CS6130 LP 12-9-19

- Formulate problems of interest as LPs
- How to solve LPs ?
  - Which LPs can be efficiently solved ?
- Certificate of optimality (Duality)
- Examples , algorithms

Product 1 & Product 2

profit per unit : P1 : 1 , P2 : 6

Every day sell : P1 : ≤ 200 , P2 : ≤ 300

Production Total : ≤ 400 (together)

Goal: Decide how many units of P1 & P2 to produce to maximize the profit ?

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x_i = # of product units of product P_i

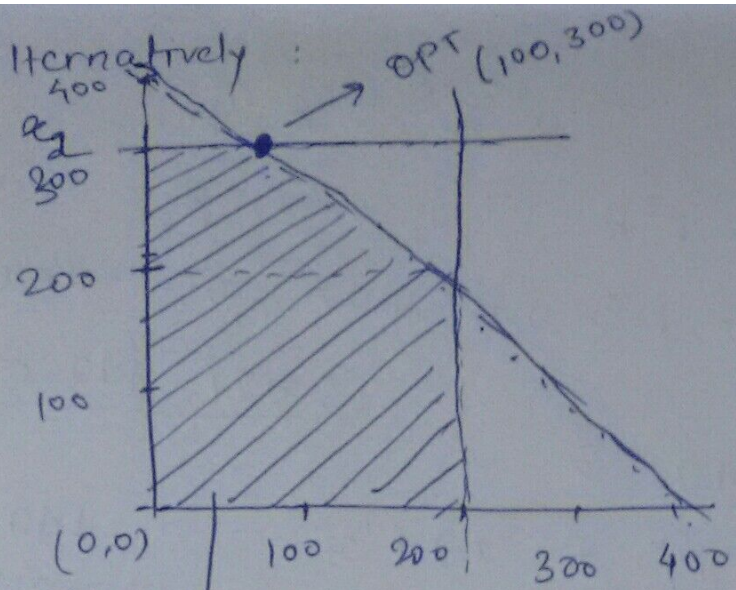
$x_1 \leq 200$, $x_2 \leq 300$, $x_1 + x_2 \leq 400$ [const-raints]

maximize $x_1 + 6x_2$ [optimization function]

OPT : P1 units = 100 , profit = 1900

P2 units = 300 ,

Alternatively :



- Positivity constraint assumed
- Integrity constraint assumed

$x_1, x_2 \geq 0$, integers

feasible region : Always a convex region if constraints are linear [not proved]

<u>Known</u> : $\exists$ optimum @ corner points of feasible region.

$\approx$ Geometrically, move the line (obj) $x_1 + 6x_2$ in the direction which increases the obj. f^n

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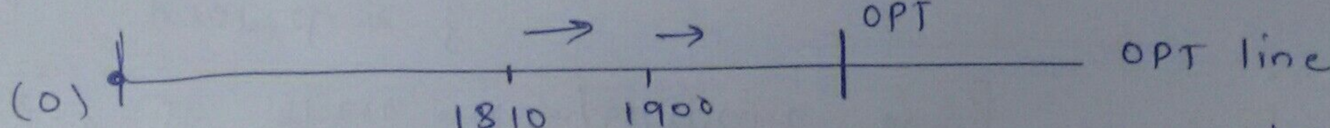
<u>Certificate</u> <u>of</u> <u>OPT</u>

if $x_1' = 10$, $x_2' = 300$, VAL = 1810



OPT $\geq$ 1900, OPT $\geq$ 1810 $\rightarrow$ Lower bounds on OPT

How to get <u>upper</u> <u>bound</u> ?

$x_1 \leq 200$, $x_2 \leq 300$ $\rightarrow$

$x_1 + 6x_2 \leq 2000$ ie OPT $\leq$ 2000 } $\exists$ gap between 1900 & 2000

How to come up with inequality that upper bounds OPT as tightly as possible!

$x_1 + 6x_2 \leq 1900 \longrightarrow$ goal.

Q: Is there a systematic way for this procedure?

$$
\left.
\begin{aligned}
x_1 &\leq 200 \quad \ldots\ * \alpha_1 \\
x_2 &\leq 300 \quad \ldots\ * \alpha_2 \\
x_1 + x_2 &\leq 400 \quad \ldots\ * \alpha_3
\end{aligned}
\right\} \alpha_i \geq 0
$$

$$
\left.
\begin{aligned}
\alpha_1 x_1 &\leq 200\alpha_1 \\
\alpha_2 x_2 &\leq 300\alpha_2 \\
\alpha_3 x_1 + \alpha_3 x_2 &\leq 400\alpha_3
\end{aligned}
\right\}
\begin{aligned}
&\alpha_1 + \alpha_3 \geq 1 \quad (*) \\
&\alpha_2 + \alpha_3 \geq 6
\end{aligned}
$$

new obj: to get tight upper bound on original OPT $\rightarrow$

minimize $200\alpha_1 + 300\alpha_2 + 400\alpha_3 \longrightarrow \overline{OPT}$

(*) Because we want upper bound on original

$OPT \equiv x_1 + 6x_2 \leq (\alpha_1 + \alpha_3)\, x_1 + (\alpha_2 + \alpha_3)\, x_2$

$\leq \overline{OPT}$

New opt / LP:

$$
\left.
\begin{aligned}
\text{min. } & 200\alpha_1 + 300\alpha_2 + 400\alpha_3 \\
\text{s.t. } & \alpha_1 + \alpha_3 \geq 1, \quad \alpha_2 + \alpha_3 \geq 6, \\
& \alpha_1, \alpha_2, \alpha_3 \geq 0
\end{aligned}
\right\} \text{Dual of original primal}
$$

variables in dual = # of constraints in primal.