Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries
Neighbors of a Pixel

• Any pixel $p(x, y)$ has two vertical and two horizontal neighbors, given by
  $(x+1, y), (x-1, y), (x, y+1), (x, y-1)$

• This set of pixels are called the 4-neighbors of $P$, and is denoted by $N_4(P)$.

• Each of them are at a unit distance from $P$. 
Neighbors of a Pixel (Contd..)

• The four diagonal neighbors of $p(x,y)$ are given by,

$$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$$

• This set is denoted by $N_D(P)$.

• Each of them are at Euclidean distance of 1.414 from $P$. 
• The points $N_D(P)$ and $N_4(P)$ are together known as 8-neighbors of the point $P$, denoted by $N_8(P)$.

• Some of the points in the $N_4$, $N_D$ and $N_8$ may fall outside image when $P$ lies on the border of image.
Neighbors of a Pixel (Contd..)

Neighbors of a pixel
a. 4-neighbors of a pixel \( p \) are its vertical and horizontal neighbors denoted by \( N_4(p) \)
b. 8-neighbors of a pixel \( p \) are its vertical horizontal and 4 diagonal neighbors denoted by \( N_8(p) \)
Neighbors of a Pixel (Contd..)

- $N_4$ - 4-neighbors
- $N_D$ - diagonal neighbors
- $N_8$ - 8-neighbors ($N_4 \cup N_D$)
Adjacency

• Two pixels are connected if they are neighbors and their gray levels satisfy some specified criterion of similarity.

• For example, in a binary image two pixels are connected if they are 4-neighbors and have same value (0/1).
Adjacency (contd.)

• Let V be set of gray levels values used to define adjacency.

• **4-adjacency**: Two pixels \( p \) and \( q \) with values from V are 4-adjacent if \( q \) is in the set \( N_4(p) \).

• **8-adjacency**: Two pixels \( p \) and \( q \) with values from V are 8-adjacent if \( q \) is in the set \( N_8(p) \).

• **m-adjacency**: Two pixels \( p \) and \( q \) with values from V are m-adjacent if,
  – \( q \) is in \( N_4(P) \).
  – \( q \) is in \( N_D(p) \) and the set \([ N_4(p) \cap N_4(q) ]\) is empty (has no pixels whose values are from V).
Connectivity:

To determine whether the pixels are adjacent in some sense.

Let V be the set of gray-level values used to define connectivity; then Two pixels p, q that have values from the set V are:

a. 4-connected, if q is in the set N₄(p)

b. 8-connected, if q is in the set N₈(p)

c. m-connected, iff

i. q is in N₄(p) or

ii. q is in N₈(p) and the set

$$N₄(p) \cap N₄(q)$$

is empty
Adjacency/Connectivity

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0 1 1
0 1 0
0 0 1
```

- **8-adjacent**
- **m-adjacent**
• Pixel $p$ is *adjacent* to pixel $q$ if they are connected.

• Two *image subsets* $S_1$ and $S_2$ are adjacent if some pixel in $S_1$ is adjacent to some pixel in $S_2$
Paths & Path lengths

- A *path* from pixel \( p \) with coordinates \((x, y)\) to pixel \( q \) with coordinates \((s, t)\) is a sequence of distinct pixels with coordinates:

\[
(x_0, y_0), (x_1, y_1), (x_2, y_2) \ldots (x_n, y_n),
\]

where \((x_0, y_0) = (x, y)\) and \((x_n, y_n) = (s, t)\); 
\((x_i, y_i)\) is adjacent to \((x_{i-1}, y_{i-1})\) \(1 \leq i \leq n\)

- Here \( n \) is the *length* of the path.

- We can define 4-, 8-, and \( m \)-paths based on type of adjacency used.
Connected Components

• If $p$ and $q$ are pixels of an image subset $S$ then $p$ is connected to $q$ in $S$ if there is a path from $p$ to $q$ consisting entirely of pixels in $S$.

• For every pixel $p$ in $S$, the set of pixels in $S$ that are connected to $p$ is called a connected component of $S$.

• If $S$ has only one connected component then $S$ is called Connected Set.
Regions and Boundaries

• A subset $R$ of pixels in an image is called a *Region* of the image if $R$ is a connected set.

• The *boundary* of the region $R$ is the set of pixels in the region that have one or more neighbors that are not in $R$.

• If $R$ happens to be entire Image?
Distance measures

Given pixels $p$, $q$ and $z$ with coordinates $(x, y)$, $(s, t)$, $(u, v)$ respectively, the distance function $D$ has following properties:

a. $D(p, q) \geq 0$ \quad [D(p, q) = 0, \text{ iff } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$
The following are the different Distance measures:

- **Euclidean Distance**: 
  \[ D_{e}(p, q) = [(x-s)^2 + (y-t)^2] \]

- **City Block Distance**: 
  \[ D_{4}(p, q) = |x-s| + |y-t| \]

- **Chess Board Distance**: 
  \[ D_{8}(p, q) = \max(|x-s|, |y-t|) \]
Relationship between pixels (Contd..)

Arithmetic/Logic Operations:

- Addition: \( p + q \)
- Subtraction: \( p - q \)
- Multiplication: \( p \times q \)
- Division: \( p/q \)
- AND: \( p \text{ AND } q \)
- OR: \( p \text{ OR } q \)
- Complement: NOT\((q)\)
Neighborhood based arithmetic/Logic:

Value assigned to a pixel at position ‘e’ is a function of its neighbors and a set of window functions.

\[ p = (w_1 a + w_2 b + w_3 c + w_4 d + w_5 e + w_6 f + w_7 g + w_8 h + w_9 i) \]

\[ = \sum w_i f_i \]
Arithmetic/Logic Operations

• Tasks done using neighborhood processing:
  
  – Smoothing / averaging
  
  – Noise removal / filtering
  
  – Edge detection
  
  – Contrast enhancement
• Issues

  – Choice of $w_i$'s ($N^2$ values)

  – Choice of $N$, window size

  – Computation at boundaries

    • Do not compute at boundaries

    • Pad with zeros and extend image boundary

    • Pad assuming periodicity of image

    • Extrapolation of image
END of Neighborhood and Connectivity