Honor code: I pledge on my honor that: I have completed all steps in the below homework on my own, I have not used any unauthorized materials while completing this homework, and I have not given anyone else access to my homework.

Name and Signature

1. (1 point) Have you read and understood the honor code?

   Solution:

Concept: System of linear equations

2. (2 points) This question has two parts as mentioned below:

   (a) Find a 2 x 3 system Ax = b whose complete solution is

   $$x = \begin{bmatrix}1\\2\\0\end{bmatrix} + w\begin{bmatrix}1\\3\\1\end{bmatrix}$$

   (b) Now find a 3 x 3 system which has these solutions exactly when $b_1 + b_2 = b_3$. (Note: $b = [b_1\ b_2\ b_3]^T$.)

   Solution:

3. (2 points) Consider the matrices A and B below

   (i) A = $\begin{bmatrix}1 & 2 & 0 & 1\\0 & 1 & 1 & 0\\1 & 2 & 0 & 1\end{bmatrix}$ (ii) B = $\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$

   (a) Write down the row reduced echelon form of matrices A and B (also mention the steps involved).

   Solution:

   (b) Find all solutions to $A\mathbf{x} = 0$ and $B\mathbf{x} = 0$.

   Solution:

   (c) Write down the basis for the four fundamental subspaces of A.

(d) Write down the basis for the four fundamental subspaces of B.

Concept: Rank

4. (1 ½ points) Consider the matrices A and B as given below:

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & x \end{bmatrix} \text{ and B} = \begin{bmatrix} 3 & 1 & 3 \\ y & 2 & y \end{bmatrix}$$

Give the values for entries x and y such that the ranks of the matrices A and B are

(a) 1

Solution:

(b) 2

Solution:

(c) 3

Solution:

Concept: Nullspace and column space

5. (½ point) State True or False and explain you answer: The nullspace of R is the same as the nullspace of U (where R is the row reduced echelon form of A and U is the matrix in LU decomposition of A).

Solution: True/False because ...

6. (1 point) Suppose column 1 + column 2 + column5 = 0 in a 4×5 matrix A.

(a) What is a special solution for $A\mathbf{x} = \mathbf{0}$

Solution:

(b) Describe the null space of A.

Solution:

7. (2 points) Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$. The column space of this matrix is a 2 dimensional plane. What is the equation of this plane? (You need to write down the steps you took to arrive at the equation)

Solution:

8. (1 point) True or false? (If true give logical, valid reasoning or give a counterexample if false)
a. If the row space equals the column space then $A^T = A$

Solution:

b. If $A^T = -A$ then the row space of A equals the column space.

Solution:

9. (1 point) Which of the four fundamental subspaces are the same for the following pairs of matrices of different sizes? (Assume A is a $m \times n$ matrix)

(a) $\begin{bmatrix} A \end{bmatrix}$ and $\begin{bmatrix} A \\ A \end{bmatrix}$

Solution:

(b) $\begin{bmatrix} A \\ A \end{bmatrix}$ and $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$

Solution:

10. (2 points) For each of the questions below, construct a matrix A which satisfies the given condition or argue why the given condition cannot be satisfied?

(a) A matrix whose row space is equal to its column space

Solution:

(b) A matrix whose null space is equal to its column space

Solution:

(c) A matrix for which all the four fundamental subspaces are equal

Solution:

11. (1 point) True or false? If A is a $m \times m$ square matrix then $\mathcal{N}(A) = \mathcal{N}(A^2)$ (If true give logical, valid reasoning or give a counterexample if false)

Solution:

12. (2 points) Consider matrices A and B and their product AB. For each of the questions below fill in the blanks with one of the following options: $<, >, =, \leq, \geq$, $can't\ say$. Explain your answer.

(a) $dim(\mathcal{C}(AB))$_______________$dim(\mathcal{C}(A))$

Solution:

(b) $dim(\mathcal{C}(AB))$_______________$dim(\mathcal{C}(B))$

Solution:

(c) $dim(\mathcal{C}((AB)^\top))$_______________$dim(\mathcal{C}(A^\top))$

Solution:

(d) $dim(\mathcal{C}((AB)^\top))$_______________$dim(\mathcal{C}(B^\top))$

Solution:

Concept: Free variables

13. (2 ½ points) True or False (with reason if true or example to show it is false).

(a) A square matrix has no free variables

Solution: True/False because ...

(b) An invertible matrix has no free variables

Solution: True/False because ...

(c) An $m \times n$ matrix has no more than n pivot variables.

Solution: True/False because ...

(d) An $m \times n$ matrix has no more than m pivot variables.

Solution: True/False because ...

(e) Matrices A and A^T have the same number of pivots.

Solution: True/False because ...

Concept: Reduced Echelon Form

14. (½ point) Suppose R is $m \times n$ matrix of rank r, with pivot columns first:

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$$

(a) Find a right-inverse B with $RB = I$ if $r = m$.

Solution: