Shape Analysis

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CS6843 Program Analysis
IIT Madras
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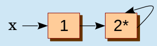
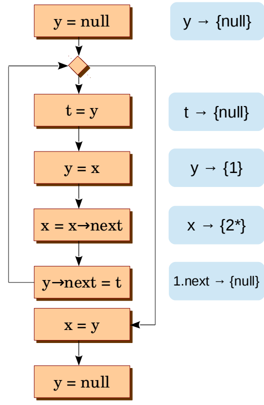
Limitations of Pointer Analysis

```
listReverse(List x) {
  assert("x is an acyclic singly linked list");

  for (y = null; x;) {
     t = y;
     y = x;
     x = x→next;
     y→next = t;
  }
  x = y;
  t = null;
  y = null;
}
```

We model the list as a two node structure.





Outline

- Limitations of pointer analysis
- Identify lists
- Identify trees, DAGs, cyclic graphs
- Identifying rotations
- List reversal and other transformations

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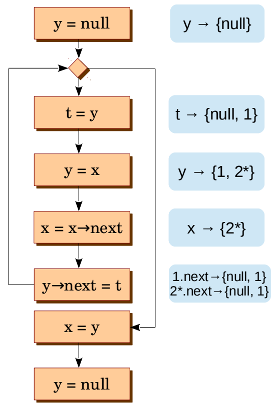

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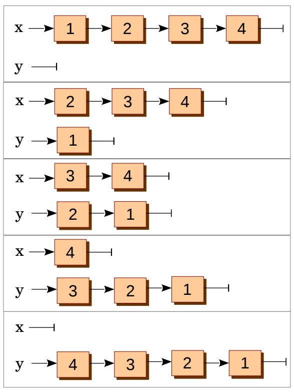
We want to check if x points to a singly linked list at the end of listReverse.
That is,
x → {4}, 4.next → {3}, ..., 1.next → {null}



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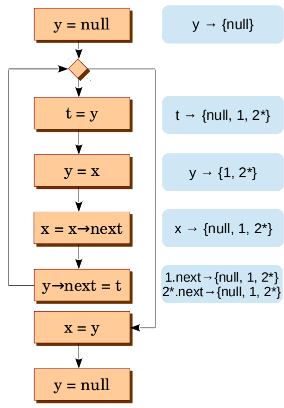

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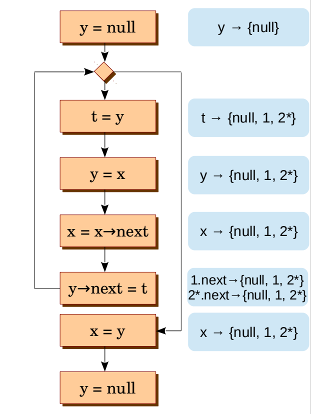

Shape Analysis

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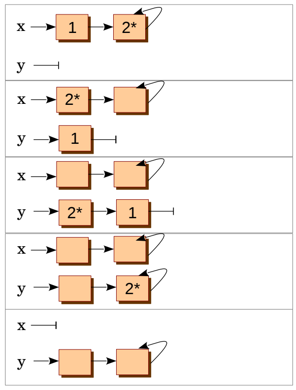
```

Maintain additional information with 2* that it is acyclic.
Use the fact that node removal maintains acyclicity.



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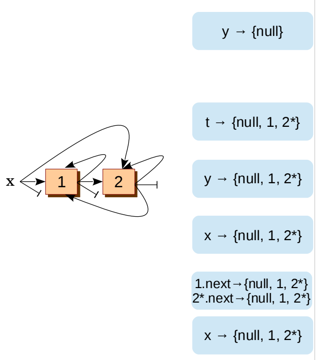


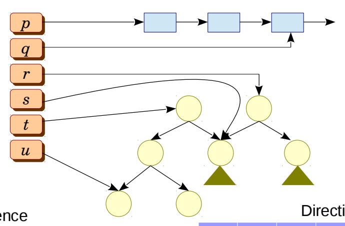
Tree, DAG, Cycle?

- Maintains three data structures:
  - Interference matrix: encodes common reachability
  - Direction matrix: encodes direct reachability
  - Shape
- Performs iterative data-flow analysis to update D, I and shape information

Shape Analysis

- Identify structural / topological properties of a data structure under manipulation.
- Usually categorized as slist, tree, DAG or cycle.
- Precision reduces along slist → tree → DAG → cycle.

Tree, DAG, Cycle?



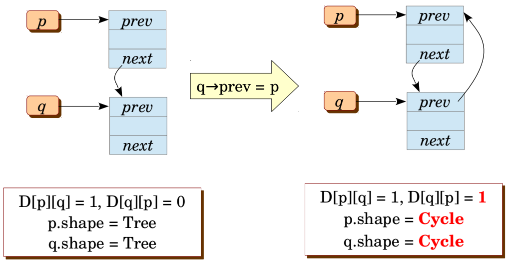
Interference

	p	q	r	s	t	u
p	1	**1**	0	0	0	0
q		1	0	0	0	0
r			1	**1**	**1**	0
s				1	**1**	0
t					1	**1**
u						1

Direction

	p	q	r	s	t	u
p	1	**1**	0	0	0	0
q	**0**	1	0	0	0	0
r	0	0	1	**1**	**0**	0
s	0	0	**0**	1	**0**	0
t	0	0	**0**	**1**	1	**1**
u	0	0	0	0	**0**	1

Shape Estimation



Inference Rules

p = malloc(...) **p = q** p = q→f p = &(q→f) p = q op k p = null p->f = q p->f = null	D_kill and I_kill sets same as for allocation statement. D_gen = {D[s][p] ∣ D[s][q] and s ≠ p} U {D[p][s] ∣ D[q][s] and s ≠ p} U {D[p][p] ∣ D[q][q]} I_gen = {I[p][s] ∣ I[q][s] and s ≠ p} U {I[p][p] ∣ I[q][q]} p.shape = q.shape

The implementation should create new D/I matrices from their current copies.
In-situ update would lead to unsound or imprecise analysis.

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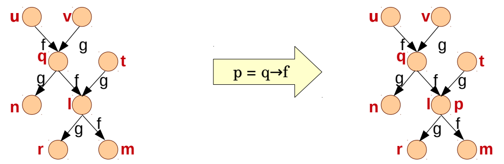


Inference Rules

p = malloc(...) p = q p = q→f p = &(q→f) p = q op k p = null p->f = q p->f = null	D_kill = {D[p][s] ∣ D[p][s] == 1} U {D[s][p] ∣ D[s][p] == 1} I_kill = {I[p][s] ∣ I[p][s] == 1} D_gen = {D[p][p]} I_gen = {I[p][p]} p.shape = Tree

Inference Rules

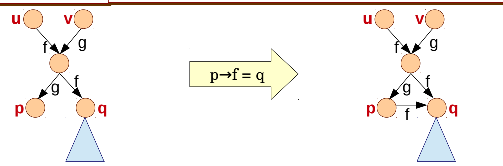
p = malloc(...) p = q p = q→f **p = &(q→f)** **p = q op k** p = null p->f = q p->f = null	Processing is the same as for p = q statement. This means the analysis loses field-sensitivity. A former work from IITK (Dasgupta, Karkare, Reddy) addresses this issue.

Inference Rules

p = malloc(...)	D_kill and I_kill sets same as for allocation statement.
p = q p = q→f p = &(q→f) p = q op k **p = null**	D_gen = { } I_gen = { } p.shape = Tree
p->f = q p->f = null	

Inference Rules

p = malloc(...)	D_kill = { }, I_kill = { }
p = q p = q→f	D_gen = {D[r][s] ∣ D[r][p] and D[q][s]} I_gen = {I[r][s] ∣ D[r][p] and I[q][s]}
p = &(q→f) p = q op k p = null	D[q][p] and D[s][q] ⇒ s.shape = Cycle D[q][p] and D[s][p] ⇒ s.shape = Cycle !D[q][p] and D[s][p] and I[s][q] and q.shape == Tree
p->f = q p->f = null	⇒ s.shape = max(s.shape, DAG) !D[q][p] and D[s][p] and q.shape != Tree ⇒ s.shape = max(s.shape, q.shape)



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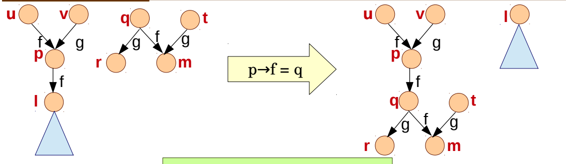

Can you improve precision?

Inference Rules

p = malloc(...)	D_kill = { }, I_kill = { }
p = q p = q→f	D_gen = {D[r][s] ∣ D[r][p] and D[q][s]} I_gen = {I[r][s] ∣ D[r][p] and I[q][s]}
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	Tree	DAG	Cycle
Tree	Tree	DAG	Cycle
DAG	DAG	DAG	Cycle
Cycle	Cycle	Cycle	Cycle

max(shape1, shape2)

Inference Rules

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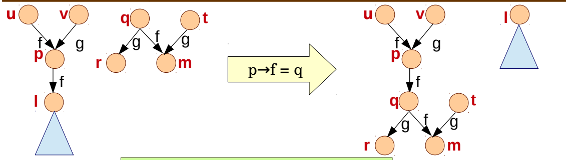

For max() consider D[v][r] == 1.

Inference Rules

p = malloc(...)	D_kill = { }, I_kill = { }
p = q p = q→f p = &(q→f) p = q op k p = null	D_gen = { } I_gen = { } No changes to the shape of p.
p->f = q **p->f = null**	

Example

```
listReverse(List x) {
  assert("x is an acyclic singly linked list");

  for (y = null; x;) {
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      y = x;
      x = x→next;
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}
```

Interference

	x	y	t
x		0	0
y			0
t			

Direction

	x	y	t
x	1	0	0
y	0	0	0
t	0	0	0

Shape

x	tree
y	tree
t	tree

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We need to assume a finite representation for the data structure.

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```

Since we do not model fields, we can't say that x is unreachable from y.

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x		1	0
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x	tree
y	cycle
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Classwork

p = malloc(10);

p->f1 = null;

q = p->f2;

q = &(r->f2);

q->f2 = p;

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Improvements

- Field-sensitivity
- Heap modeling
- Path-sensitivity

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Shape

x	cycle
y	cycle
t	cycle

Summary

- Shape analysis helps several transforms.
- Existing techniques often trade off precision for efficiency.
- We are still far away from a precise and scalable analysis.