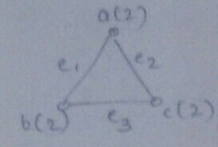
26-9-19




$e_1 \leftarrow 2$ a & b tight
VC = {a, b} OR
(i) $e_1 \leftarrow 2$ a becomes tight. VC = {a},
(ii) b is tight, VC = VC ∪ {b} | fro = {$e_1$, $e_2$}
@ $y_{e_3} = 0$ itself VC = {a, b} | fro = {$e_1$, $e_2$, $e_3$}

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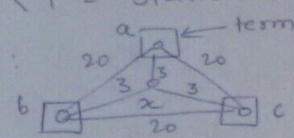
Steiner Tree Problem

G(V, E) (Undirected), $c(e) \geq 0$, T (terminal),
$T \subseteq V$.
Goal: A min-cost Tree that connects vertices in T.
$V \setminus T$ = Steiner vertices (helper vertices)

Ex:




T = {a, b, c}, Steiner = {x}
Opt = {xa, xb, xc}

Easy case: i) If V = T, → MST (not NP Hard)
ii) |T| = 2 → S.P (-u-)

Q. How does solution (OPT) look like w.r.t T set & S set?

- Leaves are always from T set.

If G: Complete Graph w/ metric property
OPT does not contain steiner vertices
Incorrect claim ↑ → In earlier ex: c(ab) = c(bc) = c(ac) = 5 : counter-ex.

Imagine OPT for such G. Double the edges & get Euler tour. Then do short-circuiting. This gives a tour only on terminal vertices, with cost at most 2 * OPT cost.
⇒ This is a H.C. on Terminal vertices.
⇒ Remove any edge to make it a tree.
So, ∃ S.T on terminal vertices s.t
cost (S.T.) ≤ 2 * OPT.

Algo: - Compute MST on T, cost (MST) ≤ cost (ST)
- Do above process

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G is given. → G' : complete & metric property with some relationship betn G & G'
T → T' (obvious)

Proposal to construct G':

Find All-pairs SP on $G$, to get $d(x,y)$

In $G'$, $c(x,y) = d(x,y)$

[$G'$ = metric closure of graph]

$OPT(G) \geq opt(G')$ because

cost (any tree in $G$) $\geq$ cost (that tree in $G'$)

But, for any tree $T'$ in $G'$, we have

cost ($T'$ in $G$) $\leq$ cost ($T'$ in $G'$)

$\Rightarrow opt(G) \leq opt(G')$

i.e. $opt(G) = opt(G')$

<u>Algo for G:</u>

- Compute $G'$
- Run previous 2-approx algo.
- Same opt on G.

<u>LP for Steiner tree Problem</u>

$\min \sum_e x_e c_e$ s.t. $x_e \geq 0$,

$\left[\forall v \in T, \sum_{e \in \delta(v)} x_e \geq 1\right]$, → redundant

Every T-separating cut has at least 1 edge crossing the cut.

i.e $\sum_{e:\, e \text{ is crossing } S,\, V \setminus S} x_e \geq 1$ where $(S, V\setminus S)$ s.t. $S \cap T \neq \phi$, $V \setminus S \neq \phi$ (or equiv. $T \supset T' \,\&\, T' \neq \phi$) where $T' = T \cap S$