

CS6848 - Principles of Programming Languages

Principles of Programming Languages

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- Structural subtyping
- Unification algorithm



Recall

$e ::= x \mid \lambda x.e \mid e_1e_2 \mid c \mid \text{succ } e$
 $x \in$ Identifier (infinite set of variables)
 $c \in$ Integer
 $v ::= c \mid \lambda x.e$
 $t ::= \text{Int} \mid t - > t$

Extending a language

- Extend the language grammar that will lead to new terms.
- Extend the allowed values.
- Extend the types.
- New operational semantics.
- New typing rules.



- Expressions

$$e ::= \dots | (e_1, e_2) | e.1 | e.2$$

- Values

$$v ::= \dots | (v_1, v_2)$$

- Types

$$t ::= \dots | t_1 \times t_2$$



Properties of pairs

- The components are evaluated left to right.
- The pair must be fully evaluated to get the components.
- A pair that is passed as an argument will be fully evaluated, before the function starts executing (in call by value semantics).



- First element:

$$(Pair \beta 1) (v_1, v_2).1 -> v_1$$

- Second element:

$$(Pair \beta 2) (v_1, v_2).2 -> v_2$$



$$[Projection 1] e -> e'.1 -> e'.1$$



$$[Projection 2] e -> e'.2 -> e'.2$$



$$[Pair Evaluation 1] e_1 -> e'_1(e_1, e_2) -> (e'_1, e_2)$$



$$[Pair Evaluation 2] e_2 -> e'_2(v_1, v_2) -> (v_1, e'_2)$$



Tuples

- Expressions

$$e ::= \dots | (e_i^{i \in 1..n}) | e.i$$

- Values

$$v ::= \dots | (v_i^{i \in 1..n})$$

- Types

$$t ::= \dots | (t_i^{i \in 1..n})$$



- Element j :

$$(\beta)(v_i^{i \in 1..n}).j - > v_j$$



[Projection 1] $e - > e' e.i - > e'.i$



[Tuple Evaluation] $e_j - > e'_j(v_i^{i \in 1..j-1}, e_j, e_k^{k \in j+1..n}) - > (v_i^{i \in 1..j-1}, e'_j, e_k^{k \in j+1..n})$



[Tuple] $A \vdash \forall i \ e_i : t_i A \vdash (l_i = e_i^{i \in 1..n}) : (l_i : t_i^{i \in 1..n})$



[Projection] $A \vdash e : (l_i : t^{i \in 1..n}) A \vdash e.l_j : t_j$

