24-9-19

Weighted Vertex Cover (min)

Cardinality min V.C

- Maximal matching M
- OPT (VC) $\geq |M|$
- Pick both endpts of edges in M
- ~~[illegible]~~
- Algo (VC) $= 2|M| \leq 2\,OPT(VC)$
- 2-approximation algorithm for VC

General primal-dual [canonical form]

... PRIMAL

min $c_1x_1 + \dots + c_nx_n$

s.t. $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \geq b_1$

m constraints

$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \geq b_m$,

$x_1, \dots, x_n \geq 0$

... DUAL

max $\sum_{i=1}^{m} y_i b_i$

s.t. $y_i \geq 0$, $\forall i = 1$ to m,

$a_{11}y_1 + a_{21}y_2 + \dots + a_{m1}y_m \leq c_1$

$a_{1n}y_1 + a_{2n}y_2 + \dots + a_{mn}y_m \leq c_n$

i.e. $\sum_{i=1}^{m} y_i a_{im} \leq c_n$

Compare primal & dual objectives

$\sum_{i=1}^{n} c_i x_i \geq \sum_{i=1}^{m} b_i y_i$: Weak-duality theorem

At optimal solution, they are $=$: Strong-duality theorem

If $(x_i > 0) \mid (y_j > 0) \rightarrow$ dual constraint / primal constraint is tight

Intuitively, if $c_1 > a_{11}y_1 + a_{21}y_2 + \dots + a_{m1}y_m$ & $x_1 > 0$ then LHS $= c_1x_1$ is strictly greater than RHS $= (a_{11}y_1 \dots)x_1$. i.e. if it's dual constraint is slack then LHS > RHS $\rightarrow$ contradicts strong duality

Claim: If $\vec{x}, \vec{y}$ are optimal, then

either for each $j = 1$ to m, $y_j = 0$ or

$$a_{j1} x_1 + a_{j2} x_2 + \dots + a_{jn} x_n = b_j$$

& for each $i = 1$ to n, either $x_i = 0$

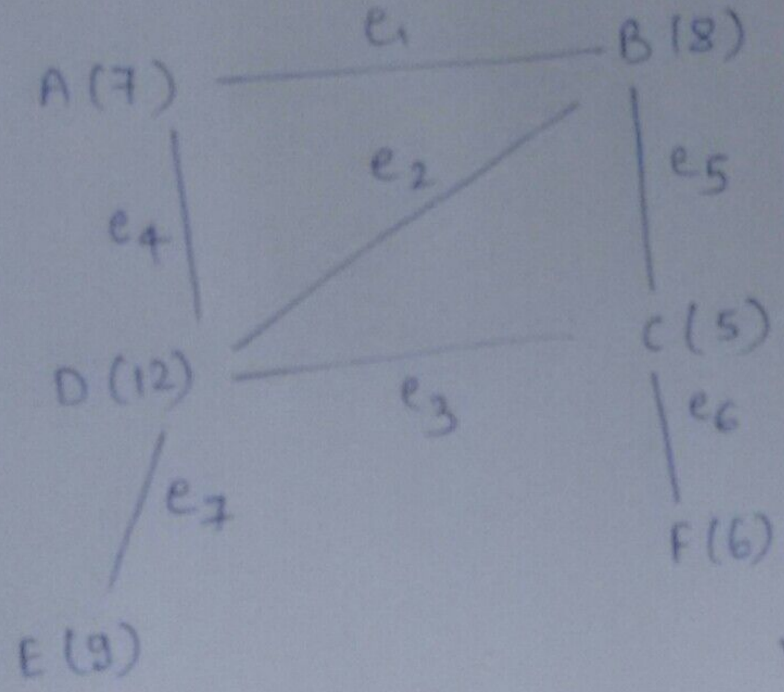
or $a_{i1} y_1 + a_{i2} y_2 + \dots + a_{im} y_m = c_i$

↳ Complementary slackness conditions

Dual of weighted VC: $(c_v \geq 0 \ \forall v \in V)$

$\sum_{e \in E} y_e$ maximize s.t.

$\forall v \in V \quad c_v \geq \sum_{e \in E \text{ is incident on } v} y_e \quad , \quad y_e \geq 0, \forall e \in E$



(i) $e_1 \leftarrow 7$, $VC = \{A\}$

Fro $= \{e_1, e_4\}$

(ii) $e_7 \leftarrow 9$, $VC = \{A, E\}$

Fro $= \{e_1, e_4, e_7\}$

(iii) $e_3 \leftarrow 3$,

$VC = \{A, E, D\}$

Fro $= \{e_1, e_4, e_7, e_2, e_3\}$

$VC = \{A, E, D, C\}$

(iv) $e_6 \leftarrow 2$,

Fro $= \{e_1, e_4, e_7, e_2, e_3, e_5, e_6\}$

Done.

$VC = \{A, E, D, C\}$, Primal $= 33$

Dual $= \{7, 0, 3, 0, 0, 2, 9\}$, Dual $= 21$