11-10-19

\# $b(s) > 0$: Is this condition enough?

- What about $b(\cdot)$ @ vertices other than s & t? - $\circ \xrightarrow[<0]{} \circ$ generates $\underset{b(\cdot)-}{\circ} \longrightarrow \underset{b(\cdot)+}{\circ}^{*}$

\# S.P. : negative cycles?

- Are S.P.s well-defined?

<u>Claim A</u>: A feasible flow f is optimal iff $\exists$ some z values $w'_{ij} \geq 0$ for every edge in residual n/w of G(f).

<u>Claim B</u>: A feasible flow f is optimal iff G(f) does not admit a negative weight cycle.

($\rightarrow$) contrapositive. If $\exists$ a neg wt cycle $\rightarrow$ we can route at least one unit flow at smaller cost $\rightarrow \perp$ that f is optimal.

($\leftarrow$) TODO using : $\exists f^*$: opt then f & f^* have m cycles s.t. $\text{cost}(f^*) + \sum_{\text{cycle } C_i} \text{cost}(C_i) * \text{flow}(C_i) = \text{cost}(f)$

<u>Proof (A)</u> : for edge $\underset{d(i)}{\overset{i}{\circ}} \xrightarrow[w_{ij}]{} \underset{d(j)}{\overset{j}{\circ}}$

$d(j) \leq d(i) + w_{ij} \rightarrow w_{ij} + d(i) - d(j) \geq 0$

→ $w_{ij} \geq 0$ for each edge.

Q: Is this maintained throughout if initially all $w_{ij} \geq 0$

→ $w'_{ij}(z_i)$: old values to compute $d(i)$

$$z_i^{new} = z_i - d(i)$$

Argue: w'_{ij} (new) ≥ 0 given $w'_{ij}(z_i) \geq 0$.

→ $w'^{(new)}_{ij} = w_{ij} - z_i^{new} + z_j^{new}$

$$= w_{ij} - z_i + z_j + (d(i) - d(j))$$

$$= w'_{ij}(z_i) + (d(i) - d(j))$$

as long as $d(i) - d(j) \geq 0 \rightarrow w'^{(new)}_{ij} \geq 0$

& $w^{(old)}_{ij} \geq 0$