CS6015: Linear Algebra and Random Processes
Quiz - 3
Course Instructor : Prashanth L.A.
Date : Sep-5, 2017 **Duration** : 30 minutes

Name of the student :
Roll No :

INSTRUCTIONS: For true/false questions, you do not have to justify the answer. For the rest, provide proper justification for the answers. Please use rough sheets for any calculations *if necessary.* Please **DO NOT** submit the rough sheets. DO NOT use pencil for writing the answers.

1. True or False? Answer any five. $(2 + 2 + 2 + 2 + 2$ marks)

   Note: 2 marks for the correct answer and -1 *for the wrong answer.*

   (a) $T(u) = v$ for some (fixed) $v \neq 0$ is a linear transformation.

   (b) Suppose P_1 and P_2 are projection matrices. Then
   $$(P_1 - P_2)^2 + (I - P_1 - P_2)^2 = I.$$

   (c) Let $\{q_1, \ldots, q_n\}$ be an orthonormal subset of vectors in $\mathbb{R}^n$ and T be a linear transformation that satisfies
   $$T(q_i)^\mathsf{T} T(q_i) = q_i^\mathsf{T} q_i,\ i = 1, \ldots, n.$$
   Then, the set $\{T(q_1), \ldots, T(q_n)\}$ is orthonormal.

   (d) If a linear transformation $T : \mathbb{R} \to \mathbb{R}$ satisfies $T(4) = 24$, then $T(x) = 6x$, for all $x \in \mathbb{R}$.

   (e) If $v_1, \ldots, v_n$ are linearly independent vectors in V and T is a linear transformation from V to V, then $T(v_1), \ldots, T(v_n)$ are linearly independent.

   (f) If $T(v_1), \ldots, T(v_n)$ are linearly independent vectors in V, where T is a linear transformation from V to V, then $v_1, \ldots, v_n$ are linearly independent.

   (g) Every orthonormal set of vectors in $\mathbb{R}^4$ must be a basis for $\mathbb{R}^4$.

2. Consider a subspace S of $\mathbb{R}^4$ spanned by the following vectors:
   $$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \text{ and } u_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$
   Using the usual dot product on $\mathbb{R}^4$, do the following:

   (a) Convert $\{u_1, u_2, u_3\}$ to an orthonormal basis for S. (5 marks)

   (b) For $b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, find the "least-squares approximation of b" in S. (3 marks)

   (c) Explain why Gram Schmidt algorithm fails when the input set of vectors is linearly dependent. (2 marks)