CS6015: Linear Algebra and Random Processes

Quiz - 1

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Date : Aug-13, 2019 **Duration** : 35 minutes

INSTRUCTIONS: Answers should be given with proper justification. Please use rough sheets for any calculations *if necessary.* Please **DO NOT** submit the rough sheets. Please DO NOT use pencil for writing the answers.

Assume standard data whenever you feel that the given data is insufficient. However, please do quote your assumptions explicitly.

1. True or False? Answer any five.

   Note: 2 *marks for the correct answer and* $-\frac{1}{2}$ *for the wrong answer.*

   (a) If A, B, C are matrices, and $AC = BC$, then $A = B$.

   Solution: False.

   (b) After Gaussian elimination, if every column of matrix A has a pivot, then $Ax = b$ is solvable for every b.

   Solution: False.

   (c) The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is elementary.

   Solution: False.

   (d) $\{(x, y) \mid x^2 + y^2 \leq 1, x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$.

   Solution: False.

   (e) If v_1, v_2, v_3, v_4 are linearly independent, then $v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4 + v_1$ are linearly independent.

   Solution: False.

   (f) If v_1, v_2, v_3, v_4 are linearly independent, then $v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4 - v_1$ are linearly independent.

   Solution: True.

2. Apply Gaussian elimination on the matrix A given below, and then express each non-pivot column as a combination of those with pivots.

$$A = \begin{bmatrix} 1 & 2 & 3 & 3 \\ 2 & 4 & 6 & 9 \\ 2 & 6 & 7 & 6 \end{bmatrix}.$$

   Hint: Use the reduced row echelon form. (10 marks)

Solution: Reduced row echelon form is

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let c_1, c_2, c_3, c_4 denote the columns of the matrix above. Then, c_1, c_2, and c_4 are pivot columns, and the non-pivot column c_3 can we written as

$$c_3 = 2c_1 + \frac{1}{2}c_2.$$