Assignment 2
Matrix Chain Multiplication

Linear Algebra and Random Processes (CS6015)
Problem Description

1 Problem Statement

Given a chain $\langle A_1, A_2, ..., A_n \rangle$ of n matrices, where matrix A_i $(i = 1, 2, ..., n)$ has the dimension $p_{i-1} \times p_i$, find the optimal sequence of pairings for multiplication of matrices $A_1, A_2 ... A_n$. Once the sequence of pairings for matrix multiplication is done, the matrix chain product can be calculated with the best computation cost (number of scalar multiplications).

2 Input

Chain $\langle A_1, A_2, ..., A_n \rangle$ of n matrices, each of which is initialized with random numbers
Assumptions. The number of matrices in the sequence chain $(n > 10)$, and the dimensions of the matrix in the chain p_i is such that $2 \leq p_i < 100$

3 Output

- Optimum pairing for calculating the product $A_1 A_2 ... A_n$. For example, if an output with $n = 4$ is $(A_1((A_2 A_3)A_4))$, then state $(1, ((2, 3), 4))$
- Number of scalar multiplications to compute the given product $A_1 A_2 ... A_n$ by performing a sequence of multiplications from left to right $(...(((A_1 A_2)A_3)A_4)...)A_n$
- Number of scalar multiplications to compute the given product $A_1 A_2 ... A_n$ by performing a sequence of non-optimal pairing from left to right $(...((A_1 A_2)(A_3 A_4))...)...(A_{n-1} A_n)$. An example with $n = 4$ is $((A_1 A_2)(A_3 A_4))$, with $n = 7$ is $(((A_1 A_2)(A_3 A_4))((A_5 A_6)A_7))$
- Optimum number of scalar multiplications to compute the product $A_1 A_2 ... A_n$
- Computation time for calculating the product by performing a sequence of multiplications from left to right
- Computation time for calculating the product by performing a sequence of non-optimal pairing from left to right
- Computation time for calculating the product using optimal pairing

4 References

- Cormen, Thomas H., et al. Introduction to algorithms. MIT press, 2009. (Section 15.2)