CS6015: Linear Algebra and Random Processes
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Quiz - 5: Solutions

1. True or False?

   (a) Union of two σ-fields is a σ-field.

   > **Solution:** False

   (b) Intersection of two σ-fields is a σ-field.

   > **Solution:** True

   (c) The probability that in a group of 25 people, at least two have same birthday is at least 0.5.

   > **Solution:** True

   (d) There exist events A, B such that $P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, and $P(A \cap B) = \frac{1}{12}$.

   > **Solution:** False

   (e) Let P_1 and P_2 be two probability measures. For any event A, let

   $$P(A) = \alpha_1 P_1(A) + \alpha_2 P_2(A), \text{ where } \alpha_1 + \alpha_2 = 1.$$

   Then, P is also a probability measure.

   > **Solution:** False

   (f) Let $B_1 \supseteq B_2 \supseteq B_3 \supseteq \dots$ be a decreasing set of events. Then,

   $$P\left(\bigcap_{i=1}^{\infty} B_i\right) = \lim_{i\to\infty} P(B_i).$$

   > **Solution:** True

2. In a supermarket near IITM, each packet of Corn Flakes may be found a plastic bust of one of the last four chairmen of CSE - abbreviated as KS, PSK, CSRM and TAG. The probability that any given packet contains any specific Chairman's bust is $\frac{1}{4}$.

   Suppose you rush to this supermarket and buy four packets of Corn flakes. Then, what is the probability that you get

   (a) at least one KS bust.

   > **Solution:** Let A_1 be the event that you get no KS bust. Then, $P(A_1) = \left(1 - \frac{1}{4}\right)^4$. The required probability of geting at least one KS bust is $1 - \left(\frac{3}{4}\right)^4$.

   (b) at least one KS bust and at least one PSK bust.

Solution: Let A_2 be the event that you get no PSK bust and A_1 be as defined in the first part. Then, $P(A_1 \cup A_2) = 2\left(1 - \frac{1}{4}\right)^4 - \left(1 - \frac{1}{2}\right)^4$. The required probability of getting at least one KS bust and at least one PSK bust is $1 - 2\left(\frac{3}{4}\right)^4 + \left(\frac{1}{2}\right)^4$.

(c) one of each four busts.

Solution: Number of ways of obtaining one of each four busts is 4! and the total number of configurations for the busts in the four packets is 4^4. So, the required probability is $\frac{4!}{4^4} = \frac{3}{32}$.