

Deterministic Protocols in the SINR Model without Knowledge of Coordinates

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Work done in collaboration with Shailesh Vaya

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- Motivate SINR model and set it up.
- Multi-Broadcast - problem statement + related work + motivation.
- Achieving multi-broadcast.
- Wakeup - problem statement + overview of solution.
- Achieving wakeup.
- Briefly mention other results obtained.
- Possible future directions.

Outline

- 1 SINR Model
 - Motivation
 - Model Description
- 2 Multi-Broadcast
 - The Problem
 - Combinatorial Tools
 - Our Solution
 - Tree Grower
 - Tree Cutter
 - Multi-Broadcast
- 3 Non-Spontaneous Wakeup
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- 5 **References**

Motivation

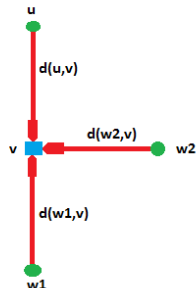


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SINR Model - In a Nutshell

Example Network



- u trying to transmit to v . Needs to overcome signals from w_1 and w_2 .
- Strength of signal of u at v
 $= \frac{P_u}{d(u,v)^\alpha}$.
Similar for w_1 and w_2 .
- In SINR model, u 's message is received by v iff

$$\frac{\frac{P_u}{d(u,v)^\alpha}}{\mathcal{N} + \left(\frac{P_{w_1}}{d(w_1,v)^\alpha} + \frac{P_{w_2}}{d(w_2,v)^\alpha} \right)} \geq \beta.$$

Model Parameters

- 1 Path loss constant - $\alpha \geq 2$
- 2 Threshold constant - $\beta \geq 1$
- 3 Ambient noise - $\mathcal{N} \geq 0$

Following inequality determines if a message from u will be received by a station v . Let \mathcal{T} be set of stations transmitting in given round.

$$\bullet \frac{\frac{P_u}{d(u,v)^\alpha}}{\mathcal{N} + \sum_{i \in \mathcal{T} \setminus u} \frac{P_i}{d(i,v)^\alpha}} \geq \beta$$

Model Parameter Added

- Sensitivity parameter - $\epsilon > 0$

An additional inequality now also helps determine when a message from u will be received by a station v .

- $$\frac{P_u}{d(u,v)^\alpha} \geq (1 + \epsilon)\beta\mathcal{N}$$

- ① Range of a station u (r) - the distance from u within which another station can hear a message from u if all other stations are silent.
- ② Communication graph $G(V,E)$ - Wireless stations are nodes. If station v within range of u , there is an edge from u to v . (*Weak links.*)
- ③ Uniform network - ranges of all stations (and by extension powers) same and equal to r .

Network & Clock Set Up

- 1 Alg. works synchronously in rounds.
- 2 Nodes located on 2D Euclidean plane.
- 3 Transmission power - fixed & uniform.
- 4 Size of message - $O((\Delta \log N + n) \log N)$ bits
 Δ - max. degree of any node
 n - no. of nodes
 N - $[N]$ is the range from which node labels (IDs) are taken
- 5 Assume even with weak devices, communication graph connected.

Properties of Stations

- 1 A node can act either as a receiver or sender in a round, but not both.
- 2 No collision detection, i.e. in a given round, a receiving node can't tell if no one sent a message or too many sent a message.
- 3 Labels (IDs) of all nodes unique and taken from $[1, N]$.
- 4 Every node knows value of N , the no. of nodes in the network n , and own label.
- 5 No idea about Euclidean coordinates. No idea about nbrs in comm. graph.
- 6 At start of multi-broadcast - all nodes initially active (awake).
- 7 At start of multi-wakeup - some nodes initially active (awake).

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Problem Statement

Problem Statement

Initially, there are several nodes, each with a different piece of information. Every piece of info must be transmitted to all other nodes within the network.

Related Work

- SINR model has been looked at quite a bit [Avin et al., 2009, Fanghänel et al., 2009, Kesselheim and Vöcking, 2010, Kesselheim, 2011].
- Specifically the problem of broadcast [Goussevskaia et al., 2008, Yu et al., 2011, Jurdzinski et al., 2013].
- [Jurdzinski et al., 2013] model is very close, except they have knowledge of coordinates. Alg. running time - $O(D\Delta \log^2 N)$ rounds, D - diameter of communication graph.

Our Contribution

- We achieve deterministic multi-broadcast w/o knowledge of coordinates in $O(n \log^2 N)$ rounds, assuming all nodes awake initially.
- To the best of our knowledge, no work before ours has been able to achieve efficient deterministic multi-broadcast w/o knowledge of coordinates or knowledge of neighbors' labels.

Outline

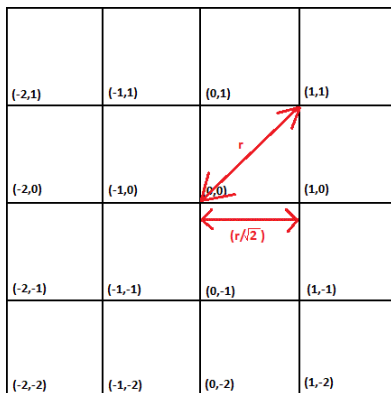
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Grid

$(-2,1)$	$(-1,1)$	$(0,1)$	$(1,1)$
$(-2,0)$	$(-1,0)$	$(0,0)$	$(1,0)$
$(-2,-1)$	$(-1,-1)$	$(0,-1)$	$(1,-1)$
$(-2,-2)$	$(-1,-2)$	$(0,-2)$	$(1,-2)$

- Consider plane as a grid.
- Length of each side x .

Pivotal Grid



- $x = \frac{r}{\sqrt{2}}$.
- Significance: If two nodes within same box, they can hear each other.

A Simple Way to Hear a Message

- According to SINR model, only if your signal at destination beats out interference plus noise can your message be heard.
- If there are lots of nodes, this becomes difficult.
- Solution: Develop a way to limit the number of people actually transmitting within some distance of you at any given time. Dilution.

Dilution

1 (-2,1)	2 (-1,1)	1 (0,1)	2 (1,1)
3 (-2,0)	4 (-1,0)	3 (0,0)	4 (1,0)
1 (-2,-1)	2 (-1,-1)	1 ^x (0,-1)	2 (1,-1)
3 (-2,-2)	4 (-1,-2)	3 (0,-2)	4 (1,-2)

- Suppose we know grid coordinates of nodes.
- Group smaller boxes together into larger boxes.
- Example is 2-dilution.

Dilution

1 (-2,1)	2 (-1,1)	1 (0,1)	2 (1,1)
3 (-2,0)	4 (-1,0)	3 (0,0)	4 (1,0)
1 (-2,-1)	2 (-1,-1)	1 ^x (0,-1)	2 (1,-1)
3 (-2,-2)	4 (-1,-2)	3 (0,-2)	4 (1,-2)

- In a given round, only one box out of 4 participates. Rest are silent.
- This chosen box is the same across all bigger boxes.

Dilution

1 (-2,1)	2 (-1,1)	1 (0,1)	2 (1,1)
3 (-2,0)	4 (-1,0)	3 (0,0)	4 (1,0)
1 (-2,-1)	2 (-1,-1)	1 ^x (0,-1)	2 (1,-1)
3 (-2,-2)	4 (-1,-2)	3 (0,-2)	4 (1,-2)

Dilution

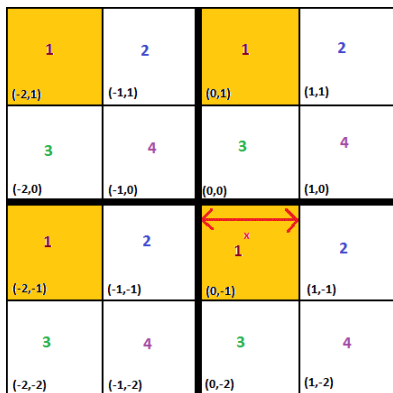
1 (-2,1)	2 (-1,1)	1 (0,1)	2 (1,1)
3 (-2,0)	4 (-1,0)	3 (0,0)	4 (1,0)
1 (-2,-1)	2 (-1,-1)	1 ^x (0,-1)	2 (1,-1)
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Dilution

1 $(-2,1)$	2 $(-1,1)$	1 $(0,1)$	2 $(1,1)$
3 $(-2,0)$	4 $(-1,0)$	3 $(0,0)$	4 $(1,0)$
1 $(-2,-1)$	2 $(-1,-1)$	1^x $(0,-1)$	2 $(1,-1)$
3 $(-2,-2)$	4 $(-1,-2)$	3 $(0,-2)$	4 $(1,-2)$

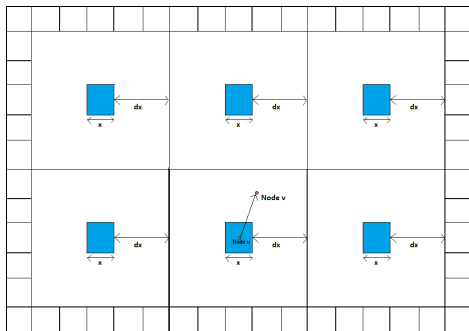
A red double-headed arrow is positioned between the cells $(0,-1)$ and $(1,-1)$ in the third row, indicating a transition or relationship between these two states.

Dilution



- And this cycle of active boxes is repeated.
- We can guarantee a box is active once every 4 rounds.

Recap of Dilution



- $(2d + 1)$ -dilution.
- Consider plane as a grid. Group $(2d + 1)^2$ boxes into a larger box.
- In one round, only one of those smaller boxes active per larger box. Same box in every larger box.
- v can now hear message of u .

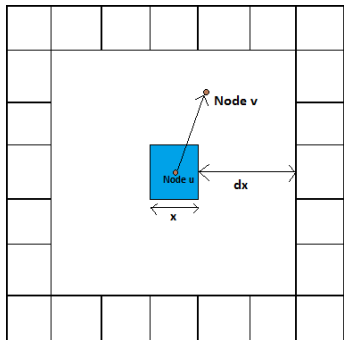
- We divide the plane into a grid. If we enforce a δ -dilution, it means that for a group of δ^2 boxes, only one of them is active at any given time. Moreover, it is the same box in each group of δ^2 boxes.
- Make the boxes small enough so that only node per box.
- Q: We're done then, right?
- A: No. Grid-based dilution is only possible when we know coordinates of each node. We don't.
- So our goal then: Silence boxes within some distance of a given box.
- How: Strongly selective families.

Strongly Selective Family

(N, c) -ssf

A family F of subsets of $[N]$ is an (N, c) strongly selective family if for every non-empty subset Z of $[N]$, such that $|Z| \leq c$, and for every $z \in Z$, there is a set $f \in F$ that intersects Z at only element z . The number of subsets in the family is $O(c^2 \log N)$.

- Each subset represents set of nodes transmitting in that round.
- To complete one execution of an (N, c) -ssf, it takes $O(c^2 \log N)$ rounds.



- If we know there is at most one node per box, it's sufficient.
- We are now able to silence area of $(2d + 1)^2$ boxes except box with u .
- v can now hear message of u .

Comparison

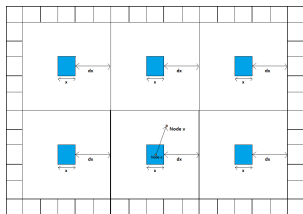


Figure: Grid-Based Dilution

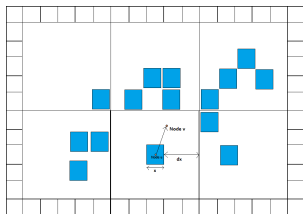


Figure: SSF-Based Dilution

Issues

- Issue 1: We don't know in which round v will be able to hear u .
- Issue 2: There may be more than one node per box.
- Issue 3: Outside interference may complicate things.

Solutions

- Issue 1 & 2 taken care of by our algorithm.
- Issue 2 & 3 addressed by our lemma (modified form of proposition from [Jurdzinski et al., 2013]).

Lemma

For stations with same range r , sensitivity $\epsilon > 0$, and transmission power, for each $\alpha > 2$, there exists a constant d , which depends only on the parameters α , β , and ϵ of the model and a constant k , satisfying the following property.

Let W be the set of stations such that at most a constant k of them are present in any grid box of the grid G_x , $x \leq \frac{r}{\sqrt{2}}$. Let u and v be two stations in different grid boxes such that the distance between them, $\sqrt{2}x$, is the minimum distance between any two stations in different grid boxes in G_x . Let A be the set of stations in u 's grid box.

If u is transmitting in a round t and no other station within its box or a box less than d box distance away from its box is transmitting in that round, then v and all stations in A can hear the message from u in round t .

Proof Sketch: If d is large enough, the interference by outside nodes will not cause problems.

Significance of Lemma

- No matter how small you make the grid boxes, the number of grid boxes you need to silence, $(2d + 1)^2 - 1$, only depends on α, β, ϵ , and k .
- The first 3 are parameters of the model. If you can ensure k is a constant, then you only need to shut down a constant number of boxes.
- $\therefore (N, c)$ -ssf, where $c = k^2(2d + 1)^2$, takes $O(c^2 \log N) = O(\log N)$ rounds to execute.
- \therefore Can replace any dilution scheme (req. knowledge of coordinates) with our scheme with an additional factor of $O(\log N)$ rounds.

Very Important Theorem

For a grid G_x , $x \leq \frac{r}{\sqrt{2}}$, let the set of all nodes that want to transmit satisfy the properties of the Lemma. Every node in this set can successfully transmit a message to its neighbors within $\sqrt{2}x$ distance of it in $O(\lg N)$ rounds by executing one (N, c) -ssf, where $c = k^2(2d + 1)^2$ where d is taken from the Lemma and k is an upper limit on the number of nodes from the set in any box of the grid.

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Overview of Approach

① Tree Grower

- Creates trees which may span multiple grid boxes with at most one root per grid box.
- Running time - $O(n \log N)$.

② Tree Cutter

- Cuts the trees to height at most 1.
- Running time - $O(n \log^2 N)$.

③ Broadcast

- Takes a message at one of the nodes and spreads it throughout the network.
- Uses Tree Grower & Tree Cutter as subroutines.
- Running time - $O(n \log^2 N)$.

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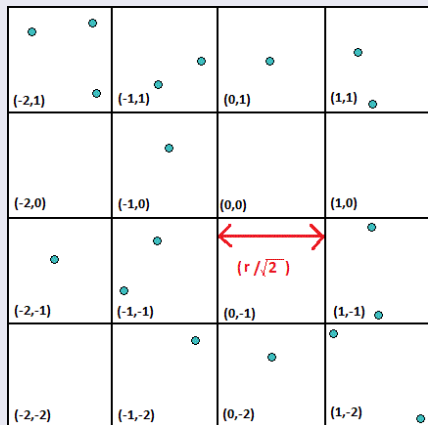
- Input: A network of connected nodes.
- Output: A forest of trees such that:
 - Every node is either a leader (root) or a child.
 - There is at most one leader per grid box of the pivotal grid (length of side of grid box = $\frac{r}{\sqrt{2}}$).

Tree Grower, run by each node u

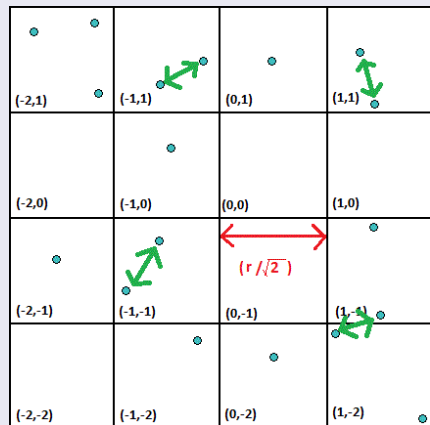
```
1: for  $cnt \leftarrow 1, n$  do
2:   if  $u$  is active then
3:     Execute SSF:
4:       Transmit  $u$ 's label. Listen for others' labels.
5:     Execute SSF:
6:       Transmit info about everyone  $u$  heard. Figure out who  $u$  can bidirectionally
       communicate with.
7:     Execute SSF:
8:       Transmit  $u$ 's label in active rounds. Listen for others who  $u$  can bidirectionally
       communicate with.
9:     Execute SSF:
10:      Transmit info about who might be  $u$ 's potential parent or children. Lower labels
       becomes parents.
11:      Form links. If  $u$  becomes a child, become inactive.
12:   end if
13: end for
14: If active, become leader.
```


Tree Grower

TG - Initially

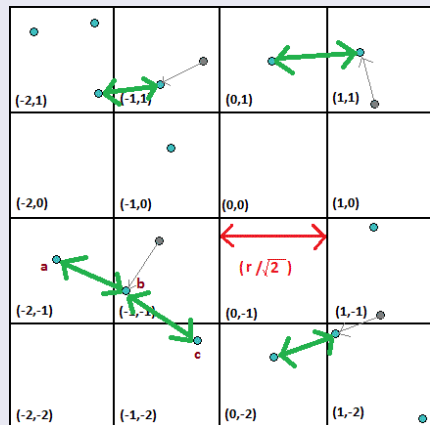


TG - Step 1



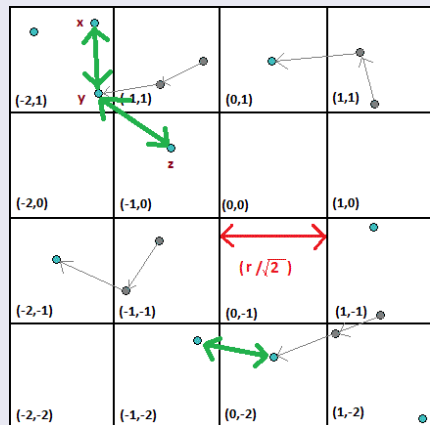
- Nodes which are close enough, hear each other thanks to SSF-based dilution and our lemma.

TG - Step 2



- A series of connections, where $a < b$ and $b > c$.

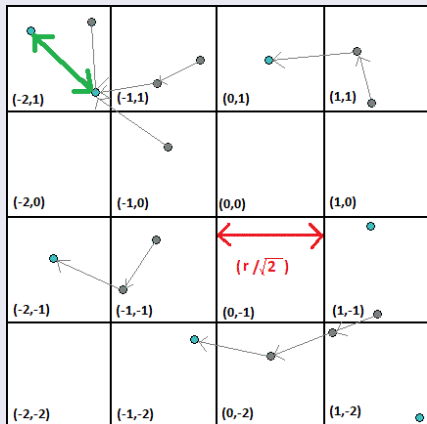
TG - Step 3



- A series of connections, where $x > y$ and $y < z$.

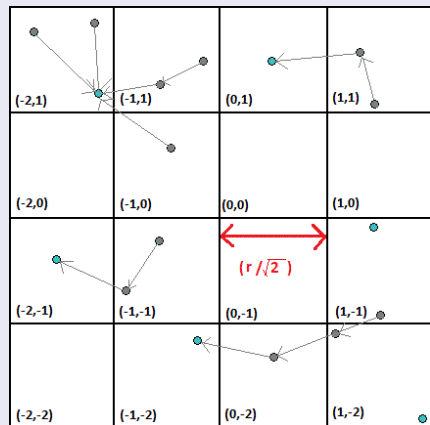
Tree Grower

TG - Step 4



Tree Grower

TG - Finally



- Every node either leader or child.
- At most one leader per grid box.

Tree Grower - Proof of Claims

Claims

- 1 Creates trees which may span multiple grid boxes with at most one root per grid box.
- 2 Running time - $O(n \log N)$ rounds.

Proof Sketch of Correctness (Claim 1)

- If two nodes are both active and distance between them is min. among all distances in graph, they will be able to communicate because of ssf-based dilution.
- At the beginning of every phase i of the alg., either at least $i - 1$ nodes have become children or all nodes which are not children will become leaders.

Claims

- 1 Creates trees which may span multiple grid boxes with at most one root per grid box.
- 2 Running time - $O(n \log N)$ rounds.

Proof of Running Time (Claim 2)

- Each execution of (N, c) -ssf takes $O(c^2 \log N) = O(\log N)$ rounds, since c is a constant.
- There are n such executions of 2 ssfs, so running time = $O(n \log N)$ rounds.

Now what?

- Why did we build trees?
- So that at most there are a constant number of nodes that want to transmit per grid box.
- What could we do now to broadcast?
- Use token passing, DFS style. Tree defined by leader. One token per tree. Transmit when you get token.
- Will it work? No.
- Why? Several trees pass through single grid box. No bound on them. What if all tokens end up in one grid box at same time. (N, c) -ssf-based dilution with constant c fails.
- Solution: Cut the trees down to height 1 or less.

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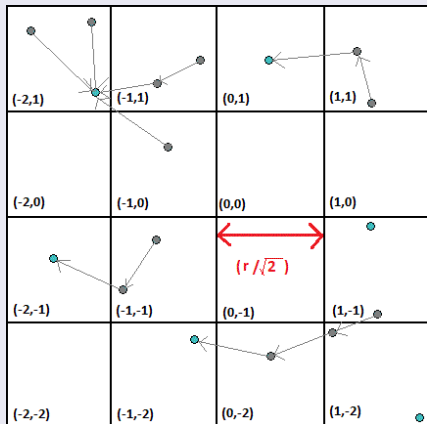
- Input: Trees such that there is at most one leader per grid box.
- Output: Trees of height at most 1 such that there is at most one leader per grid box.

Tree Cutter, run by each node u

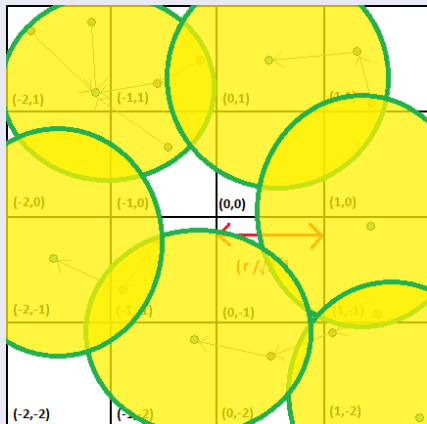
- 1: Initially, leaders have tokens.
- 2: **for** $cnt \leftarrow 1, 2 \cdot 947 \cdot (n + 1) - 1$ **do**
- 3: Execute Potential Leader Election.
- 4: Execute SSF:
- 5: Transmit u 's status (leader/follower) and update u 's tree.
- 6: Execute SSF:
- 7: Pass token, if any, in DFS manner.
- 8: **end for**

Tree Cutter

TC - Initially



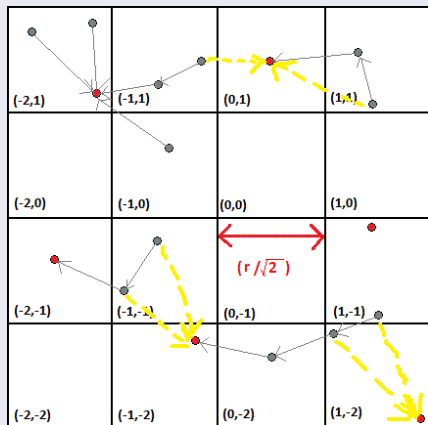
TC - Step 1



- Leaders transmit.

Tree Cutter

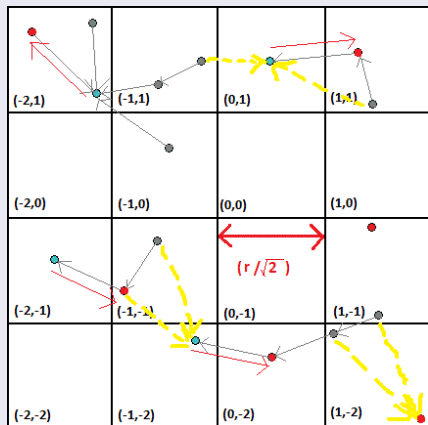
TC - Step 2



- Non-leaders orient themselves to whichever leader they hear first. If they hear from someone other than original leader first, then they reorient themselves to new leader.
- Note that non-leaders may hear from no leader initially, in which case they stay unoriented.

Tree Cutter

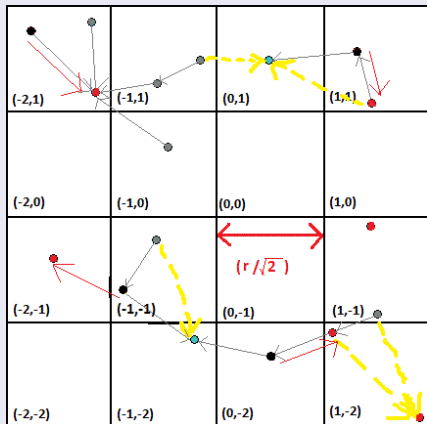
TC - Step 3



- Once a node who reorients itself gets a token from its old leader, it can declare its new allegiance.

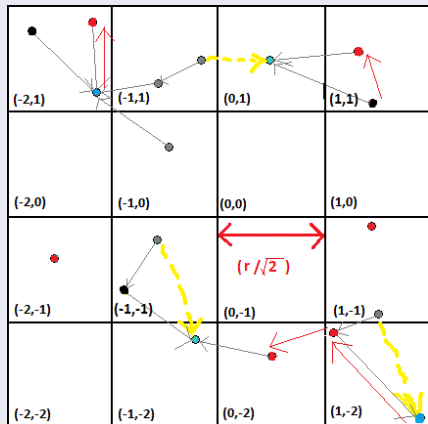
Tree Cutter

TC - Step 4



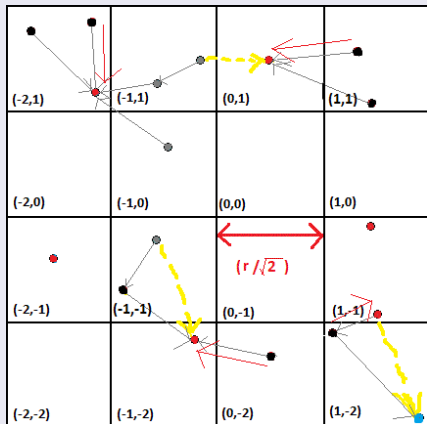
Tree Cutter

TC - Step 5



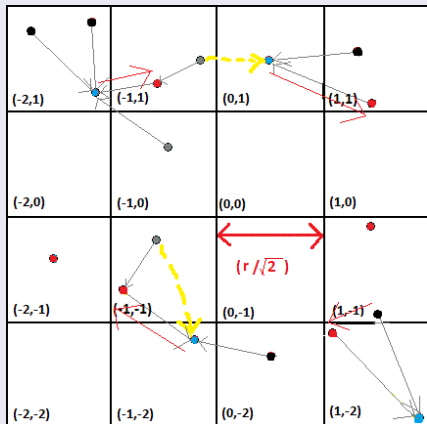
Tree Cutter

TC - Step 6



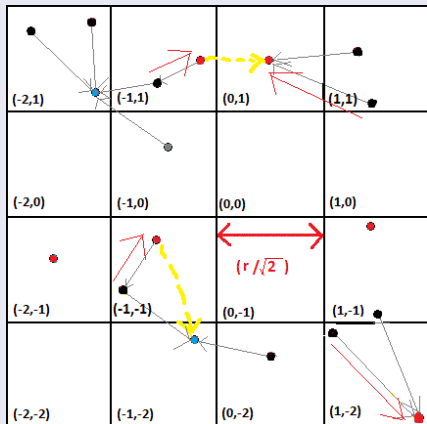
Tree Cutter

TC - Step 7



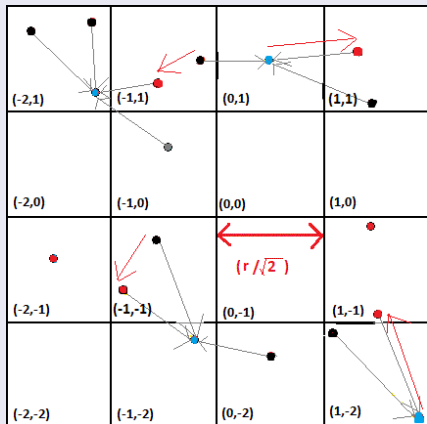
Tree Cutter

TC - Step 8



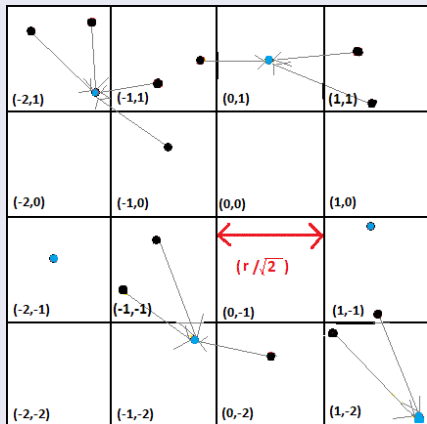
Tree Cutter

TC - Step 9



Tree Cutter

TC - Finally



- All nodes either leaders or followers.
- At most one leader per grid box.
- All trees of height 1 or less.

Tree Cutter - Potential Leader Election

- Till now we have not made use of the Potential Leader Election part of the algorithm.
- We present an example which shows how it works.
- It is only used by nodes who get the token but have not oriented themselves to a leader yet.

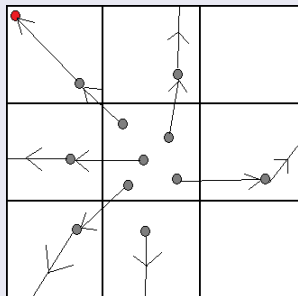
Tree Cutter - Potential Leader Election

Potential Leader Election, run by each node u

- 1: Execute SSF:
- 2: Transmit u 's label. Record other nodes heard in every round.
- 3: Execute SSF:
- 4: Transmit info about which nodes u heard and which round heard in during previous ssf.
- 5: Determine which round is the round in which u **alone** transmits in the ssf. Call it R .
- 6: **for** $i \leftarrow 1, c_1 \log N$ **do**
- 7: Execute SSF:
- 8: If $i = R$ and u is not follower, transmit, else stay silent.
- 9: If u transmits, become leader. If u hears from another node v and u is not a leader/follower, become v 's follower.
- 10: **end for**

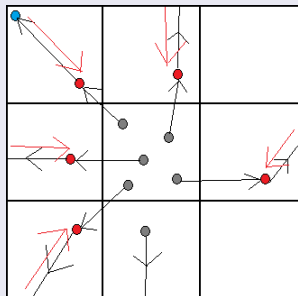
Potential Leader Election

PLE - Initially



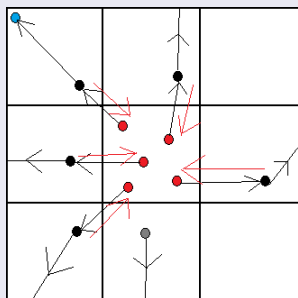
Potential Leader Election

PLE - Step 1



Potential Leader Election

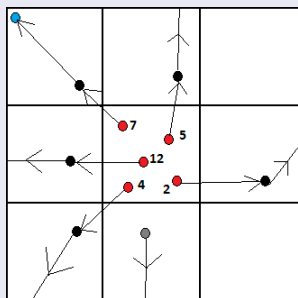
PLE - Step 2



- Now, these nodes with tokens haven't oriented themselves to any leader. So now they participate in Potential Leader Election.

Potential Leader Election

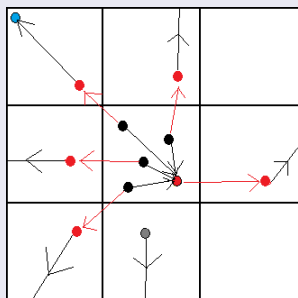
PLE - Step 3



- They've figured out the number of the round in which they alone transmit to the others.
- Notice that the grey node outside does not get involved. Reason - no token.

Potential Leader Election

PLE - Finally



- Tokens sent back. Now the leader has his own token. Others recognize him as leader and have already transmitted the same.

Claims

- 1 There is at most one leader per grid box of pivotal grid.
- 2 Cuts the trees to height at most one.
- 3 Running time - $O(n \log^2 N)$.

Proof Sketch of Correctness (Claim 1)

- **Goal:** At most one leader per grid box of pivotal grid (**Invariant 1** across phases of Tree Cutter).
- When is this true? When Potential Leader Election works as desired, i.e. no more than one leader created per grid box (Lemma 8).
- But for Potential Leader Election and Tree Cutter to work properly, there must be a constant no. of tokens in any grid box of pivotal grid at the beginning of every phase of Tree-Cutter (**Invariant 2**).

- Intertwined proof.

Step 1: Assume Inv. 2 holds in every phase $\leq i$ of Tree-Cutter, then Inv. 1 holds in phase i (Lemma 9).

Step 2: To prove Inv. 2, we need to bound the number of tokens that can get into a grid box. We do this by bounding the distance a token can move away from its leader (Lemma 10).

Step 3: Strong induction on Inv. 2, to show it holds with help from Lemma 9 and Lemma 10.

Proof Sketch of Correctness (Claim 2)

- **Goal:** All trees cut down to height at most one.
- Using Tree Cutter & Potential Leader Election, a node will be either a leader or declare itself a follower of a leader within range of it.
- We show that after $2 \cdot 947 \cdot (n + 1) - 1$ phases of Tree Cutter, every node has a chance to transmit its status (i.e. gets to participate in Potential Leader Election if necessary).
- \therefore After so many phases, all trees cut down to size.

Proof of Running Time (Claim 3)

- Potential Leader Election performs $O(c^2 \log N)$ executions of an (N, c) -ssf of size $O(c^2 \log N)$. Since c is a constant, it takes $O(\log^2 N)$ rounds.
- We perform $O(n)$ executions of Potential Leader Election. Total running time = $O(n \log^2 N)$ rounds.

Now what?

- Now we can actually perform multi-broadcast.

Outline

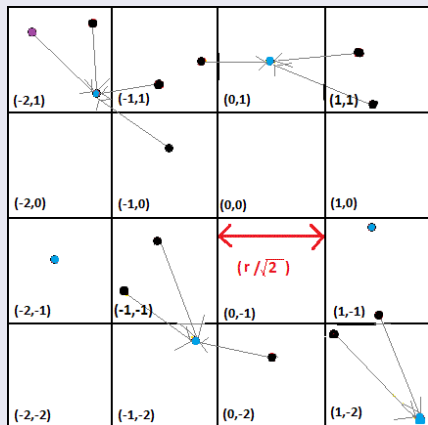
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- Input: Several nodes, each with a different piece of information.
- Output: Every node has every piece of information.
- First we look at the problem of Broadcast, where only one node has info and it needs to be spread.
- After showing how this is done, we show how Multi-Broadcast is achieved in a similar manner.

Broadcast, run by each node u

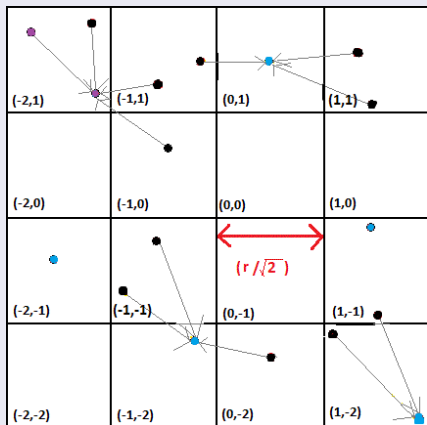
- 1: Run Tree Grower.
- 2: Run Tree Cutter.
- 3: Initially leaders have tokens.
- 4: **for** $cnt \leftarrow 1, 4 * n$ **do**
- 5: Execute SSF:
- 6: If u has token and info, transmit it. Else stay silent.
- 7: Execute SSF:
- 8: Pass token, if any, in DFS manner.
- 9: **end for**

Broadcast - Initially



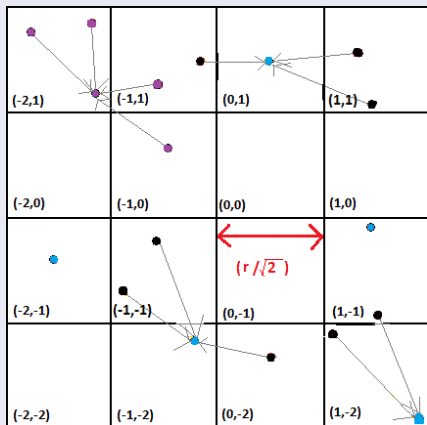
- Initially one node has info.

Broadcast - Step 1



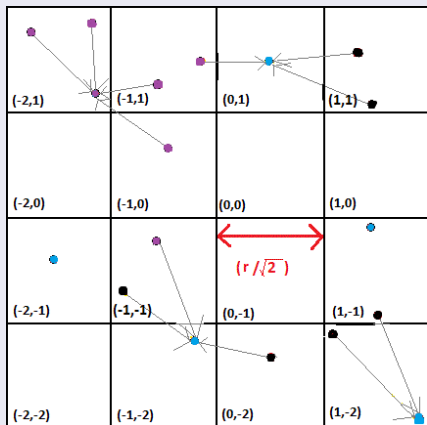
- When token comes to node, it transmits and its leader hears the info.

Broadcast - Step 2



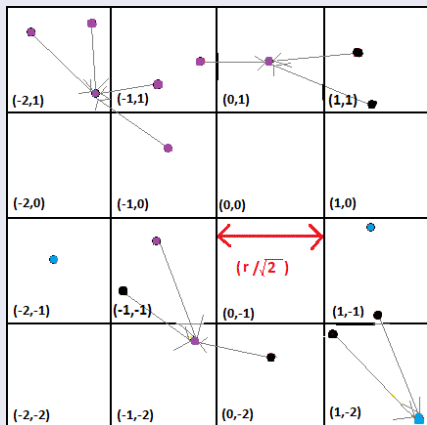
- Leader gets token, transmits, and all its children get info.

Broadcast - Step 3



- Once all children transmit, since connected graph, someone new will hear.

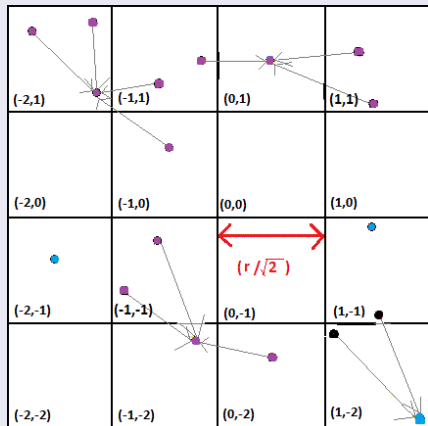
Broadcast - Step 4



- And the cycle continues...

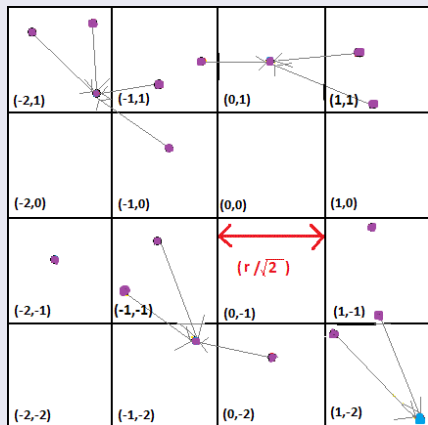
Broadcast

Broadcast - Step 5

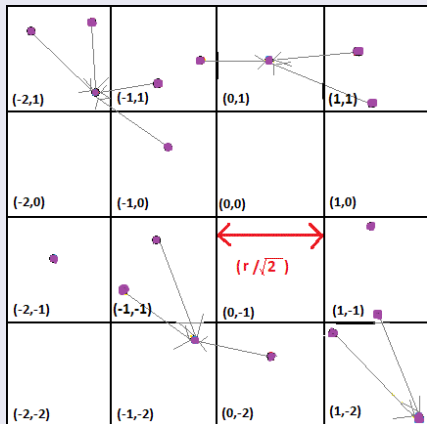


Broadcast

Broadcast - Step 6



Broadcast - Finally



- Until finally all nodes have info.

Claims

- 1 Takes a message at one of the nodes and spreads it throughout the network.
- 2 Running time - $O(n \log^2 N)$.

Proof Sketch of Correctness (Claim 1)

- If a node with message belongs to a tree of size s_i , it takes at most $O(s_i \log N)$ time for the node to get a token.
- Once that node transmits, it takes at most $O(s_i \log N)$ time for all nodes in that tree to transmit.
- Let us say that a tree has a message if one of its nodes has a message. If all nodes belonging to trees with messages transmit, and if there still exist nodes in the network without the message, then at least one node belonging to a new tree will now have the message because the underlying communication graph is connected.

Claims

- 1 Takes a message at one of the nodes and spreads it throughout the network.
- 2 Running time - $O(n \log^2 N)$.

Proof of Running Time (Claim 2)

- Each node executes two (N, c) -ssfs $4n$ times. Total running time = $O(n \log N)$ rounds.
- Overall running time from start to finish = $O(n \log N) + O(n \log^2 N) + O(n \log N) = O(n \log^2 N)$ rounds.

- In Multi-Broadcast, more messages.
- Large enough message size ensures that even if there are n pieces of information, still only one round reqd. to transmit them.
- Each piece of info takes same time to traverse network.
- Therefore, just have nodes always transmit messages (if they have one) when they get token.
- Running time is same, i.e. $O(n \log^2 N)$ rounds from start to finish.

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Problem Statement

Problem Statement

Initially, only a subset of the total nodes are awake. Our goal is to wake them all up.

Our Contribution

We achieve deterministic non-spontaneous wakeup w/o knowledge of coordinates in $O(n \log^2 N \log n)$ rounds.

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- We use Tree Grower, Tree Cutter, and another algorithm called Token Passing Transfer as subalgorithms in our main algorithm, which we call Multi-Wakeup.
- We also use techniques of slotting and multiplexing and the idea of epochs.

Token Passing Transfer

- Input: The trees obtained from running Tree Grower and Tree Cutter on participating nodes.
- Output: Any sleeping nodes within range are woken up.

Token Passing Transfer

Token Passing Transfer, run by each node u

- 1: Initially leaders have tokens.
- 2: **for** $cnt \leftarrow 1, 4 \cdot n$ **do**
- 3: Execute SSF:
- 4: If u has token, transmit wakeup message. Else stay silent.
- 5: Execute SSF:
- 6: Pass token, if any, in DFS manner.
- 7: **end for**

- When we run an algorithm in slot i , it means that we are assuming at most 2^i nodes will participate in that algorithm in that slot.
- We have $\lfloor \log n \rfloor + 1$ slots in Multi-Wakeup.

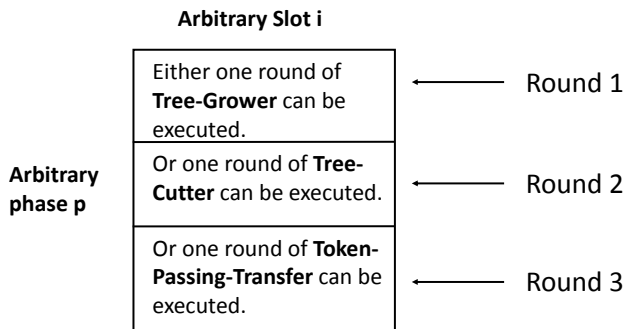


Figure: One phase of one slot for a given awake node.

Epochs

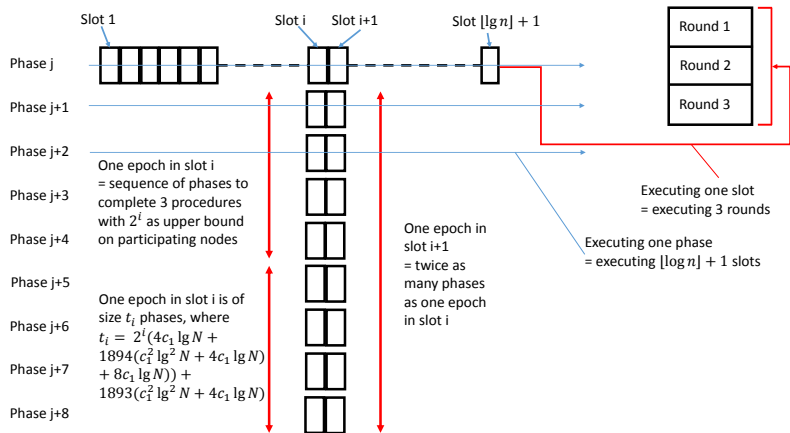


Figure: One epoch across multiple phases for a given slot.

Epochs (contd.)



Figure: Phases, slots, & epochs together.

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Overview of Approach

- Consider Tree Grower, Tree Cutter and Token Passing Transfer as one combo pack $T3$.
- As soon as a node wakes up, it begins $T3$ in each slot at the first opportunity (next epoch of that slot).
- In one of the slots, the correct assumption on no. of nodes participating will be made.
- After $T3$ finishes executing in that slot, if there are any nodes within range that are asleep, they will be woken up.
- The cycle continues until all nodes are woken up.

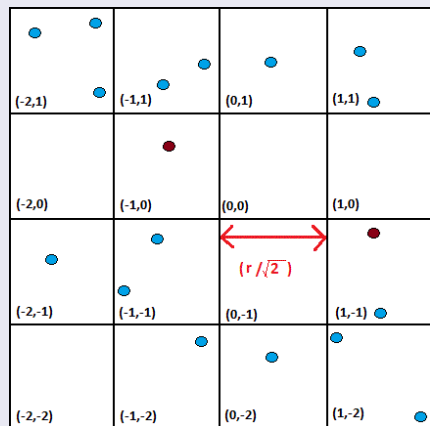
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Multi-Wakeup, run by each node u

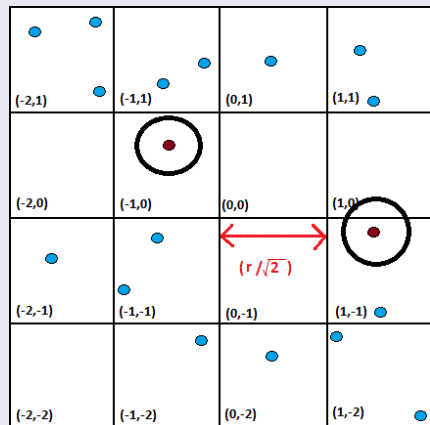
```
1: Initially leaders have tokens.
2: for  $phase \leftarrow 0, 4nt_1 - 1$  do
3:   for  $slot \leftarrow 1, \lfloor \log n \rfloor + 1$  do
4:     If asleep, stay silent for 3 rounds.
5:     Else if awake and Tree Grower not started yet or going on
6:       If not executing anything yet, wait (stay silent) till the start of next epoch.
7:       If waiting over, start executing Tree Grower. Execute one round of it.
8:     Else if Tree Grower over, start executing Tree Cutter. Execute one round of it.
9:     Else if Tree Cutter over, start executing Token Passing Transfer. Execute one round
of it.
10:    Else if Token Passing Transfer over, stay silent for 3 rounds.
11:   end for
12: end for
```

Multi-Wakeup - Initially



- Initially some nodes awake.

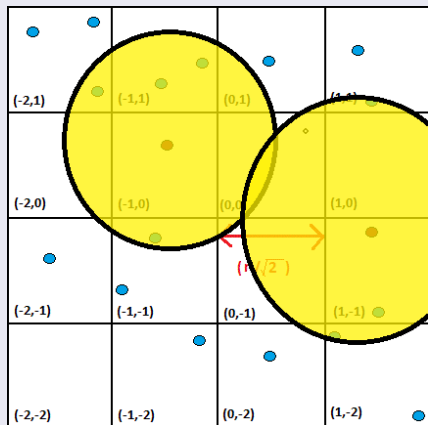
Multi-Wakeup - Step 1



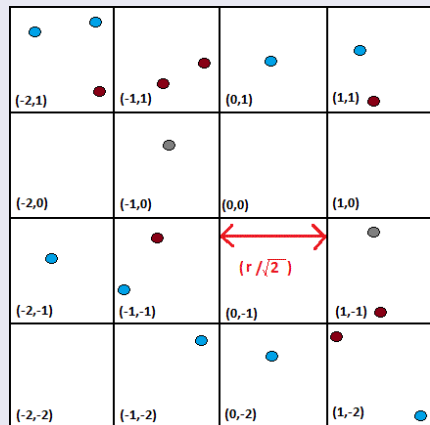
- They execute in slot 1.

Multi-Wakeup

Multi-Wakeup - Step 2



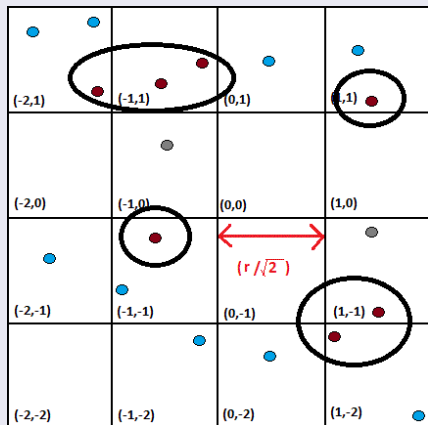
Multi-Wakeup - Step 3



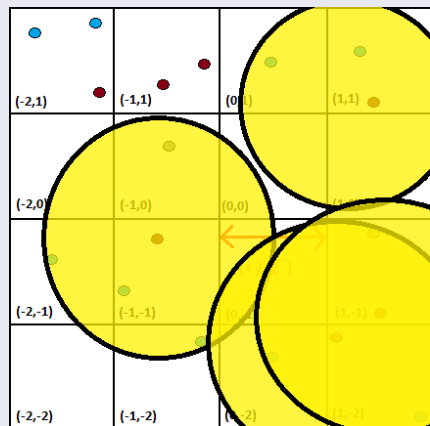
- New nodes woken up.

Multi-Wakeup

Multi-Wakeup - Step 4



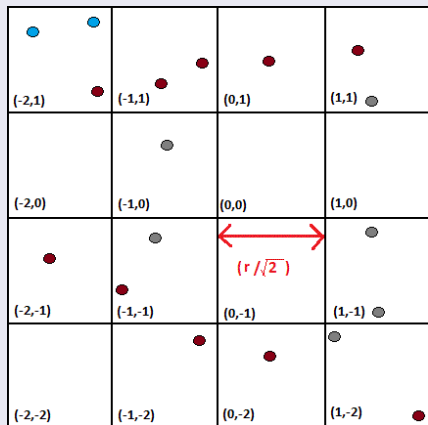
Multi-Wakeup - Step 5



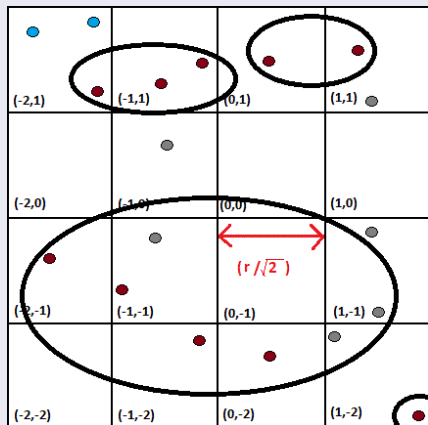
- Only those in slot 1 can execute immediately.

Multi-Wakeup

Multi-Wakeup - Step 6

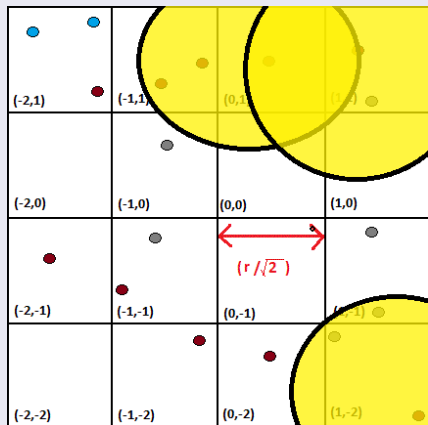


Multi-Wakeup - Step 7



- Larger group of woken up nodes absorbs nodes which haven't executed in a higher slot yet.

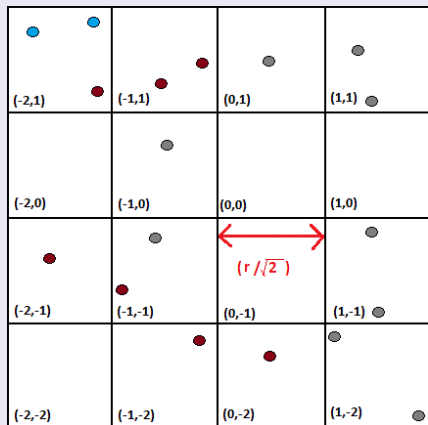
Multi-Wakeup - Step 8



- Slot 1 execution.

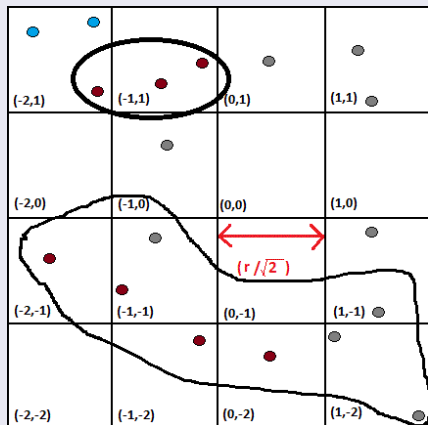
Multi-Wakeup

Multi-Wakeup - Step 9

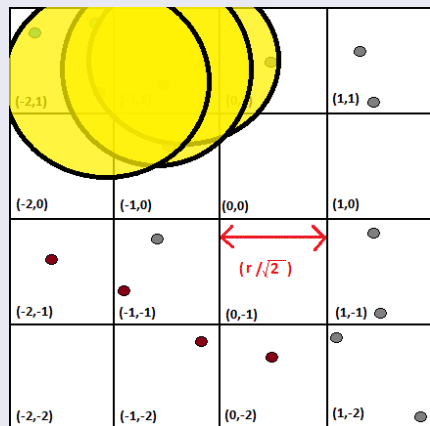


Multi-Wakeup

Multi-Wakeup - Step 10



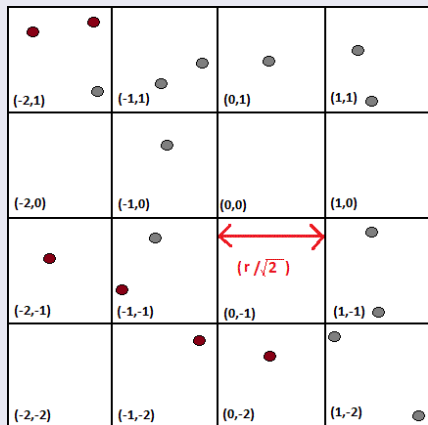
Multi-Wakeup - Step 11



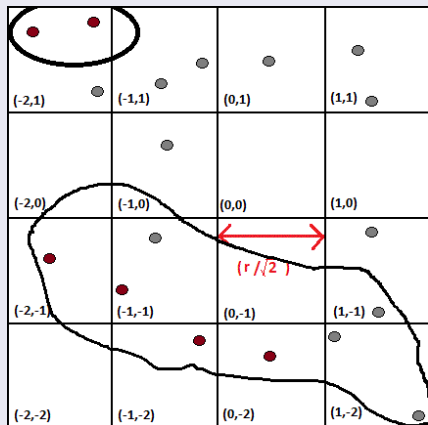
- Slot 2 execution.

Multi-Wakeup

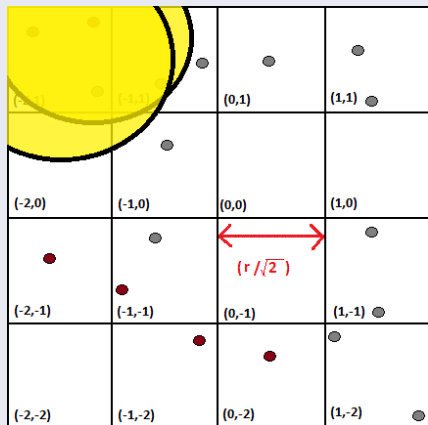
Multi-Wakeup - Step 12



Multi-Wakeup - Step 13



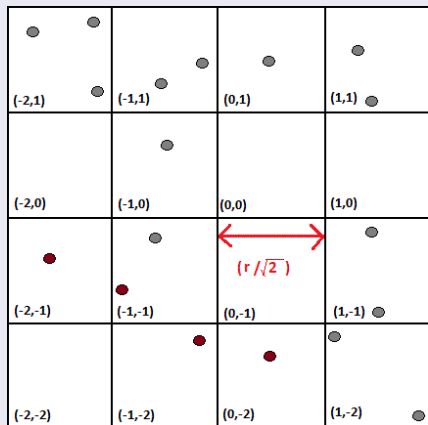
Multi-Wakeup - Step 14



- Slot 1 execution.

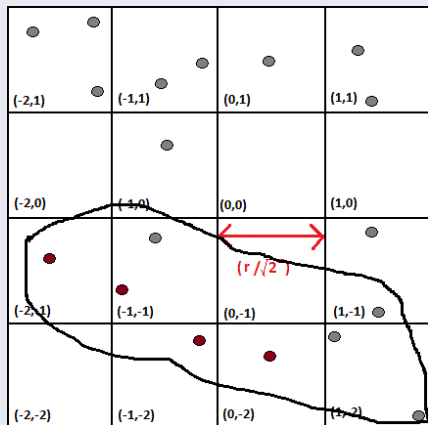
Multi-Wakeup

Multi-Wakeup - Step 15

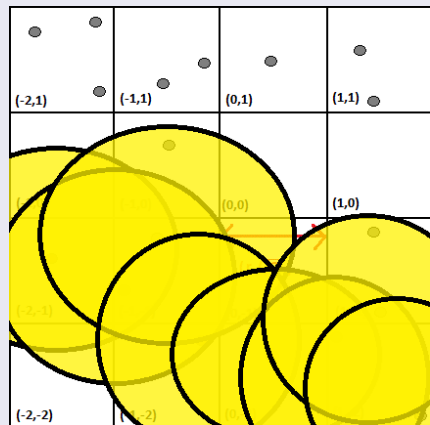


Multi-Wakeup

Multi-Wakeup - Step 16

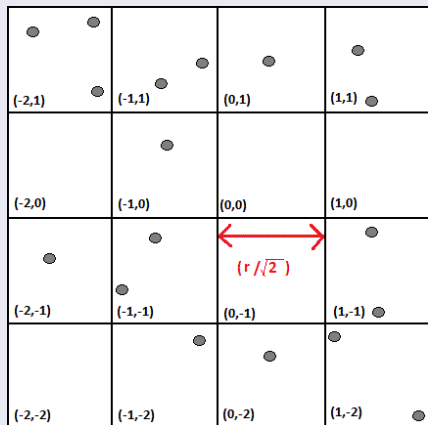


Multi-Wakeup - Step 17



- Next epoch for slot 3 arrives. Slot 3 execution.

Multi-Wakeup - Finally



- All nodes awake.

Proof Sketch of Correctness & Running Time

- The worst case scenario for running time is when nodes wake up in increasingly larger batches and cause other nodes to keep participating in larger slots.
- This worst case is covered by waiting the time period required for 2 executions of T3 in the largest slot ($\lfloor \log n \rfloor + 1$) to complete.
- This takes at most $O(n \log^2 N \log n)$ rounds.
- Running our algorithm that long ensures all nodes wake up. Also, running time of algorithm is fixed.

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Other Results Obtained using SSF-based Dilution





- When knowledge of N , n , and your label is known, solved problem of deterministic creation of backbone. Running time = $O(n \log^2 N)$ rounds.
- When knowledge of N , n , your own label and your neighborhood is known, solved problem of deterministic creation of backbone. Running time = $O(\Delta \log^2 N)$ rounds.

Possible Future Research

- When only knowledge of N and your label is known, solve the following:
 - 1 Broadcast.
 - 2 Backbone creation.
 - 3 Non-spontaneous wake-up.

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The End