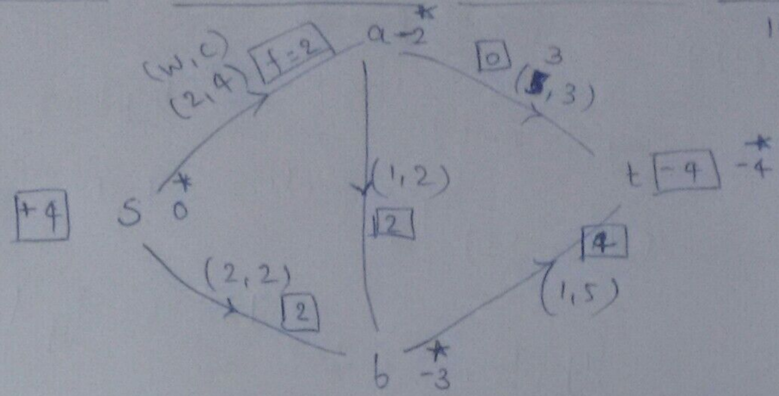
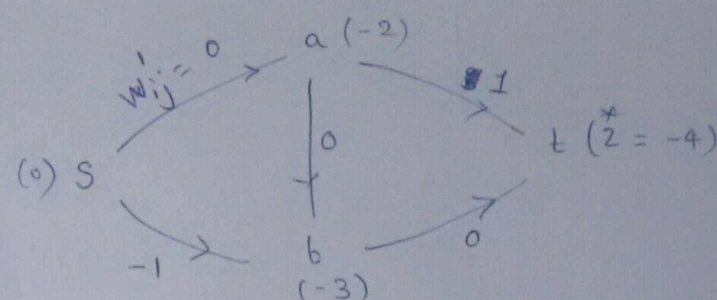
10-10-19



cost of flow = $2*2 + 2*2 + 2*1 + 4*1 = 14 = \sum_{i \in E} w_i f_i$

If we have opt sol^n for dual, $\overset{*}{z}$, then find reduced costs $w'_{ij} = w_{ij} - z_i + z_j$

Z-values in above fig : $\overset{*}{z}$

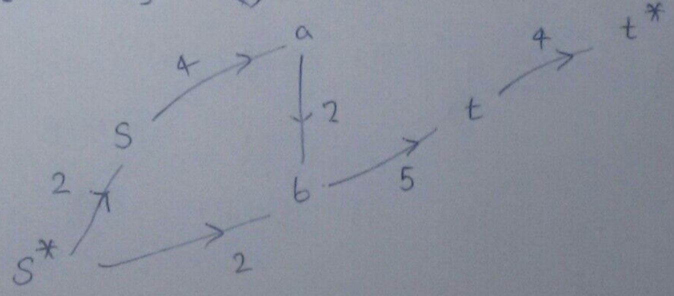


C.S. condition implies $f_{at} = 0$

C.S. — u — $f_{sb} = c_{sb} = 2$ & change

$b(s) = 2$, $b(b) = 2$ [$b(b) + f_{sb}$]
[$b(s) - f_{sb}$]

⇓



Reduction to max flow network

Q: If primal sol^n $\overset{*}{f_{ij}}$ are given, can we compute corresponding $\overset{*}{z}$ values?

Algo to compute min cost flow :

$f_{ij} = 0 \quad \forall (i, j) \in E$

$z_i = 0 \quad \forall i \in V$ →

f ← a particular edge f(i,j) = 0

while ($b(s) > 0$)

- Calculate reduced costs w'_{ij}
- Compute shortest path (need to be well defined) from s to all vertices v_i as d_i (distance)
- Set $z_i \leftarrow z_i - d_i \quad \forall v_i$
- Define admissible n/w wrt modified duals : G'
- Compute max flow f^* from s to t in G'
- Modify $b(s), b(t)$: $b(s) - f^*$, $b(t) + f^*$

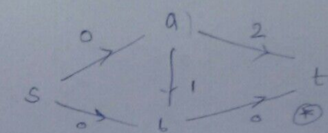
In 1st iteration, $w^*_{ij} = w_{ij} \quad \forall (i,j) \in E$

$d(s,a) = 2$, $d(s,b) = 2$, $d(s,t) = 3$

& $d(s,s) = 0$

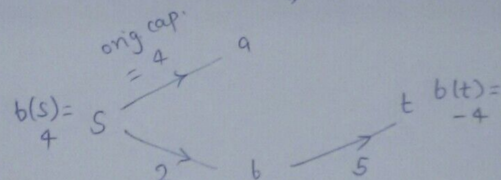
$z_s = 0$, $z_a = -2$, $z_b = -2$, $z_t = -3$

Admissible n/w :



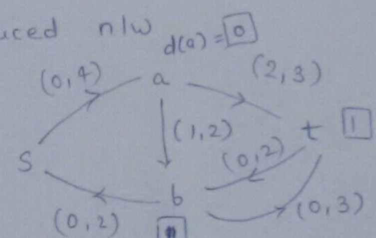
, no saturated edges i.e $w'_{ij} < 0$

⇓



max flow = 2 : s-b-t

Reduced n/w



$w \leftarrow w'_{ij}$ from admissible n/w (*)

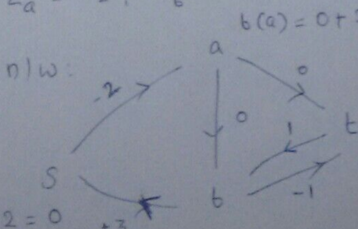
$c \leftarrow$ normal capacities in max flow reduced n/w

$b(s) = 4 - 2 = 2$, $b(t) = -4 + 2 = -2$

$z_s = 0$, $z_a = -2$, $z_b = -3$, $z_t = -4$

$b(a) = 0 + 2 = 2$

Admissible n/w :



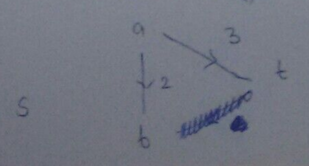
$b(s) = 2 - 2 = 0$

$f_{sa} = c_{sa} = \min\{4, 2\} = 2$

$f_{bt} = c_{bt} = 3$

$f_{bt} = 0$, $f_{bs} = 0$

⇓



No s-t path

$b(s) = 0$

terminate