**Definition** An argument is a sequence of propositions that ends with a conclusion. All but the last statements are called premises. An argument is **valid** if the truth of the premises implies that the conclusion is valid.

**Example**

If it rains, there will be no class.

It rains.

∴ There is no class.

**Modus ponens**

\[ p \rightarrow q \]

\[ p \]

∴ \[ q \]
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Example  Modus ponens

If it rains, there will be no class.  \( p \rightarrow q \)
It rains.  \( p \)

\[ \therefore \text{There is no class.} \]  \( \therefore q \)
Example

If a number is even, then its square is even.
6 is an even number.

∴ 36 is even.
Rules of inference

Example

If a number is even, then its square is even.
6 is an even number.

∴ 36 is even.

Universal modus ponens

\[ \forall x \ P(x) \rightarrow Q(x) \]
\[ P(a) \]

for an arbitrary “a” in the domain.

∴ Q(a)
All healthy people eat an apple a day. Tarak is not healthy.

∴ Tarak does not eat an apple a day.
Example

All healthy people eat an apple a day.
Tarak is not healthy.

∴ Tarak does not eat an apple a day.
Example

All healthy people eat an apple a day.
Tarak does not eat an apple a day.

∴ Tarak is not healthy.
Example

All healthy people eat an apple a day.
Tarak does not eat an apple a day.

∴ Tarak is not healthy.

Universal modus tollens

\[ \forall x \ P(x) \rightarrow Q(x) \]
\[ \neg Q(a) \]

for an arbitrary “a” in the domain.

∴ \[ \neg P(a) \]
Rules of inference

Example

If the box is red, it contains apples.
If the box contains apples, it is heavy.

\[
\begin{align*}
\text{p} & \rightarrow \text{q} \\
\text{q} & \rightarrow \text{r} \\
\therefore & \text{p} \rightarrow \text{r}
\end{align*}
\]
Example

If the box is red, it contains apples.
If the box contains apples, it is heavy.

∴ If the box is red, it is heavy.
Example

If the box is red, it contains apples.
If the box contains apples, it is heavy.

∴ If the box is red, it is heavy.

Hypothetical syllogism

\[ p \rightarrow q \]
\[ q \rightarrow r \]

∴ \[ p \rightarrow r \]
Example

The box is red or it contains fruits.
The box does not contain fruits

∴ The box is red.

Disjunctive syllogism

\( p \lor q \)
\( \neg q \)
\( \therefore p \)
Example

The box is red or it contains fruits.
The box does not contain fruits

∴ The box is red.
Rules of inference

Example

The box is red or it contains fruits.
The box does not contain fruits

∴ The box is red.

Disjunctive syllogism

\[ p \lor q \]
\[ \neg q \]

∴ \[ p \]
Example

The box is red or it contains fruits.
The box does not contain fruits or apples are blue.

∴ The box is red or apples are blue.
Example

The box is red or it contains fruits.
The box does not contain fruits or apples are blue.

∴ The box is red or apples are blue.
Example

The box is red or it contains fruits.
The box does not contain fruits or apples are blue.

∴ The box is red or apples are blue.

Resolution

\[ p \lor q \]
\[ \neg p \lor r \]

∴ \[ q \lor r \]
Is this a valid argument?

Cheap food is not good.
Therefore good food is not cheap.

1. $\forall x \ (\text{cheap}(x) \rightarrow \neg \text{good}(x))$ premise
Cheap food is not good.
Therefore good food is not cheap.

1. \( \forall x \ (\text{cheap}(x) \rightarrow \neg \text{good}(x)) \) premise
2. \( \text{cheap}(a) \rightarrow \neg \text{good}(a) \) for an arbitrary food item “a”
3. \( \text{good}(a) \rightarrow \neg \text{cheap}(a) \) using (2) and modus tollens
4. \( \forall a \ (\text{good}(a) \rightarrow \neg \text{cheap}(a)) \) using (3) and univ. gen.
Is this a valid argument?

Cheap food is not good.
Therefore good food is not cheap.

1. $\forall x \ (cheap(x) \rightarrow \neg good(x))$ premise
2. $cheap(a) \rightarrow \neg good(a)$ for an arbitrary food item “a”
3. $good(a) \rightarrow \neg cheap(a)$ using (2) and modus tollens
4. $\forall a \ (good(a) \rightarrow \neg cheap(a))$ using (3) and univ. gen.

Caution! To use universal generalization it is important not to assume anything extra about “a”.

All people in Chennai read “The Hindu”. If you are not well read or you do not know about politics, then you do not read “The Hindu”. Ramesh either stays in Bangalore or he stays in Chennai. All people in Bangalore read “Deccan Herald”. Ramesh knows about politics but is not well read.
Where does Ramesh stay?

- $C(x)$: $x$ stays in Chennai.
- $B(x)$: $x$ stays in Bangalore.
- $H(x)$: $x$ reads Hindu.
- $D(x)$: $x$ reads Deccan Herald.
- $R(x)$: $x$ is well-read.
- $P(x)$: $x$ knows about politics.
All people in Chennai read “The Hindu”. If you are not well read or you do not know about politics, then you do not read “The Hindu”. Ramesh either stays in Bangalore or he stays in Chennai. All people in Bangalore read “Deccan Herald”. Ramesh knows about politics but is not well read. Where does Ramesh stay?

\[
\begin{align*}
C(x) &: \ x \text{ stays in Chennai.} \\
B(x) &: \ x \text{ stays in Bangalore.} \\
H(x) &: \ x \text{ reads Hindu.} \\
D(x) &: \ x \text{ reads Deccan Herald.} \\
R(x) &: \ x \text{ is well-read.} \\
P(x) &: \ x \text{ knows about politics.}
\end{align*}
\]

1. \( \forall x (C(x) \to H(x)) \).
2. \( \forall x (R(x) \lor P(x) \to H(x)) \).
3. \( C(\text{Ramesh}) \lor B(\text{Ramesh}) \).
4. \( \forall x (B(x) \to D(x)) \).
5. \( P(\text{Ramesh}) \land \neg R(\text{Ramesh}) \)
Tony, Mike, and John belong to a Club. Every club member is either a skier or a mountain climber or both. No mountain climber likes rain, and all skiers like snow. Mike dislikes what Tony likes and likes whatever Tony dislikes. Tony likes rain and snow. Is there a member of the Club who is a mountain climber but not a skier?

- $C(x)$: $x$ is a club member
- $S(x)$: $x$ is a skier.
- $M(x)$: $x$ is a mountain climber
- $L(x, y)$: $x$ likes $y$. 