

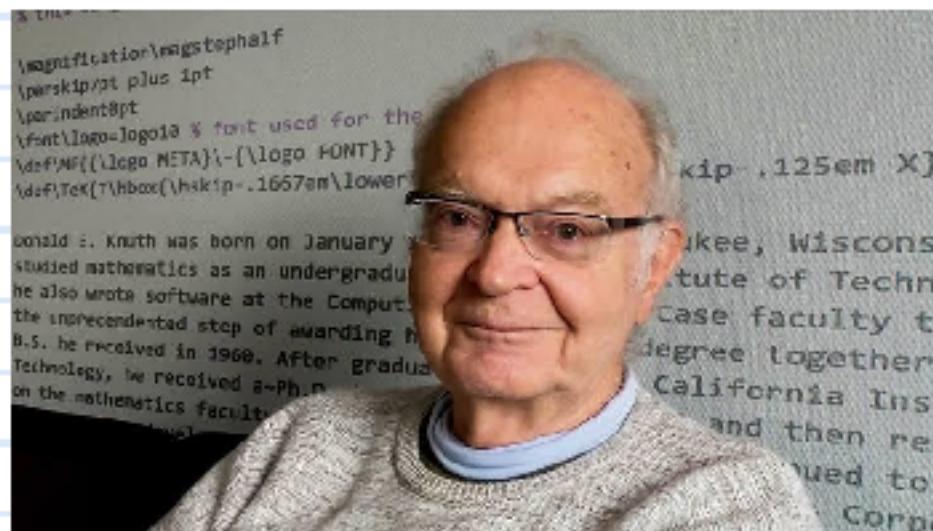
CS2700 : PROGRAMMING and DATA STRUCTURES.

TODAY : TWO SORTING ALGORITHMS.

1. HEAPSORT

2. QUICKSORT

SORTING : A FUNDAMENTAL PRIMITIVE.



THE CLASSIC WORK
NEWLY UPDATED AND REVISED

The Art of
Computer
Programming

VOLUME 3
Sorting and Searching
Second Edition

DONALD E. KNUTH

- * NEED TO OPTIMIZE SORTING ALGOs

CANT BE EMPHASISED ENOUGH

- A RANGE OF SORTING ALGOs

- different properties
satisfied

- what algos have you studied?

SORTING : A FUNDAMENTAL PRIMITIVE.

INPUT ; AN ARRAY OF INTEGERS

OUTPUT : INPUT SORTED IN (INCREASING) ORDER

POSSIBLE CONSTRAINTS / VARIANTS

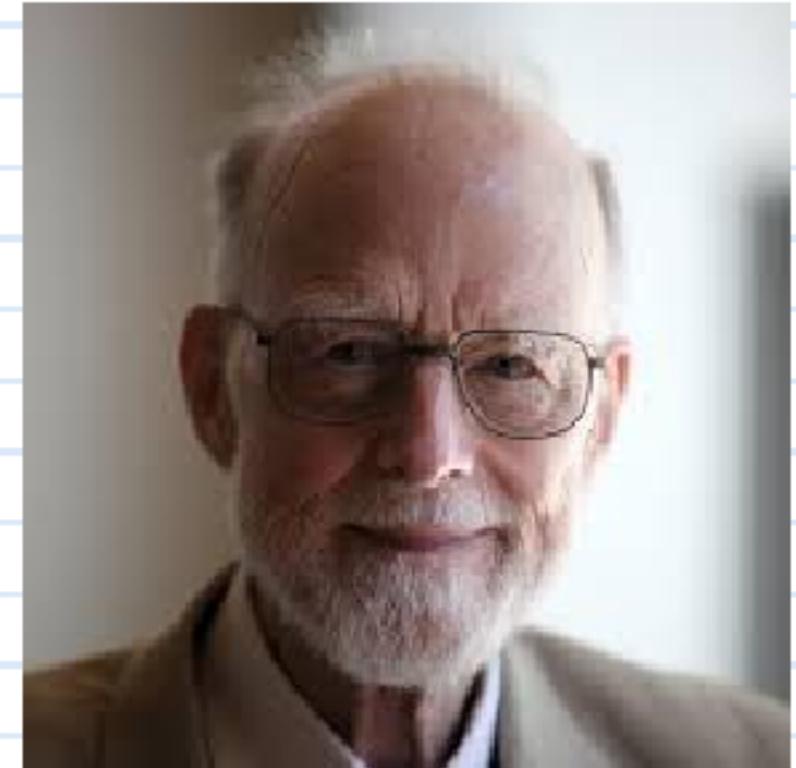
HEAP SORT

- Initialize a min heap with array elements.
- Repeatedly remove min element
(using deletemin)
and store it in an aux. array
(which is the output)

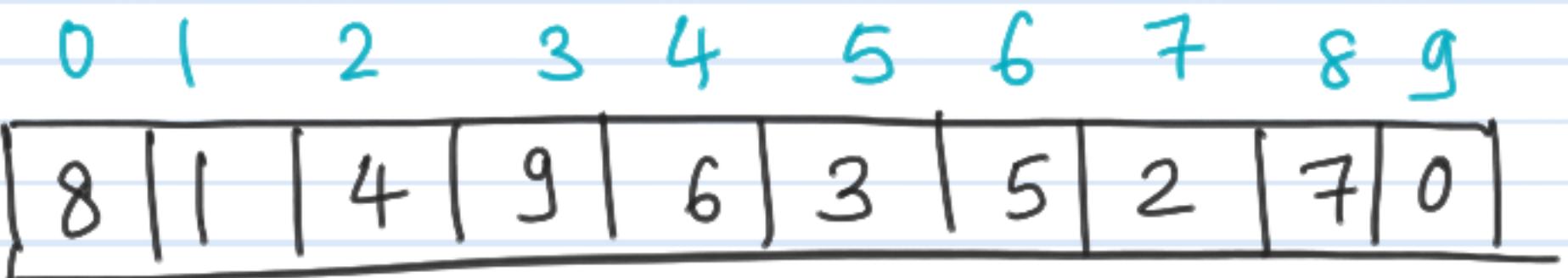
Running time : $O(n \log n)$

QUICK SORT

- A very fast in practice algo
- Needs careful implementation
- Has $O(n^2)$ worst case running time
 $O(n \log n)$ average case complexity.

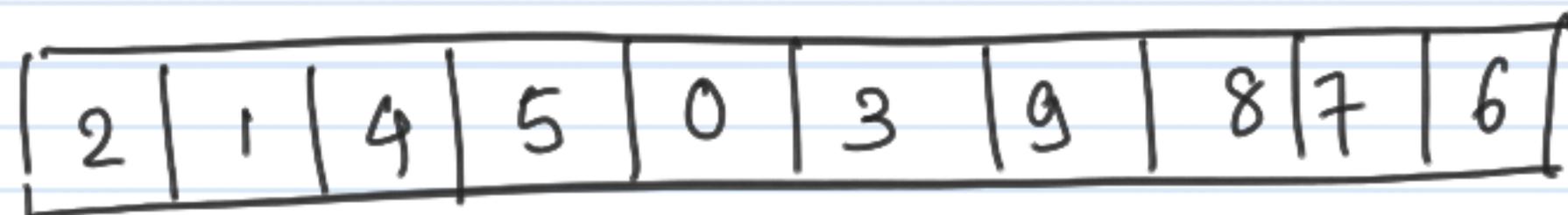
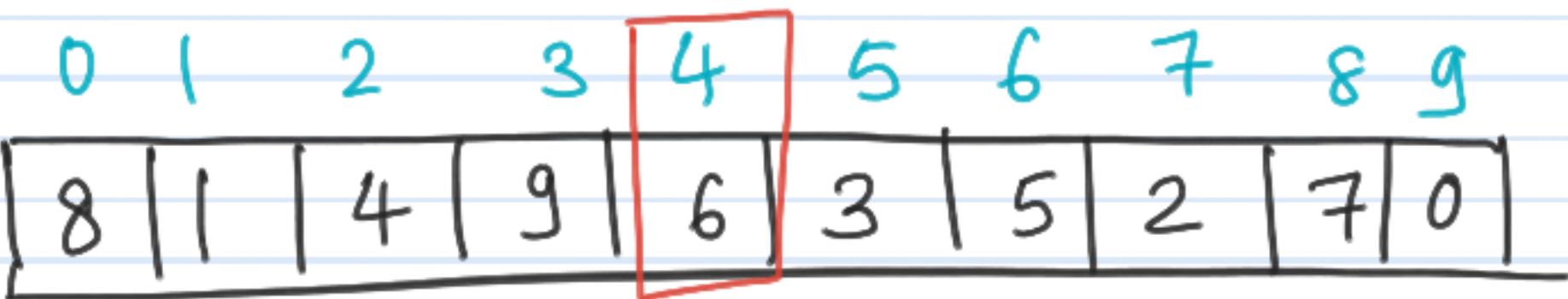


QUICK SORT : BASIC IDEA



- pick a pivot [let us say $A[4]$]
- find position of pivot in sorted array
[how??]

QUICK SORT : BASIC IDEA

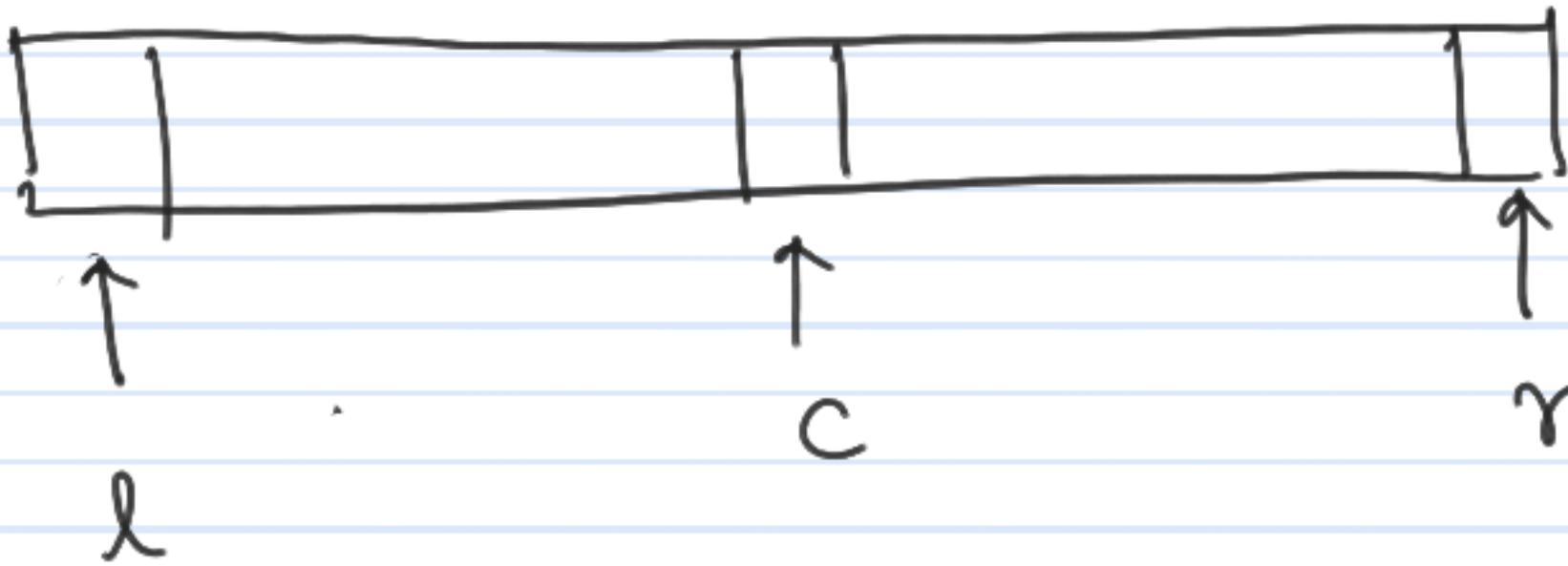


- pick a pivot [let us say $A[4]$]
- find position of pivot in sorted array [how??]
 - partition the array
- recursively sort left, sort right.

QUICK SORT

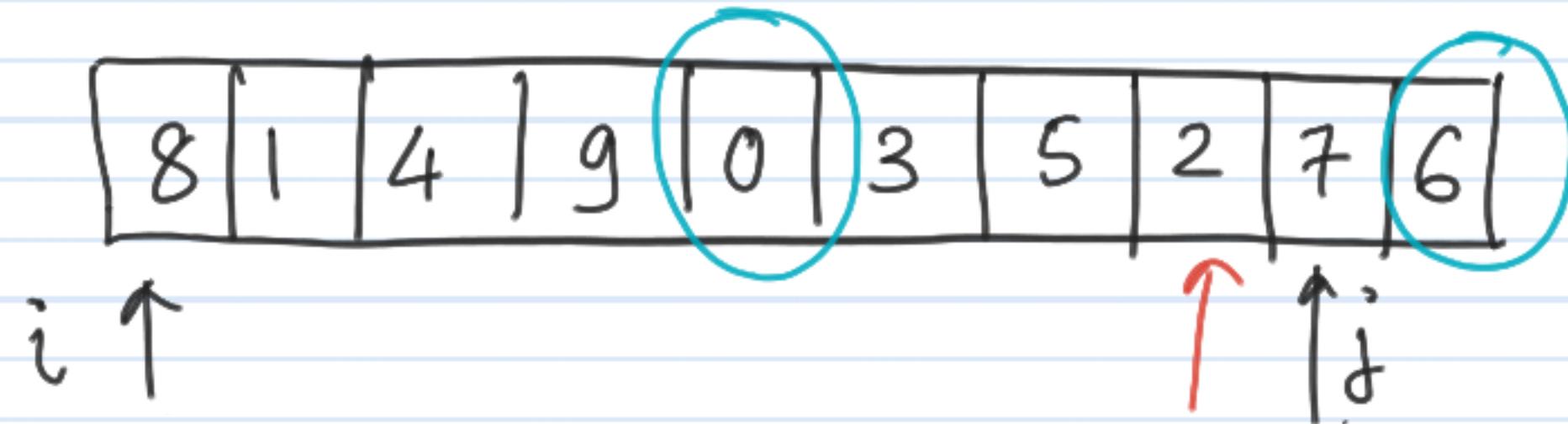
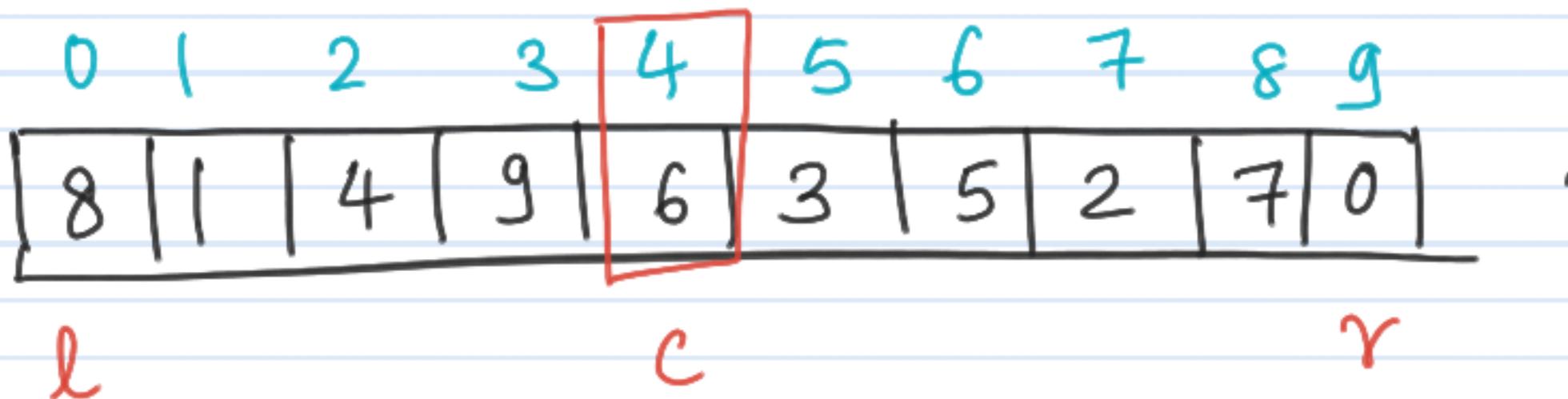
- Find a pivot
- partition the array based on pivot
 - multiple choices for selecting pivot,
 - first / last element
 - center element
 - random index
 - median of 3 elements

QUICK SORT : Selecting a pivot



- sort $A[l], A[c], A[r]$
- pick middle of the 3 as pivot
- get pivot out of the way.
 → this is a side effect.

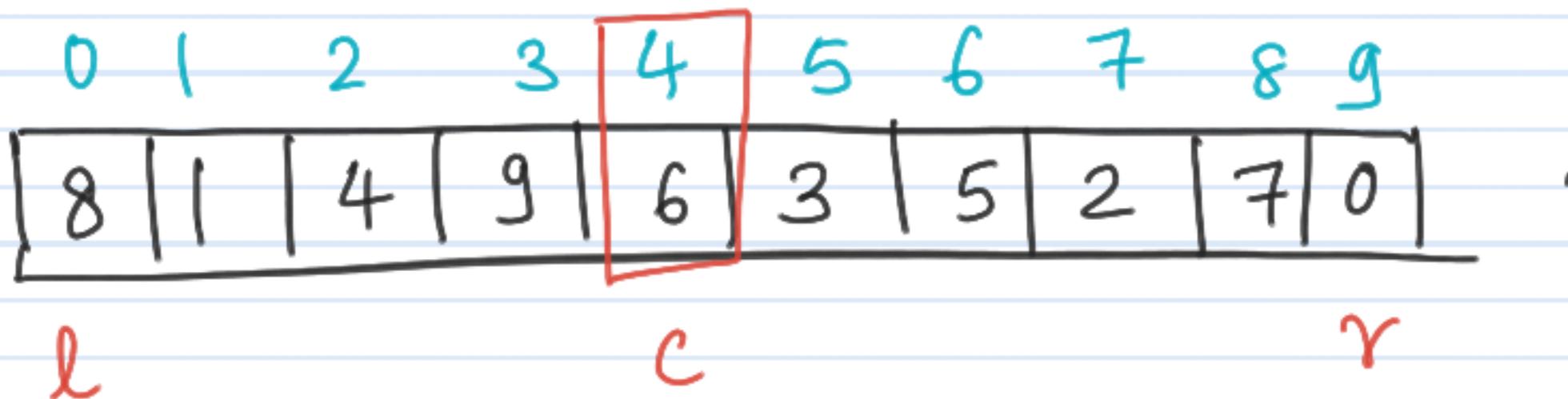
QUICK SORT : Selecting a pivot



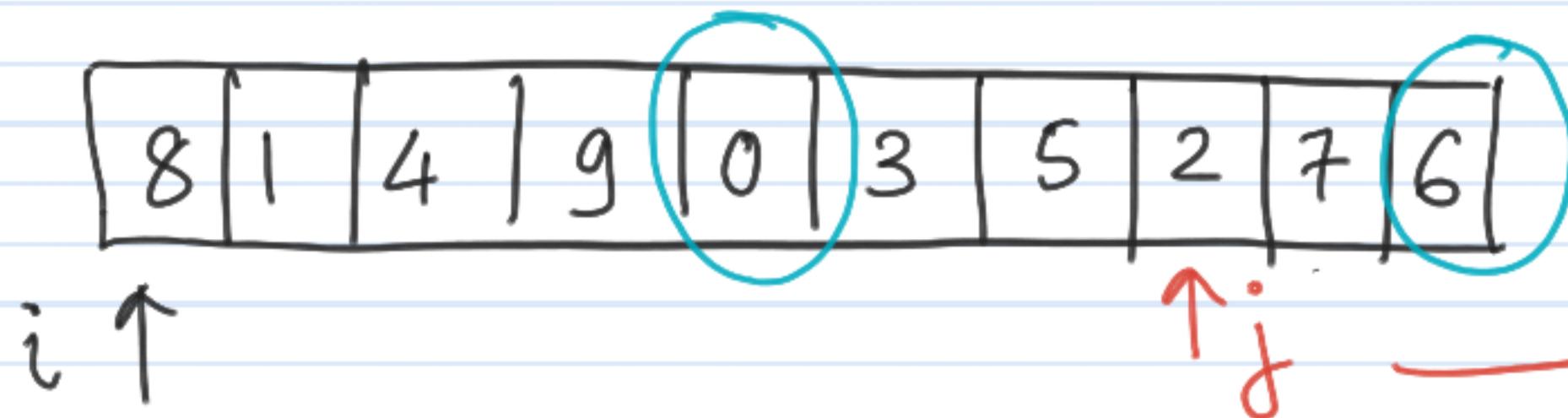
Partition :

- move all "small" elements to "left"
- move all "large" elements to "right"

QUICK SORT : Selecting a pivot



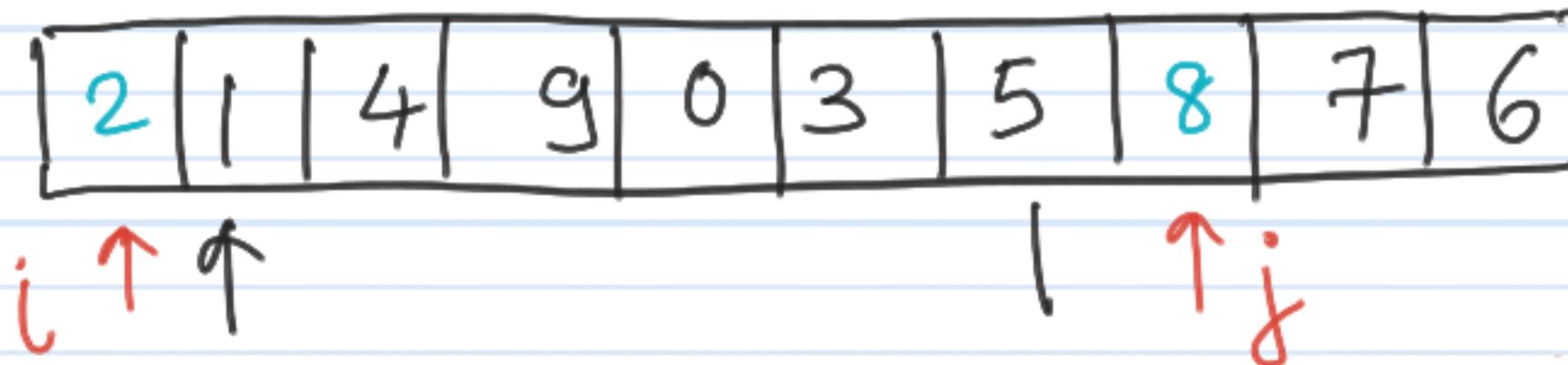
after selecting
pivot



Partition :

- move all "small" elements to "left"
- move all "large" elements to "right"

QUICK SORT : Selecting a pivot



```
while ( A[i] < pivot) { i++; }  
while ( A[j] > pivot) { j--; }  
if (i < j)  
    ( swap A[i], A[j] )  
else _____ ??
```

```
QSort ( A[], left, right) {
```

```
Pivot = getPivot (A, left, right)  
↳ has side effect.
```

```
i = left; j = right - 1;
```

```
while ( ) {
```

```
    while (A[i] < pivot) i++;
```

```
    while (A[j] > pivot) j--;
```

```
    if (i < j) swap (— —) else —
```

```
}
```

```
{
```

```
swap (A[i], A[right]); QSort (A, —, —);  
QSort (A, —, —);
```

```
QSort ( A[], left, right) {
```

```
Pivot = getPivot (A, left, right)  
↳ has side effect.
```

```
i = left; j = right - 1;
```

```
while ( ) {
```

```
    while (A[i] < pivot) i++;
```

```
    while (A[j] > pivot) j--;
```

```
    if (i < j) { swap (---) } else —
```

```
}
```

```
} swap (A[i], A[right]); QSort (A, —, —);  
QSort (A, —, —);
```

Running time analysis for quicksort

$$T(n) = T(i) + T(n-i-1) + c \cdot n$$

$i = |S| \rightarrow$ determined by the pivot

Best case : pivot happens to be the median

Worst case : pivot happens to be smallest or largest

Write down recurrences for both .

Running time analysis for quicksort

Best case : $T(n) = 2T(n/2) + c \cdot n$

Worst case : $T(n) = T(n-1) + c \cdot n$

One more sorting algorithm : Mergesort

Basic idea : Suppose we have 2 sorted arrays

$$A[] = 1, 13, 24, 26$$

$$B[] = 2, 15, 27, 38$$

Can we create a single "merged" array out
of these ?

One more sorting algorithm : Mergesort

Basic idea : Suppose we have 2 sorted arrays

$$A[] = \begin{matrix} & \star & \star & \star \\ 1, & 13, & 24, & 26 \end{matrix}$$

$$B[] = \begin{matrix} & \star & \star & \star \\ 2, & 15, & 27, & 38 \end{matrix}$$

$$C[] = [1, 2, 13, 15, \dots]$$

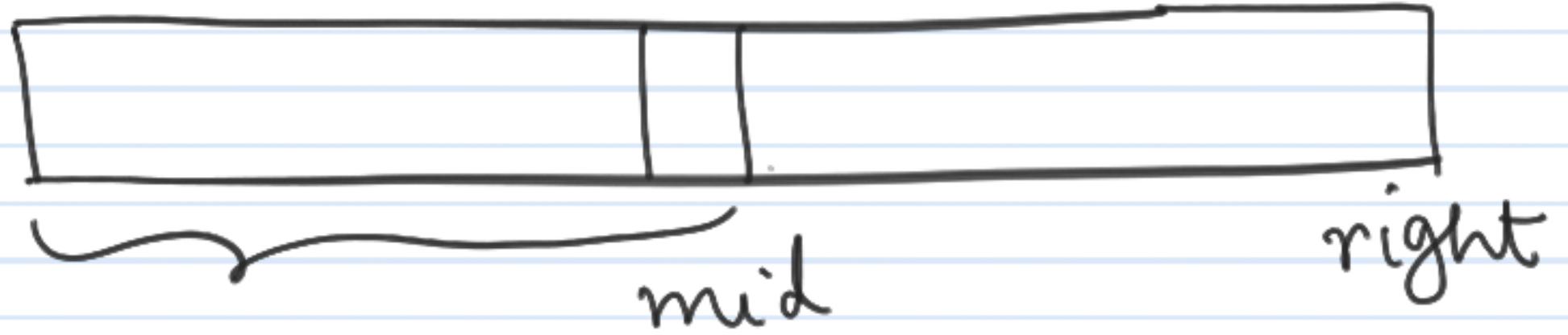
- Compare $A[i]$, $B[j]$
- Populate $C[k]$
- Which indices advance?

Mergesort : Recursive part

```
void msort ( int A [ ] , int left , int right ) {  
    if ( left >= right ) return ;  
    mid = ( left + right ) / 2 ;  
    msort ( A , left , mid ) ;  
    msort ( A , mid + 1 , right ) ;  
    merge ( A , left , mid , right ) ;  
}
```

Merge function

```
void merge( A[], left, mid , right) {
```



$$T(n) = 2 T(n/2) + c \cdot n$$