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# Hamiltonicity and Longest Path Problem on Special Classes of Graphs

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### Fault Tolerance and Hamiltonicity of the Optical Transpose Interconnection System of Non-Hamiltonian Base Graphs



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# Optical Transpose Interconnection System(OTIS)

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### Definition (Marsden et al.- 1993)

- OTIS is a widely studied interconnection network topology
- In this architecture, processors are divided into groups (called clusters)
- Processors within the same group are connected using electronic interconnects, while optical interconnects are used for intercluster communication.
- Power consumption is minimized and the bandwidth rate is maximized when the number of processors in a cluster equals the number of clusters



# **OTIS** Arrangement

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### Figure: (a) Base Network(b) OTIS network



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### Optical Transpose Interconnection System

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# The Hamiltonian Cycle problem

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### The problem

A Hamiltonian cycle is a spanning cycle in a graph, i.e., a cycle through every vertex, and a Hamiltonian path is a spanning path. A graph containing a Hamiltonian cycle is said to be Hamiltonian.

### Importance

- Hamiltonicity is important to ensure deadlock freedom in some routing algorithms and to allow efficient emulation of linear-array and ring algorithms.
  - All-to-all broadcasting or total exchange algorithms relies on a Hamiltonian cycle for its efficient execution.
- Hamiltonicity is closely related to fault tolerance.



# Fault Tolerance and Independent Spanning Tree

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### Major Faults in Distributed Systems

- Dead processor fault(due to failure of processor or support chip).
- Dead interprocessor communication(due to failure of communication hardware).
- Independent Spanning Trees ensure the presence of parallel, node-disjoint paths between nodes of the network.

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# Independent Spanning Tree(IST)





# Hamiltonian Cycle and IST count



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Figure: Illustration

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# Previous Work done

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# Hamiltonicity of OTIS graphs

- Parhami proved that OTIS networks built of Hamiltonian basis networks are Hamiltonian.
- Hoseinyfarahabady et al. showed that the OTIS-Network is Pancyclic and hence Hamiltonian, if its base network is Hamiltonian-connected.





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# Our Contribution

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### Workdone

- We prove that Hamiltonicity of base graph is not a necessary condition for the OTIS to be Hamiltonian.
- We further prove that it is not sufficient for the base graph to have Hamiltonian path, for the OTIS to be Hamiltonian.
- We consider the generalization of this butterfly/bowtie graph, BF(n,m) as base network, give constructive proofs for Hamiltonicity, on Bowtie-OTIS of BF(2m + 1, 2n + 1) and BF(2m + 1, 2k), where  $m, n, k \in \mathbb{N}$ .
- This construction leads to an efficient alternate linear time rooted Independent Spanning Tree construction algorithm on this class of Bowtie-OTIS graphs.



# Butterfly/Bowtie Base Graph and OTIS

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Figure: (a) Butterfly or Bowtie Graph BF(3,3) (b) OTIS on BF(3,3)



# Hamiltonian OTIS on Non-Hamiltonian Base Graphs

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Figure: Hamiltonian edges shown in red



# Hamiltonian Path on Base Graph does not gurantee Hamiltonian Cycle on OTIS

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Figure: Total number of edges in OTIS(BF(4,4)) is  $\frac{i=1}{2} = 77$ .

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# Inference Rules

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### Inference Rules

IR 1: If a vertex of degree  $\geq 3$ , gets saturated, the rest of its edges, not used in the saturation, becomes Non-Hamiltonian edges and are deleted from the graph.

IR 2: If  $(\langle g_1, u \rangle, \langle g_2, v \rangle)$  is an edge between the vertices  $\langle g_1, u \rangle$ and  $\langle g_2, v \rangle$ , both of degree 3, and if the edge  $(\langle g_1, u \rangle, \langle g_2, v \rangle)$  is identified as Non-Hamiltonian, then all other edges incident to the vertices  $\langle g_1, u \rangle$  and  $\langle g_2, v \rangle$  are forced to be Hamiltonian.



# Algorithm Overview

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### Algorithm

- 1: We identify the key Non-Hamiltonian edges whose endpoints lies within the same cluster, explicitly and delete them.
- 2: In this process some vertices becomes saturated; we apply IR 1 on these vertices.
- 3: The previous step, in turn decides Hamiltonian edges of the remaining vertices(due to IR 2).

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# Construction for OTIS(BF(3, 2k))

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### Step 1:

Cluster 1: (3, 2), (3, 4), (3, i)Cluster 2: (3, 1), (3, 4) And the set  $S_2 = \{(5, 6), (7, 8), \dots, (i - 1, i)\}$ Cluster 3: (3, 1), (3, 4)Cluster 4: (3, 1), (3, 2), (3, i)Cluster 5, 6, ..., (i - 1): (3, 2), (3, i)Cluster i: (3, 1), (3, 2) and the set  $S_i = \{(4, 5), (6, 7), \dots, (i - 2, i - 1)\}$ 



Complexity

# Illustration with OTIS(BF(3,2k)), k = 2



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# Step 1

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Figure: After execution of Step 1

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# Step 2

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Figure: After execution of Step 2



# Independent Spanning Tree Construction

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### IST Construction

- Pick any vertex as root and denote it as r
- Delete one edge incident to r. This gives a spanning Tree  $T_1$
- Now, retain the edge previously deleted, and delete the other edge incident to r. This gives another spanning tree  $T_{\rm 2}$



# Time Complexity

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### Complexity Analysis

- Consider OTIS(BF(2m + 1, 2n + 1)) and (OTIS(BF(2m + 1, 2k))); the number of intracluster edges, deleted per cluster, in these constructions, is of O(m), assuming m > n, m > k
- Hence the Hamiltonian Cycle construction takes time  $O(m|V_B|)$ , which is of  $O(m^2)$ , i.e., linear in number of vertices of the OTIS graph.



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### A Polynomial Time Algorithm for Longest Paths in Biconvex Graphs

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# Doubly Convex or Biconvex Graphs

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### Definition (Glover-1967)

A bipartite graph  $G = (S \cup T, E)$  is doubly convex or biconvex

if  $\exists$  a numbering  $1,2,\ldots,|S|$  of the vertices in S and a numbering of the vertices  $1,2,\ldots,|T|$  in T such that

 $\forall v \in S \cup T$ , N(v) is a set of consecutive integers.



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S-Partition

**T-Partition** 

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# The Longest Path problem

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### The problem

The longest path problem is the problem of finding a simple path of maximum length in a given graph.

### Importance

- The well-known Travelling Salesman problem and Hamiltonian Path problem are special cases of Longest Path problem.
- The longest path in program activity graph is known as critical path, which represents the sequence of program activities that take the longest time to execute.



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# Polynomial Solution for longest path problem on some graph classes

- Polynomial time algorithms for longest path exist for graph classes like Trees, Cacti, Ptolemic graphs, Bipartite Permutation graphs, proposed by Uehera et al.
- Recently polynomial time solutions for the longest path problem is proposed by loannidou et al. on interval graphs and cocomparability graphs.



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# Our Contribution

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### Workdone

• We have proposed a  $O(n^6)$  time algorithm to find the longest path on biconvex graphs, where n is the number of vertices of the input graph.

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• We have used Dynamic Programming approach.



# The Ordering

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### Ordering the vertices

• Let  $\pi_1 = (s_1, s_2, \cdots, s_{|S|})$  be the labeled vertices of partition S, and  $\pi_2 = (t_1, t_2, \cdots, t_{|T|})$  be that of partition T

- Initialize  $\sigma_S = \pi_1$ .
- Update σ<sub>S</sub> as follows: For all t<sub>i</sub> in π<sub>2</sub>, insert t<sub>i</sub> immediately after its rightmost neighbor.



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Figure: 
$$\pi_1 = (s_1, s_2, s_3, s_4, s_5)$$
 and  $\pi_2 = (t_1, t_2, t_3, t_4)$   
 $\sigma_s = (s_1, s_2, t_1, s_3, s_4, t_3, t_4, s_5, t_2)$ 



# Monotonic Path

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### Definition

A S-Monotone path of a Biconvex graph  $G = (S \cup T, E)$  is a simple path  $P = \{s_{\alpha_1}, t_{\beta_1}, s_{\alpha_2}, \dots, t_{\beta_{j-1}}, s_{\alpha_j}, t_{\beta_j}\}$  such that  $s_{\alpha_k} \prec_{\sigma_S} s_{\alpha_{k+1}} \forall \ k \ge 1 \le k \le j$ .

Symmetrically, we define T-Monotone path.



### Lemma

Lemma

Let

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•  $P = \{s_{\alpha_1}, t_{\beta_1}, s_{\alpha_2}, t_{\beta_2} \dots s_{\alpha_{j-1}}, t_{\beta_{j-1}}, s_{\alpha_j}, t_{\beta_j}\}$  be a simple path of a biconvex graph  $G = (S \cup T, E)$ .

• Let  $P_{max}$  denote the longest S-S sub path of P.

Then, the vertices on the path  $P_{max}$  can be reordered to get a path  $P'_{max}$  on the same set of vertices, which is S-Monotone.



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Figure: Case  $\beta \prec \alpha \prec \gamma$ (a)Non S-Monotone Path (b)S-Monotone Path



Case  $\beta \prec \gamma \prec \alpha$ 

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Figure: (a) Non S-Monotone Path (b) S-Monotone Path

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# **Proof Sketch**

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### Inductive hypothesis:

For the longest S-S sub path of a simple path  $P = \{s_{\alpha_1}, t_{\beta_1}, s_{\alpha_2}, \dots, t_{\beta_{j-1}}, s_{\alpha_j}, t_{\beta_j}\}$ , there exists a path  $P' = \{s_{\gamma_1}, t_*, s_{\gamma_2}, \dots t_*, s_{\gamma_j}\}$  satisfying the *S*-Monotonicity Property.

### Inductive Step

We prove the lemma by induction on |S|. Hence we consider a path  $P_1=P \cdot s_{\alpha_{j+1}}$  and prove by principle of induction that the longest S-S subpath satisfying the S-Monotonicity property exists.



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# Algorithm Overview

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### First Part of the Algorithm

Given the ordering  $\sigma_S = (u_1, u_2, \cdots, u_n)$ , for all  $s_i, s_j$ , where,  $1 \le i < j \le |S|$ , do the following:

- Choose the subsequence  $\sigma_{Sij} = (u_k, u_{k+1}, \cdots, u_m)$  such that:
  - $u_k$  is the vertex  $s_i$
  - $u_m$  is  $s_j$ , if  $s_{j+1}$  immediately succeeds  $s_j$  in  $\sigma_S$ .
  - Otherwise,  $u_m$  is the rightmost T-vertex that lies between  $s_j$  and  $s_{j+1}$  in  $\sigma_S$ .

• Run the "Longest Path" routine and remember the maximum path length obtained over these iterations and all the paths of that maximum length.



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Figure:  $\sigma_S = (s_1, s_2, t_1, s_3, s_4, t_3, t_4, s_5, t_2)$ 

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Figure:  $\sigma_S = (s_1, s_2, t_1, s_3, s_4, t_3, t_4, s_5, t_2)$  and  $\sigma_{S14} = (s_1, s_2, t_1, s_3, s_4, t_3, t_4)$ Longest S-Bimonotone Path is  $P = \{s_3, t_3, s_4, t_4\}$ 



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### Second Part of the Algorithm

Symmetric to Part 1, this part is executed for vertices of T-partition with the initial ordering  $\sigma_T = (u_1, u_2, \cdots, u_n)$ .



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### Third Part of the Algorithm

- Choose the maximum of the two lengths obtained as output from the first and second part of the algorithms. Let us denote this length as max
- For all paths of length *max*, check if any of the end vertices have unvisited neighbor.
- If such a neighbor exists, extend the path till that neighbor and declare it as a longest path and max + 1 as longest path length.
- Else declare *max* as the longest path length and all paths of this length as longest paths.



# The "Longest Path" Routine

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### Description

- ${\small \bigcirc}$  Iterate through all the vertices of  $\sigma_{S1n}$  from indices 1 to n
- 2 Initialize  $P(u_i; i, i)$  with  $u_i$  and  $l(u_i; i, i)$  with 1.
- For all  $u_k$ ,  $i \le k \le j-1$ , which are S-vertices, initialize  $P(u_k; i, j)$  with  $P(u_k; i, j-1)$  and  $l(u_k; i, j)$  with  $l(u_k; i, j-1)$
- ${\small \bigcirc}~$  If  $u_j$  is also a S-vertex , initialize  $l(u_j;i,j)$  with 1 and  $P(u_k;i,j)$  with  $u_j$
- **(**) If  $u_j$  is a T-vertex then execute process(G(i, j))
- Compute the max $\{(u_k; 1; n) : u_k \in S(G)\}$  and the corresponding path  $P(u_k; 1; n)$ .



# The process(G(i, j)) subroutine

process(G(i, j))

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# • For $y = f(u_j) + 1$ to j - 1, iterate through $x = f(u_j)$ to y - 1 if both $u_x$ and $u_y$ are S-vertices, then update the path as follows:

• 
$$w_1 \leftarrow l(u_x; i, j-1); P'_1 \leftarrow P(u_x; i, j-1)$$
  
•  $w_2 \leftarrow l(u_y; x+1, j-1); P'_2 \leftarrow P(u_y; x+1, j-1)$   
If  $w_1 + w_2 + 1 > l(u_y; i, j)$ , then  
•  $l(u_y; i, j) \leftarrow w_1 + w_2 + 1;$ 

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• 
$$P(u_y; i, j) = (P'_1, u_j, P'_2);$$

Return the value  $\{I(u_k; i, j)\}$  and the path  $\{P(u_k; i, j), \forall u_k \in S(G(f(u_j) + 1, j - 1))\}$ 



# Illustration

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Figure:  $\sigma_s = (s_1, s_2, t_1, s_3, s_4, t_3, t_4, s_5, t_2)$ 

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# Illustration



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Figure: Path getting updated at call process(G(4,6)) and x = 4, y = 5



# Illustration(Cont.)



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Figure: Path getting updated at call process(G(1,9)) with x = 5, y = 8



# Time Complexity

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### Complexity Analysis

- Generating the ordering  $\sigma_S$  and  $\sigma_T$  will take O(|S||T|) time.
- The subroutine process() takes  $O(n^2)$  time and is executed at most once for each subgraph G(i, j) of G. Hence takes  $O(n^4)$  time.

• Since the routine "Longest Path" is called for each ordered pair  $s_i, s_j$  (and  $t_i, t_j$ ), and there can be  $O(n^2)$  such ordered pairs, so the total running time is  $O(n^6)$ .



# Publications from this Thesis

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- Optical Transpose Interconnection System
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- Biconvex Graphs
- Longest Path problem
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- Esha Ghosh, N. S. Narayanaswamy, and C. Pandu Rangan. A Polynomial Time Algorithm for Longest Paths in Biconvex Graphs. To appear in: *Proceedings of Workshop* on Algorithms and Computation(WALCOM) 2011, pp.191-201, LNCS 6552, Springer-Verlag 2011.
- Esha Ghosh, Subhas K. Ghosh, and C. Pandu Rangan. On the Fault Tolerance and Hamiltonicity of the Optical Transpose Interconnection System of Non-Hamiltonian Base Graphs. under Review



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