(CS24) (Quiz 1)

Directed graph $G(V,E)$, c_e (capacities) ≥ 0 $\forall e \in E$
& 2 vertices s,t : given

Goal: Max flow (max s-t flow)

LP1: $\max \sum_{e:\, e \text{ out of } s} f_e$ s.t. $0 \leq f_e \leq c_e \;\forall e \in E$,

$$\sum_{v:(u,v)\in E} f(u,v) = \sum_{v:(v,u)\in E} f(v,u) \quad \forall u \in V-\{s,t\}$$

(Assume: No incoming edges to s)

(Assume: No directed cycles in G)

Another way to write LP:

Let $\mathcal{P}$ = {set of s-t paths (simple) in G}

For each $P \in \mathcal{P}$, we let x_P be a variables,
s.t. x_P denotes flow from s to t
along P, $x_P \geq 0$

LP2: $\max \sum_{P \in \mathcal{P}} x_P$ s.t. $x_P \geq 0$,

$$\forall e \in E, \quad \sum_{P\in\mathcal{P}:\, e \text{ is on } P} x_P \leq c_e$$

is redundant

$$\left[\forall u \in V-\{s,t\}, \quad \sum_{P\in\mathcal{P}} \sum_{(u,v):(u,v)\in P} x_P = \sum_{P\in\mathcal{P}} \sum_{(v,u)\in P} x_P\right]$$

Show LP1 & LP2 are equivalent:

(a) For a feasible sol^n f to LP1 → get an equivalent feasible sol^n x for LP2.

Algo: (*) - Delete edges if $f_e = 0$ for that edge.
- Find an s-t path - P. If no path, exit.
- For all edges $e \in P$, set $x_P = \epsilon$,
set $f_e = f_e - \epsilon$ where
$\epsilon = \min_{e\in P} f_e$
- Repeat (*)

iterations → ≤ Flow value, better bound: ≤ $m = |E|$

Dual for LP2 : $y_{u,v}$: per edge

$$\min \sum_{e=(u,v)\in E} y_e c_e \quad \text{s.t.} \quad y_e \geq 0 \;\; \forall e \in E,$$

$$\sum_{e=(u,v)\in P} y_e \geq 1 \qquad \forall P \in \mathcal{P}$$

Interpretation : select edges s.t. for each path, at least 1 edge is selected [cut] & total capacity is minimum [min.cut].

Dual for LP 1 : TODO