

$b(s) > 0$: Is this condition enough?

- What about $b(\cdot)$ @ vertices other than s & t ? - $\circ \xrightarrow{<0} \circ$ generates $\circ \xrightarrow{>0} \circ^*$
 $b(\cdot) -$ $b(\cdot) +$

S.P. : negative cycles?

- Are S.P.s well-defined?

Claim A: A feasible flow f is optimal iff \exists some z values $w'_{ij} \geq 0$ for every edge in residual n/w of $G(f)$.

Claim B: A feasible flow f is optimal iff $G(f)$ does not admit a negative weight cycle.
 (\rightarrow) contrapositive. If \exists a neg. wt cycle \rightarrow we can route at least one unit flow at smaller cost $\rightarrow \perp$ that f is optimal.
 (\leftarrow) TODO using: $\exists f^*$: opt then f & f^* have m cycles s.t. $\text{cost}(f^*) + \sum_{\text{cycle } C_i} \text{cost}(C_i) = \text{cost}(f)$

Proof (A): for edge $i \xrightarrow{w_{ij}} j$
 $d(j) \leq d(i) + w_{ij} \rightarrow w_{ij} + d(i) - d(j) \geq 0$

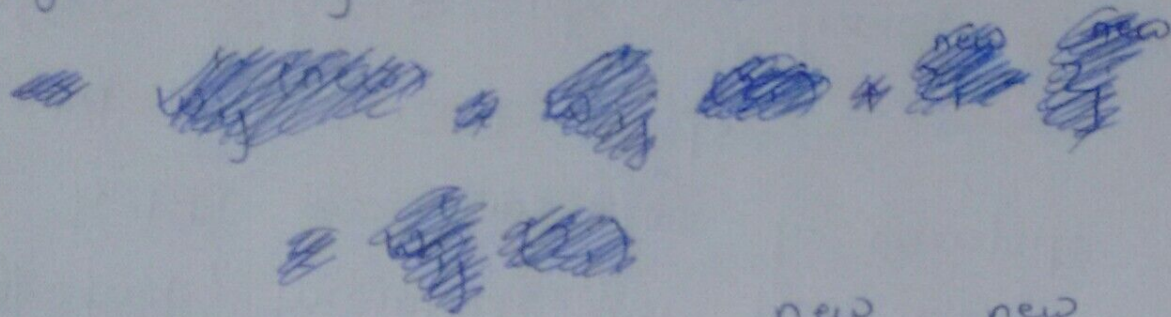
→ $w_{ij} \geq 0$ for each edge.

Q: Is this maintained throughout if initially all $w_{ij} \geq 0$

→ $w_{ij}^{\text{old}}(z_i)$: old values to compute $d(i)$

$$z_i^{\text{new}} = z_i - d(i)$$

Argue: $w_{ij}^{\text{new}} \geq 0$ given $w_{ij}^{\text{old}}(z_i) \geq 0$.



$$\rightarrow w_{ij}^{\text{new}} = w_{ij} + z_i^{\text{new}} + z_j^{\text{new}}$$

$$= w_{ij} + z_i + z_j + (d(i) - d(j))$$

$$= w_{ij}^{\text{old}}(z_i) + (d(i) - d(j))$$

as long as $d(i) - d(j) \geq 0 \rightarrow w_{ij}^{\text{new}} \geq 0$
& $w_{ij}^{\text{old}} \geq 0 \rightarrow$