IITM-CS1200 : Discrete Mathematics for CS | Tutorial #5

Duration: 30 mins | Marks: 0

Instructions:

- This is a ungraded tutorial which was conducted online on moodle.

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1. How many pairs of subsets (S, T) of $\{1, 2, ..., n\}$ are there such that $S \cap T = \emptyset$?

2. The English alphabet has 26 letters of which 5 are vowels (A,E,I,O,U). How many seven letter words, with all letters distinct, can be formed that start with B, end with the letters ES, and have exactly three vowels? The "words" for this problem are just strings of letters and need not have linguistic meaning.

3. Determine the number of permutations of $\{1, 2, \ldots, 7\}$ in which at least one odd integer is in its natural position.

4. An exam has 4 multiple choice questions each having 3 options (a), (b), and (c). Every student attempting the exam, answers all 4 questions and marks exactly one answer for each of the four questions.

   Call the following event as (E): During evaluation the teacher finds that there are 3 answer sheets with exactly same answers for all the four questions.

   What is the smallest number of students in the class if the above event (E) must be guaranteed to occur?

5. We wish to compute the following sum.
   $S = \binom{16}{1} + 2 \cdot \binom{16}{2} + 3 \cdot \binom{16}{3} + \cdots + 16 \cdot \binom{16}{16}$
   Write the value of a which will allow us to compute S.
   $S = 16 \cdot a$

6. A prime number is called a "strange" if either it is a one digit prime or if each of the numbers obtained by removing its first digit or its last digit are themselves strange prime numbers. For instance 13 is not a strange prime number because after removing 3 we get 1 which is not a strange prime number.

   How many two digit strange prime numbers are there?

   Can there be a three digit strange prime number? (you will not be able to answer this here, but think of this offline).

7. Four boys participate in a running race. How many outcomes are possible for the race if there can be ties in the outcome of the race; however at most two boys can be tied at a position? That is, an outcome of the form 3 boys come second and one boy comes first is forbidden.

8. A domino is made up of two squares each of which is marked with one of one, two, three, four, five, or six spots or is left blank. How many different dominoes can we have?

   Note that the two squares of the dominoes may have the same number of spots.

9. What is the coefficient of x^8 in the expansion of $(x+2)^{11}$?

10. How many numbers must be selected from the set 1, 3, 5, 7, 9, 11, 13, 15 to guarantee that at least one pair of these numbers add up to 16?