Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Neighbors of a Pixel

- Any pixel p(x, y) has two vertical and two horizontal neighbors, given by

  $$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$

- This set of pixels are called the 4-neighbors of P, and is denoted by N_4(P).

- Each of them are at a unit distance from P.

Neighbors of a Pixel (Contd..)

- The four diagonal neighbors of p(x,y) are given by,

$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$

- This set is denoted by $N_D(P)$.

- Each of them are at Euclidean distance of 1.414 from P.

Neighbors of a Pixel (Contd..)

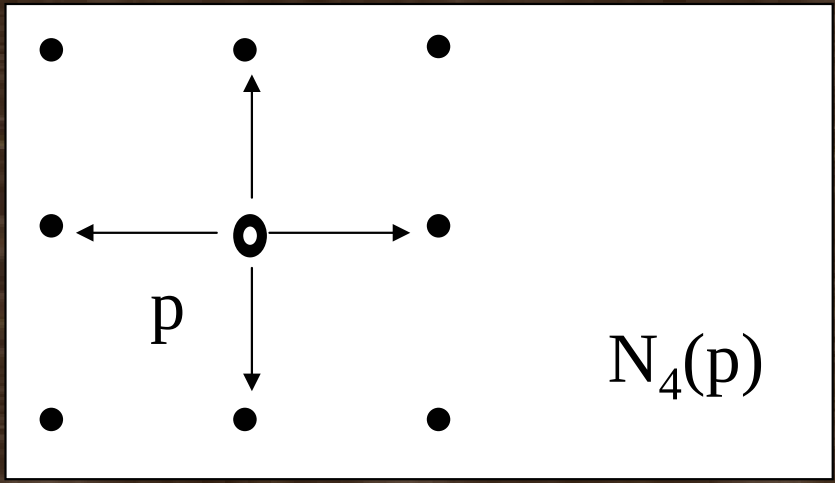
- The points $N_D(P)$ and $N_4(P)$ are together known as 8-neighbors of the point P, denoted by $N_8(P)$.

- Some of the points in the N_4, N_D and N_8 may fall outside image when P lies on the border of image.

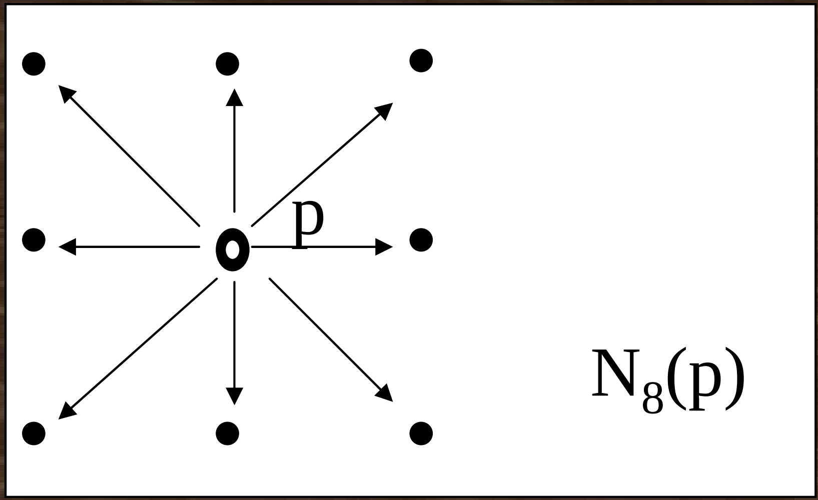
Neighbors of a Pixel (Contd..)

Neighbors of a pixel

a. 4-neighbors **of a pixel p are its vertical and horizontal neighbors denoted by $N_4(p)$**



b. 8-neighbors **of a pixel p are its vertical horizontal and 4 diagonal neighbors denoted by $N_8(p)$**



Neighbors of a Pixel (Contd..)

N_D	N_4	N_D
N_4	**P**	N_4
N_D	N_4	N_D

- N_4 - 4-neighbors
- N_D - diagonal neighbors
- N_8 - 8-neighbors (N_4 U N_D)

Adjacency

- Two pixels are connected if they are neighbors and their gray levels satisfy some specified criterion of similarity.

- For example, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1).

Adjacency (contd.)

- **Let V be set of gray levels values used to define adjacency.**
- **<u>4-adjacency</u>: Two pixels *p* and *q* with values from V are 4-adjacent if q is in the set $N_4(p)$.**
- **<u>8-adjacency</u>: Two pixels *p* and *q* with values from V are 8-adjacent if q is in the set $N_8(p)$.**
- **<u>m-adjacency</u>: Two pixels *p* and *q* with values from V are m-adjacent if,**
  - **q is in $N_4(P)$.**
  - **q is in $N_D(p)$ and the set [$N_4(p) \cap N_4(q)$] is empty (has no pixels whose values are from V).**

Connectivity :

To determine whether the pixels are adjacent in some sense.

Let V be the set of gray-level values used to define connectivity; then Two pixels p, q that have values from the set V are:

a. 4-connected, **if q is in the set $N_4(p)$**

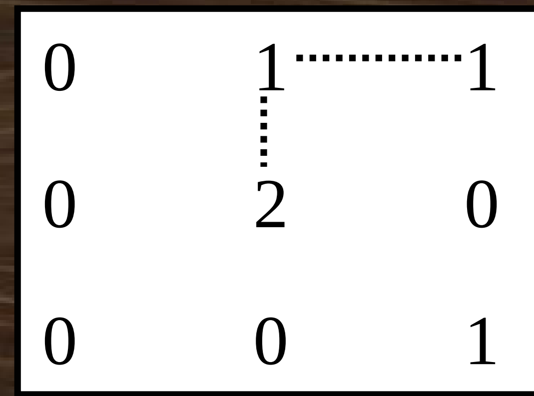
b. 8-connected, **if q is in the set $N_8(p)$**

c. m-connected, **iff**

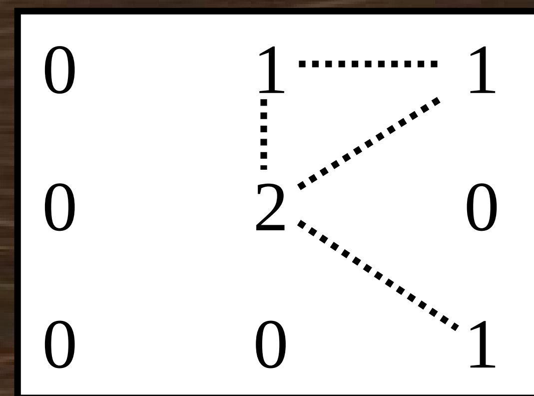
   i. q is in $N_4(p)$ or

   ii. q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty

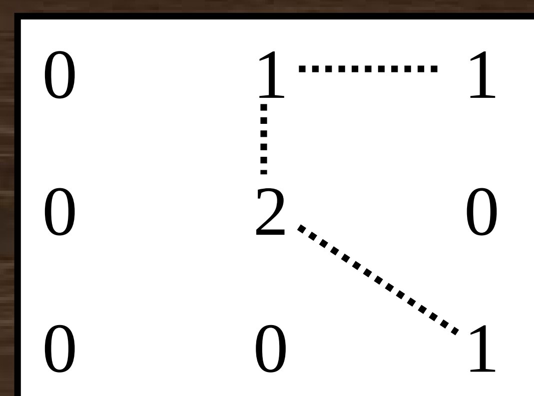
V = {1, 2}



a.

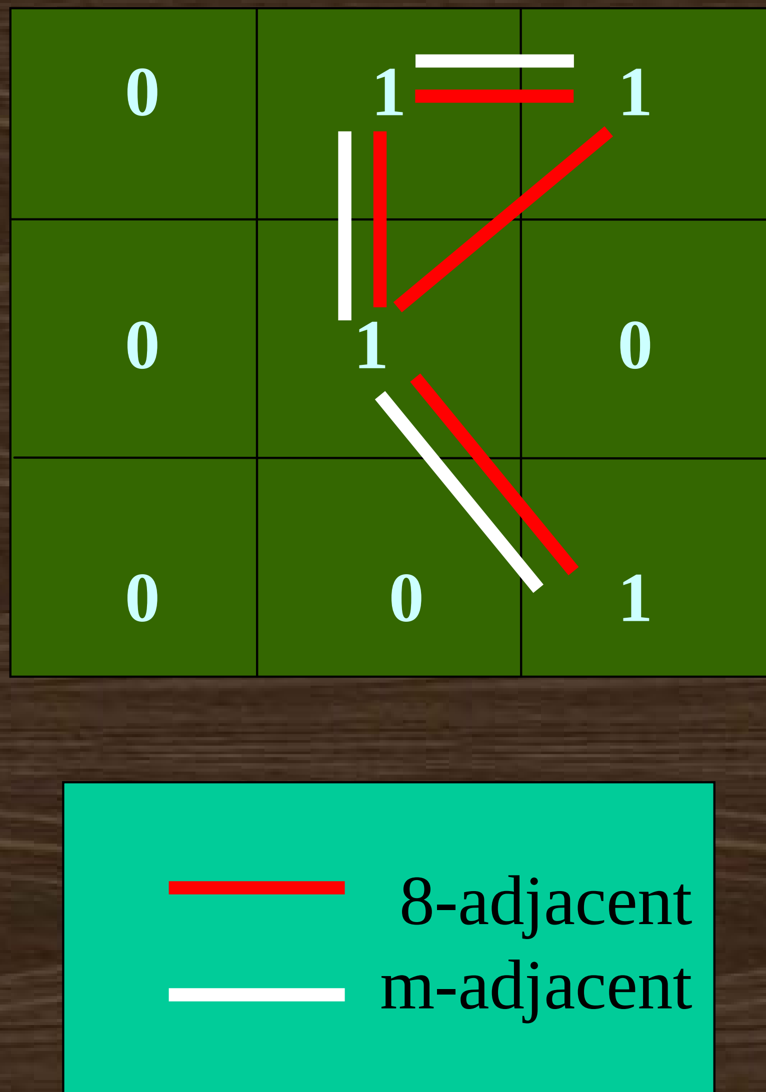


b.



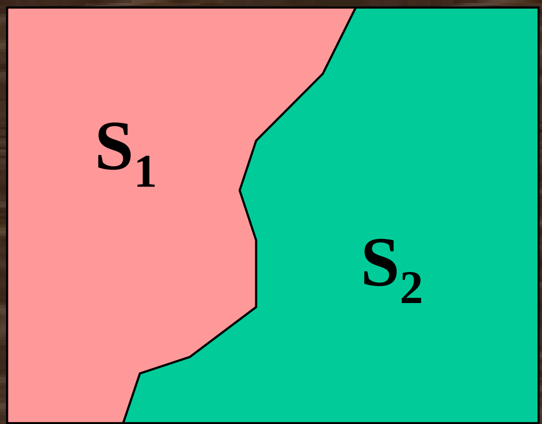
c.

Adjacency/Connectivity



Adjacency/Connectivity

- **Pixel p is *adjacent* to pixel q if they are connected.**
- **Two *image subsets* S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2**



Paths & Path lengths

- **A *path* from pixel p with coordinates *(x, y)* to pixel *q* with coordinates *(s, t)* is a sequence of distinct pixels with coordinates:**
  $(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n)$,
  where $(x_0, y_0)=(x, y)$ **and** $(x_n, y_n)=(s, t)$;
  (x_i, y_i) **is adjacent to** (x_{i-1}, y_{i-1}) $1 \le i \le n$
- **Here *n* is the *length* of the path.**
- **We can define 4-, 8-, and m-paths based on type of adjacency used.**

Connected Components

- **If *p* and *q* are pixels of an image subset S then *p* is *connected* to *q* in S if there is a path from *p* to *q* consisting entirely of pixels in S.**
- **For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a *connected component* of S.**
- **If S has only one connected component then S is called *Connected Set.***

Regions and Boundaries

- **A subset R of pixels in an image is called a *Region* of the image if R is a connected set.**
- **The *boundary* of the region R is the set of pixels in the region that have one or more neighbors that are not in R.**
- **If R happens to be entire Image?**

Distance measures

Given pixels *p*, *q* and *z* with coordinates *(x, y), (s, t), (u, v)* respectively, the distance function D has following properties:

a. $D(p, q) \geq 0$ [$D(p, q) = 0$, iff $p = q$]

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

The following are the different Distance measures:

- Euclidean Distance :

  $D_e(p, q) = [(x-s)^2 + (y-t)^2]$

b. City Block Distance: →

$D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c. Chess Board Distance: →

$D_8(p, q) = max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Relationship between pixels (Contd..)

Arithmetic/Logic Operations:

- Addition : $p + q$
- Subtraction: $p - q$
- Multiplication: $p*q$
- Division: p/q
- AND: p AND q
- OR : p OR q
- Complement: NOT(q)

Neighborhood based arithmetic/Logic :

Value assigned to a pixel at position 'e' is a function of its neighbors and a set of window functions.

		:		
	a	b	c	
...	d	e	f	..
	g	h	i	
		:		

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$p = (\mathrm{w}_1\mathrm{a} + w_2\mathrm{b} + w_3\mathrm{c} + w_4\mathrm{d} + w_5\mathrm{e} + w_6\mathrm{f} + w_7\mathrm{g} + w_8\mathrm{h} + w_9\mathrm{i})$$

$$= \sum w_i f_i$$

Arithmetic/Logic Operations

- Tasks done using neighborhood processing:
  - Smoothing / averaging
  - Noise removal / filtering
  - Edge detection
  - Contrast enhancement

- **Issues**
  - **Choice of w_i's (N^2 values)**
  - **Choice of N, window size**
  - **Computation at boundaries**
    - **Do not compute at boundaries**
    - **Pad with zeros and extend image boundary**
    - **Pad assuming periodicity of image**
    - **Extrapolation of image**

END of Neighborhood

and Connectivity