams can you form?
3. Amongst these 12 people, 2 people will cannot work together so at most one of them can be in the team. How many teams can you form?
4. Suppose the 12 people consists of 5 men ans 7 women.
   - How many 5 person teams can be chosen that consist of three men and two women?
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The answer to first one simply $\binom{12}{5}$.

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If N denotes the total number of teams (with desired property), we can see that

$$N = N_1 + N_2$$

- N_1 = number of teams that contain both A and B.
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- For counting N_2: We need to select 5 people from 10 people (because we must be avoid A and B). Thus, $N_2 = \binom{10}{5} = 252$.

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Make sure that the calculations in the two different ways match

The previous approach was sum rule, this approach is using subtraction rule

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  Sol: We view it as a two step process. We select 1 man from the 5 men, this can be done in 5 ways. Now, amongst the remaining 11 people we select 4 people. This can be done in $\binom{11}{4}$ ways. Thus, total number of ways is $5 \cdot \binom{11}{4}$.

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  Calculate the exact answer and note it down.
  Revisit the solution and verify that this is indeed the answer.

  Checklist

  - Am I counting every way?
  - Am I double counting anything? That is, is the same way counted twice?

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In fact the above approach counts the same selection multiple times. Say the men are A, B, C, D, E.

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These two ways although leading to the same selection of team, are counted as two different ways.
Thus this is an incorrect solution to the above problem.

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Try the approach that uses the sum rule. Verify that you get the same answer as above.

Ex:

- How many pairs of two distinct integers chosen from the set $\{1, 2, \ldots, 101\}$ have a sum that is even?
- How many 16 length bit strings contain at least one 1? How many contain at most one 1?
- Suppose a dept. has 10 men and 15 women. How many ways are there to form a committee of six members if it must have the same number of men and women?
- In how may ways can we order the letters of the word MISSISSIPPI, to obtain distinguishable orderings?

How about these?

Ex:

- How many pairs of two distinct integers chosen from the set $\{1, 2, \ldots, 101\}$ have a sum that is even?
- How many 16 length bit strings contain at least one 1? How many contain at most one 1?
- Suppose a dept. has 10 men and 15 women. How many ways are there to form a committee of six members if it must have the same number of men and women?
- In how may ways can we order the letters of the word MISSISSIPPI, to obtain distinguishable orderings?

References: Section 6.3 [KR]