sets $X_1 = X_2$. [$\Rightarrow$ Every S.M. has same size]

18-10-19

Instance with > 1 Stable M :

$a_1 : b_2\ b_1$	$b_1 : a_1\ a_2$
$a_2 : b_1\ b_2$	$b_2 : a_2\ a_1$

$M_A = \{(a_1, b_2), (a_2, b_1)\}$, $M_B = \{(a_1, b_1), (a_2, b_2)\}$

Both are stable & have same set of vertices matched.

| Popular Matchings —

Votes for above matchings:

	M_A	M_B
a_1	✓	
a_2	✓	
b_1		✓
b_2		✓

votes (M_1) V_1 # votes (M_2) V_2

$V_1 > V_2$: M_1 is more popular than M_2

& so on ... (= , <)

M is popular if $\nexists$ M' s.t. M' beats M ie

$\nexists$ M' s.t. votes (M') > votes (M)

Q: If $M_1 > M_2$, $M_2 > M_2$, does $M_1 > M_3$?

Q: If not: whats the implication?

A: $\exists$ instances with no popular matching.

Q: Is stable M popular?

i.e. stable M is not beaten by any other arbitrary M'?

Q : How to show M is not popular ?

A : Show another M' that beats M.

Q : How to show M is popular ?

- Do we need to check all possible (exp. # of) matchings ?

Is stable M popular ?

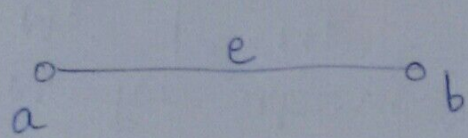
: Let M_S : stable M, M : any other matching.

Look at $M_S \oplus M$:

If $e \in M_S \cap M$: both endpts are indifferent.

$\longrightarrow$ Ignore such edges.

Consider $e \in M \setminus M_S = (M \oplus M_S) \cap M$



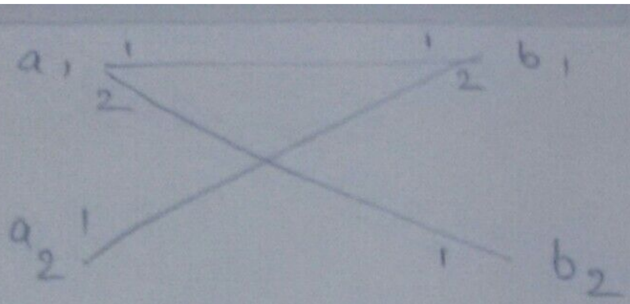
What can be votes of a & b ?

If a prefers M over M_S $\rightarrow$ b prefers M_S over M [o/w (a, b) blocks M_S]

$\Rightarrow$ For each vote M gets, there is at least 1 vote that M_S gets.

If v not matched in M $\rightarrow$ vote by v to $M_S \geq$ vote by v to M

$\Rightarrow$ votes (M_S) $\geq$ votes (M) $\Rightarrow$ M_S is popular.



$M_S = \{(a_1, b_1)\}$

$M' = \{(a_2, b_1), (a_1, b_2)\}$

Votes:

	M_S	M'
a_1	✓	
a_2		✓
b_1	✓	
b_2		✓

M_S & M' both popular but M' is not stable

$\Rightarrow$ [stable $\rightarrow$ popular but popular $\not\Rightarrow$ stable]

Also, $|M'| > |M_S|$ ie $\exists$ popular M larger than stable M.

Q: Compute a maximum size popular M.

: Only interesting if bipartite graph is incomplete

Ties: B.P (Blocking Pair) (a, b) if a & b both strictly prefers each other i.e. b, M(a) : not in tie & a, M(b) : not in tie.

$\Rightarrow$ Weakly stable matchings.

Q: Do weakly stable matchings always exist?

A: Yes : ~~~~ Break ties arbitrarily & compute S.M. $\rightarrow$ it's weakly stable in original G.

$M_1 = \{(a_1, b_1)\}$

$M_2 = \{(a_1, b_2), (a_2, b_1)\}$

↳ not stable but weakly stable.

Both weakly stable but of different sizes

<u>Q</u>: Compute a maximum size weakly stable M.