13-9-19

MAXIMUM MATCHING

x_e = variable per edge

$$= \begin{cases} 0 & \text{if } e \text{ is not in } M \\ 1 & \text{if } e \text{ is in } M \end{cases}$$

maximize $\sum_{e \in E} x_e$

s.t. $\sum_{e:\, e \text{ is incident on } v} x_e \leq 1$, for all $v \in V$

Linear $\longrightarrow$ $1 \geq x_e \geq 0$, x_e : integers. $\forall e \in E$

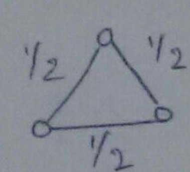
Not linear $\longrightarrow$ or $x_e \in \{0, 1\}$ $\forall e \in E$

ILP : Integer LP with $x_e \in \{0, 1\}$ constraint.

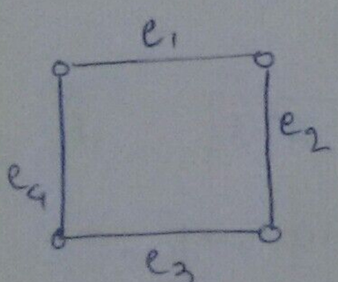
Q: How are LP & ILPs related? [for a fixed problem]

ILP feasibility $\rightarrow$ LP feasibility

Example where LP is feasible but ILP is not.



$LP_{-OPT} = \frac{3}{2}$ but $|M^*| = 1 = ILP_{-OPT}$



ILP (feasible) : $x_{e_1} = 1$, $x_{e_3} = 1$
also LP feasible

LP (feasible) : $x_{e_1} = x_{e_2} = x_{e_3} = x_{e_4} = 1/2$
but not ILP feasible

Q: In bipartite G, can $LP_{obj} > ILP_{obj}$?

A: $|ILP = LP|^*$ for bipartite G, [i.e.* extreme points are same in both sets]
(proof pending)

Q: For general case, can we add more constraints s.t. $|ILP = LP|$?

VERTEX COVER

$$x_v = \begin{cases} 1 & \text{if } v \in V.C. \\ 0 & \text{o/w} \end{cases}$$

minimize $\sum_{v \in V} x_v$

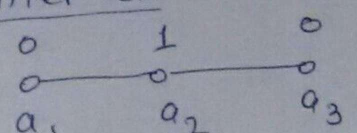
s.t. $x_u + x_v \geq 1 \quad \forall (u,v) \in E$

$x_u \in \{0,1\} \quad \forall u \in V$

Exercise: $\sum_{u \in N(v)} x_u \geq 1 \quad \forall v \in V$

Is this constraint enough for feasibility?

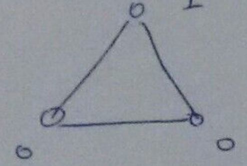
Counter-ex:



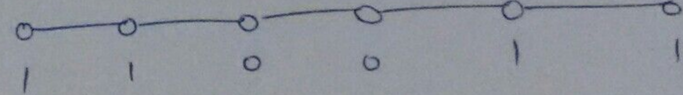
valid VC, but constraint $\sum_{u \in N(a_2)} x_u \geq 1$ violated

Even if we let N(v) to include v itself,

Counter-ex:



shows that constraints are not necessary



shows that original constraints (w/o including 'v' in N(v)) are not sufficient.