4-10-19

Min Cost Flow

Given : Directed $G(V,E)$,

$\forall e$: $c(e) \geq 0$ → capacity

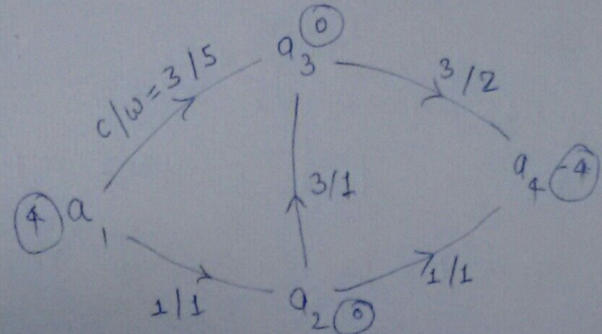
$w(e)$ → (weight / cost) / unit flow

$\forall v$: $b(v)$, $b(v) > 0$ → supply node

$b(v) < 0$ → demand node

$b(v) = 0$ → neither sup / dem. node.

Ex :



LP^* : $f(e) \geq 0$, $f(e) \leq c(e)$ → $\forall e \in E$

$$\sum_{(v,w)\in E} f(v,w) - \sum_{(u,v)\in E} f(u,v) = b(v) \ \forall v \in V$$

objective : $\min \sum_{e\in E} w(e) f(e)$

i.e. minimize the cost among all feasible flows.

Is this a generalization of .

1) MAX FLOW problem

2) Shortest s-t paths on directed graph.

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(1) Q : Is there a feasible flow in a given Min Cost flow instance ?

⇒ Necessary condition : $\sum_{v\in V} b(v) = 0$ but not sufficient :

$\boxed{2}\ o \xrightarrow[\text{cap}=+1]{} o\ \boxed{-2}$: infeasible though $\sum_v b(v) = 0$

A : Add s^*, t^* For $v : b(v) > 0$, add (s^*, v) with capacity $c(s^*, v) = b(v)$. For $v : b(v) < 0$, add (v, t^*) edge with capacity $-b(v)$.

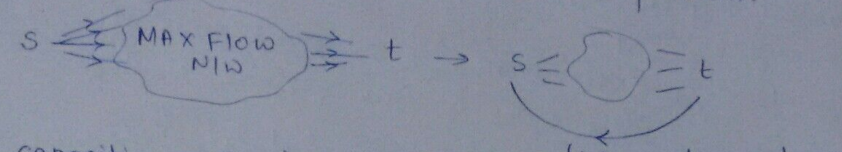
Ignore cost & run a max flow.

If max flow $= \sum_{v:(s^*,v)\in E} b(v)$ then it's a feasible flow in given Min cost flow instance

e.g. $\boxed{s} \xrightarrow{1/2} o \xrightarrow{1/+1} o \xrightarrow{1/2} \boxed{t}$

max flow $1 \neq \sum_{(s,v)\in E} b(v)$

Q. Max Flow < Min cost flow problem.



capacities remain same, cost (e) = 0 } for MAX FLOW edges.

b(v) = 0 ∀v incl. s & t.

If f_1 & f_2 are 2 feasible flows in MAX FLOW N/W s.t. $f_1 > f_2$, we get $-f_1$ & $-f_2$ costs so MIN COST choses $-f_1$.

TODO: If cost of internal edge = -1 & cost (t - s) = 0 does it work?

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(2)

$\boxed{1}$ s c / w = 1/w for internal edge t $\boxed{-1}$

Path of weight w_1 ↔ cost w_1 unit flow.
↳ TODO.

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(3) Max flow w/ costs:

- Find amongst all max flows, the one that is min-cost.

TODO: Write dual for LP*.

Successive S.P. Algo. for Min cost flow problem:

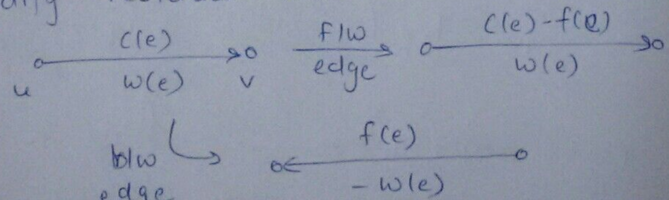
while all b(v)s are not zero,

Find a simple s-t path [where s = any v : b(v) > 0, t = any v : b(v) < 0] of shortest distance / smallest weight.

Send appropriate flow f along this path.

Update b(v)'s.

Modify residual n/w as follows:



Repeat.

Next step: Get rid of negative weights.

9-10-19