CS6015 : Linear Algebra and Random Processes
Tutorial #1

Deadline: None

- This tutorial covers topics already covered in class as of 18th September 2020.
- While this is optional, it is strongly recommended that students solve this tutorial.
- Questions marked with an asterisk are hard questions which thoroughly test your concepts.

Name :
Roll Number :

1. Consider the following system of linear equations

$$
\begin{aligned}
a_1x_1 + b_1y_1 + c_1z_1 &= 1 \\
a_2x_2 + b_2y_2 + c_2z_2 &= 0 \\
a_3x_3 + b_3y_3 + c_3z_3 &= -1
\end{aligned}
$$

Each equation represents a plane, so find out the values for the coefficients such that the following conditions are satisfied:

1. All planes intersect at a line
2. All planes intersect at a point
3. Every pair of planes intersects at a different line.

Solution:

2. Consider a system of linear equations which has k linear equations in n variables, written in matrix form as $AX = Y$
Give proof or counter example for the following statements:

1. If $n = k$, there is always at most one solution.
2. If $n > k$, you can always solve $AX = Y$.
3. If $n < k$, then for some Y, there is no solution for $AX = Y$

Solution:

3. Consider the system of linear equations:

$$\begin{aligned} x + ky &= 1 \\ kx + y &= 1 \end{aligned}$$

Find values of k for which the system has:

1. no solution
2. exactly 1 solution, and find the solution
3. infinitely many solutions

Solution:

4. Solve the following system of linear equations using Gaussian Elimination:

$$\begin{aligned} 2x_1 - 3x_2 + x_3 - 2x_4 &= 3 \\ -2x_1 + 3x_2 - x_3 + 4x_4 &= -1 \\ x_1 + 3x_2 + 3x_3 + 2x_4 &= -5 \\ x_1 + 6x_2 + 4x_3 + 7x_4 &= -5 \end{aligned}$$

Solution:

5. Consider the matrix A,

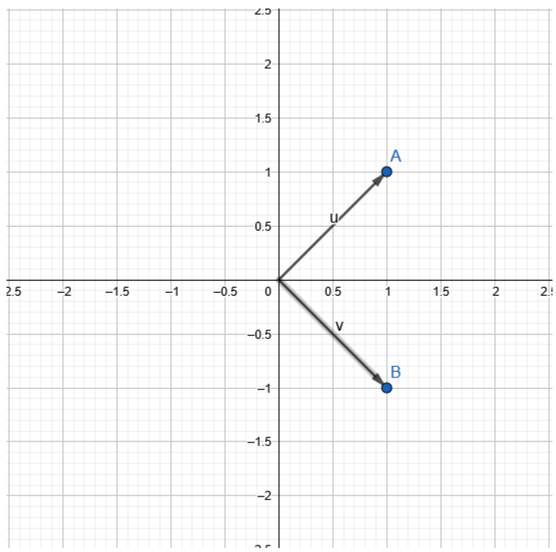
$$\begin{bmatrix} 3 & 1 & 0 & 0 \\ 2 & 2 & -1 & -1 \\ -3 & 1 & -2 & 2 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

Can the matrix A, be decomposed into a product of Elementary matrices as shown below $A = E_1 * E_2 * E_3 * ... * E_k$
If yes, find out the elementary matrix factorisation. If no, state the reason why it cannot be done.

Solution:

6. * Consider a 2 dimensional space which has the following two vectors:

$$u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



The generic form of their linear combination is $c = pu + qv$ where, p and q are scalar coefficients of the vectors u and v respectively. The result is also a vector in the 2 dimensional space.

1. For $p, q \in \mathbb{R}$, if we plot each possible vector c as a point in this 2 dimensional space, what is the resultant object/space that we will get?
2. Now if we restrict the values of p and q as $p, q \in [-1, 1]$, and then plot every possible vector c, what is the resultant object/space that we will get? Can you describe this object geometrically or algebraically? If yes, do so.

Solution:

7. * Find a 2 X 2 matrix A ($A \neq I$), such that $A^5 = I$.

Solution:

8. * Is Transpose operation a valid Linear Transformation? Provide an explanation for your answer.

Solution: