CS7015 (Deep Learning) : Lecture 10

Learning Vectorial Representations Of Words

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Acknowledgments

- 'word2vec Parameter Learning Explained' by Xin Rong
- 'word2vec Explained: deriving Mikolov et al.'s negative-sampling word-embedding method' by Yoav Goldberg and Omer Levy
- Sebastian Ruder's blogs on word embeddings[a]

[a] Blog1, Blog2, Blog3

Module 10.1: One-hot representations of words

- Let us start with a very simple motivation for why we are interested in vectorial representations of words

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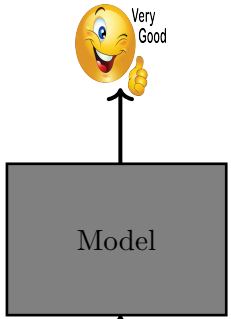
- Suppose we are given an input stream of words (sentence, document, etc.) and we are interested in learning some function of it (say, $\hat{y} = sentiments(words)$)

This is by far AAMIR KHAN's best one. Finest casting and terrific acting by all.



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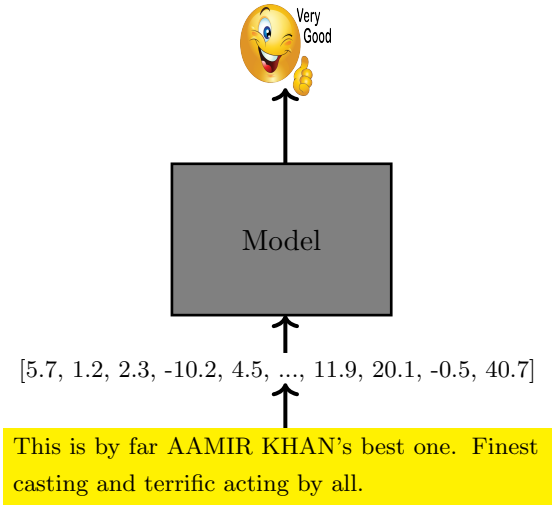
- Suppose we are given an input stream of words (sentence, document, etc.) and we are interested in learning some function of it (say, $\hat{y} = sentiments(words)$)
- Say, we employ a machine learning algorithm (some mathematical model) for learning such a function ($\hat{y} = f(\mathbf{x})$)



- Let us start with a very simple motivation for why we are interested in vectorial representations of words
- Suppose we are given an input stream of words (sentence, document, etc.) and we are interested in learning some function of it (say, $\hat{y} = sentiments(words)$)
- Say, we employ a machine learning algorithm (some mathematical model) for learning such a function ($\hat{y} = f(\mathbf{x})$)
- We first need a way of converting the input stream (or each word in the stream) to a vector $\mathbf{x}$ (a mathematical quantity)

- Given a corpus,

Corpus:

- Human machine interface for computer applications
- User opinion of computer system response time
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V = [human,machine, interface, for, computer, applications, user, opinion, of, system, response, time, management, engineering, improved]

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- V is called the **vocabulary** of the corpus (*i.e.*, all sentences or documents)

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machine:

0	1	0	...	0	0	0

- Given a corpus, consider the set V of all unique words across all input streams (*i.e.*, all sentences or documents)
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- One very simple way of doing this is to use one-hot vectors of size $|V|$

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- We need a representation for every word in V
- One very simple way of doing this is to use one-hot vectors of size $|V|$
- The representation of the i-th word will have a 1 in the i-th position and a 0 in the remaining $|V|-1$ positions

cat:	0	0	0	0	0	1	0
dog:	0	1	0	0	0	0	0
truck:	0	0	0	1	0	0	0

Problems:

- V tends to be very large (for example, 50K for PTB, 13M for Google 1T corpus)

cat:	0	0	0	0	0	1	0
dog:	0	1	0	0	0	0	0
truck:	0	0	0	1	0	0	0

Problems:

- V tends to be very large (for example, 50K for PTB, 13M for Google 1T corpus)
- These representations do not capture any notion of similarity

cat:	0	0	0	0	0	1	0
dog:	0	1	0	0	0	0	0
truck:	0	0	0	1	0	0	0

Problems:

- V tends to be very large (for example, 50K for PTB, 13M for Google 1T corpus)
- These representations do not capture any notion of similarity
- Ideally, we would want the representations of cat and dog (both domestic animals) to be closer to each other than the representations of cat and truck

cat:	0	0	0	0	0	1	0
dog:	0	1	0	0	0	0	0
truck:	0	0	0	1	0	0	0

$$euclid\_dist(\textbf{cat}, \textbf{dog}) = \sqrt{2}$$

$$euclid\_dist(\textbf{dog}, \textbf{truck}) = \sqrt{2}$$

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truck:	0	0	0	1	0	0	0

$$euclid\_dist(\textbf{cat}, \textbf{dog}) = \sqrt{2}$$
$$euclid\_dist(\textbf{dog}, \textbf{truck}) = \sqrt{2}$$
$$cosine\_sim(\textbf{cat}, \textbf{dog}) = 0$$
$$cosine\_sim(\textbf{dog}, \textbf{truck}) = 0$$

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- V tends to be very large (for example, 50K for PTB, 13M for Google 1T corpus)
- These representations do not capture any notion of similarity
- Ideally, we would want the representations of cat and dog (both domestic animals) to be closer to each other than the representations of cat and truck
- However, with 1-hot representations, the Euclidean distance between **any two words** in the vocabulary in $\sqrt{2}$
- And the cosine similarity between **any two words** in the vocabulary is 0

Module 10.2: Distributed Representations of words

- *You shall know a word by the company it keeps - Firth, J. R. 1957:11*

A bank is a **financial** institution that accepts **deposits** from the public and creates **credit**.

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A bank is a **financial** institution that accepts **deposits** from the public and creates **credit**.

The idea is to use the accompanying words (financial, deposits, credit) to represent bank

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- A co-occurrence matrix is a **terms** $\times$ **terms** matrix which captures the number of times a term appears in the context of another term

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- The context is defined as a window of k words around the terms
- Let us build a co-occurrence matrix for this toy corpus with $k = 2$

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	human	machine	system	for	...	user
human	0	1	0	1	...	0
machine	1	0	0	1	...	0
system	0	0	0	1	...	2
for	1	1	1	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	2	0	...	0

Co-occurence Matrix

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human	0	1	0	1	...	0
machine	1	0	0	1	...	0
system	0	0	0	1	...	2
for	1	1	1	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	2	0	...	0

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	human	machine	system	for	...	user
human	0	1	0	1	...	0
machine	1	0	0	1	...	0
system	0	0	0	1	...	2
for	1	1	1	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	2	0	...	0

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- The context is defined as a window of k words around the terms
- Let us build a co-occurrence matrix for this toy corpus with $k = 2$
- This is also known as a **word** $\times$ **context** matrix
- You could choose the set of **words** and **contexts** to be same or different
- Each row (column) of the co-occurrence matrix gives a vectorial representation of the corresponding word (context)

	human	machine	system	for	...	user
human	0	1	0	1	...	0
machine	1	0	0	1	...	0
system	0	0	0	1	...	2
for	1	1	1	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	2	0	...	0

Some (fixable) problems

- Stop words (a, the, for, etc.) are very frequent $\rightarrow$ these counts will be very high

	human	machine	system	...	user
human	0	1	0	...	0
machine	1	0	0	...	0
system	0	0	0	...	2
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
user	0	0	2	...	0

Some (fixable) problems

- Stop words (a, the, for, etc.) are very frequent → these counts will be very high
- Solution 1: Ignore very frequent words

	human	machine	system	for	...	user
human	0	1	0	x	...	0
machine	1	0	0	x	...	0
system	0	0	0	x	...	2
for	x	x	x	x	...	x
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	2	x	...	0

Some (fixable) problems

- Stop words (a, the, for, etc.) are very frequent $\rightarrow$ these counts will be very high
- Solution 1: Ignore very frequent words
- Solution 2: Use a threshold t (say, t = 100)

$$X_{ij} = min(count(w_i, c_j), t),$$

where w is word and c is context.

Some (fixable) problems

- Solution 3: Instead of $count(w, c)$ use $PMI(w, c)$

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$$PMI(w, c) = \log \frac{p(c|w)}{p(c)}$$
$$= \log \frac{count(w, c) * N}{count(c) * count(w)}$$

N *is the total number of words*

Some (fixable) problems

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N *is the total number of words*

	human	**machine**	**system**	**for**	...	**user**
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

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- If $count(w, c) = 0$, $PMI(w, c) = -\infty$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

Some (fixable) problems

- Solution 3: Instead of $count(w, c)$ use $PMI(w, c)$

$$PMI(w, c) = \log \frac{p(c|w)}{p(c)}$$
$$= \log \frac{count(w, c) * N}{count(c) * count(w)}$$

N *is the total number of words*

- If $count(w, c) = 0$, $PMI(w, c) = -\infty$

Instead use,

$$PMI_0(w, c) = PMI(w, c) \quad if\ count(w, c) > 0$$
$$= 0 \quad otherwise$$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

Some (fixable) problems

- Solution 3: Instead of $count(w, c)$ use $PMI(w, c)$

$$PMI(w, c) = \log \frac{p(c|w)}{p(c)}$$
$$= \log \frac{count(w, c) * N}{count(c) * count(w)}$$

N *is the total number of words*

- If $count(w, c) = 0$, $PMI(w, c) = -\infty$

Instead use,

$$PMI_0(w, c) = PMI(w, c) \quad if\ count(w, c) > 0$$
$$= 0 \quad otherwise$$

or

$$PPMI(w, c) = PMI(w, c) \quad if\ PMI(w, c) > 0$$
$$= 0 \quad otherwise$$

Some (severe) problems

- Very high dimensional ($|V|$)

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

Some (severe) problems

- Very high dimensional ($|V|$)
- Very sparse

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

Some (severe) problems

- Very high dimensional ($|V|$)
- Very sparse
- Grows with the size of the vocabulary

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

Some (severe) problems

- Very high dimensional ($|V|$)
- Very sparse
- Grows with the size of the vocabulary
- **Solution:** Use dimensionality reduction (SVD)

Module 10.3: SVD for learning word representations

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m \times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k \times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k \times n}$$

- Singular Value Decomposition gives a rank k approximation of the original matrix

$$X = X_{PPMI_{m \times n}} = U_{m \times k} \Sigma_{k \times k} V^T_{k \times n}$$

X_{PPMI} (simplifying notation to X) is the co-occurrence matrix with PPMI values

$$\underbrace{\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}}_{m \times n} = \underbrace{\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}}_{m \times k} \underbrace{\begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}}_{k \times k} \underbrace{\begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}}_{k \times n}$$

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- SVD gives the best rank-k approximation of the original data (X)

$$\underset{m\times n}{\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}} = \underset{m\times k}{\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}} \underset{k\times k}{\begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}} \underset{k\times n}{\begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}}$$

- Singular Value Decomposition gives a rank k approximation of the original matrix

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X_{PPMI} (simplifying notation to X) is the co-occurrence matrix with PPMI values

- SVD gives the best rank-k approximation of the original data (X)
- Discovers latent semantics in the corpus (let us examine this with the help of an example)

- Notice that the product can be written as a sum of k rank-1 matrices

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m\times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m\times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k\times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k\times n}$$

$$= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_k u_k v_k^T$$

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m \times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k \times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k \times n}$$

$$= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_k u_k v_k^T$$

- Notice that the product can be written as a sum of k rank-1 matrices
- Each $\sigma_i u_i v_i^T \in \mathbb{R}^{m \times n}$ because it is a product of a $m \times 1$ vector with a $1 \times n$ vector

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m \times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k \times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k \times n}$$

$$= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_k u_k v_k^T$$

- Notice that the product can be written as a sum of k rank-1 matrices
- Each $\sigma_i u_i v_i^T \in \mathbb{R}^{m \times n}$ because it is a product of a $m \times 1$ vector with a $1 \times n$ vector
- If we truncate the sum at $\sigma_1 u_1 v_1^T$ then we get the best rank-1 approximation of X

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m \times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k \times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k \times n}$$

$$= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_k u_k v_k^T$$

- Notice that the product can be written as a sum of k rank-1 matrices
- Each $\sigma_i u_i v_i^T \in \mathbb{R}^{m \times n}$ because it is a product of a $m \times 1$ vector with a $1 \times n$ vector
- If we truncate the sum at $\sigma_1 u_1 v_1^T$ then we get the best rank-1 approximation of X (By SVD theorem! But what does this mean? We will see on the next slide)

$$\underset{m\times n}{\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}} = \underset{m\times k}{\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}} \underset{k\times k}{\begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}} \underset{k\times n}{\begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}}$$

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- If we truncate the sum at $\sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$ then we get the best rank-2 approximation of X and so on

- What do we mean by approximation here?

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m \times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k \times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k \times n}$$

$$= \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T + \cdots + \sigma_k u_k v_k^T$$

- What do we mean by approximation here?
- Notice that X has $m \times n$ entries

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

$$\begin{bmatrix} \uparrow & \cdots & \uparrow \\ u_1 & \cdots & u_k \\ \downarrow & \cdots & \downarrow \end{bmatrix}_{m \times k} \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_k \end{bmatrix}_{k \times k} \begin{bmatrix} \leftarrow & v_1^T & \rightarrow \\ & \vdots & \\ \leftarrow & v_k^T & \rightarrow \end{bmatrix}_{k \times n}$$

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- What do we mean by approximation here?
- Notice that X has $m \times n$ entries
- When we use he rank-1 approximation we are using only $n + m + 1$ entries to reconstruct $[u \in \mathbb{R}^m, v \in \mathbb{R}^n, \sigma \in \mathbb{R}^1]$

$$\begin{bmatrix} & & \\ & X & \\ & & \end{bmatrix}_{m \times n} =$$

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- But SVD theorem tells us that u_1,v_1 and σ_1 store the most information in X (akin to the principal components in X)

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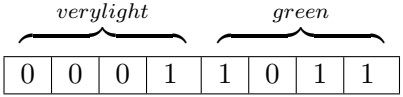
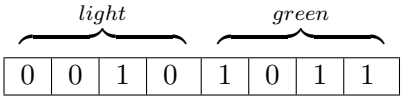
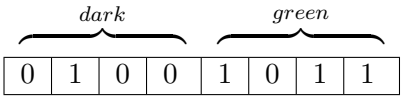
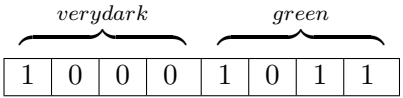
- But SVD theorem tells us that u_1,v_1 and σ_1 store the most information in X (akin to the principal components in X)
- Each subsequent term ($\sigma_2 u_2 v_2^T$, $\sigma_3 u_3 v_3^T$, $\ldots$) stores less and less important information

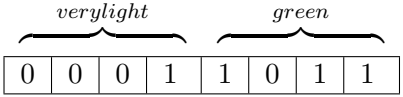


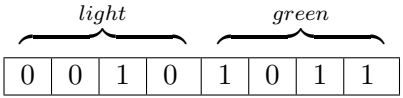


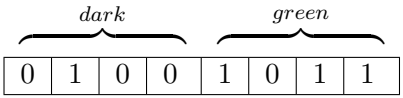


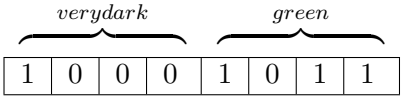


- As an analogy consider the case when we are using 8 bits to represent colors

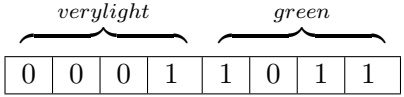


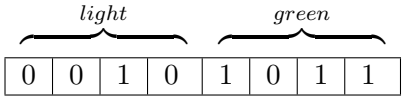


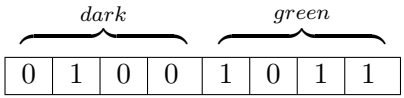


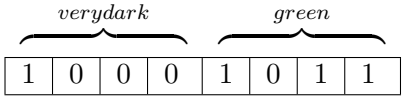


- As an analogy consider the case when we are using 8 bits to represent colors
- The representation of very light, light, dark and very dark green would look different









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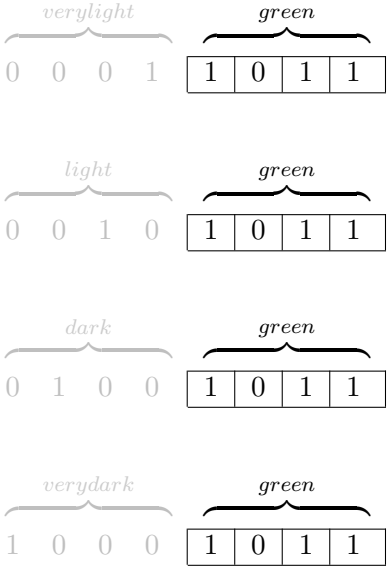
- But now what if we were asked to compress this into 4 bits? (akin to compressing $m \times m$ values into $m + m + 1$ values on the previous slide)



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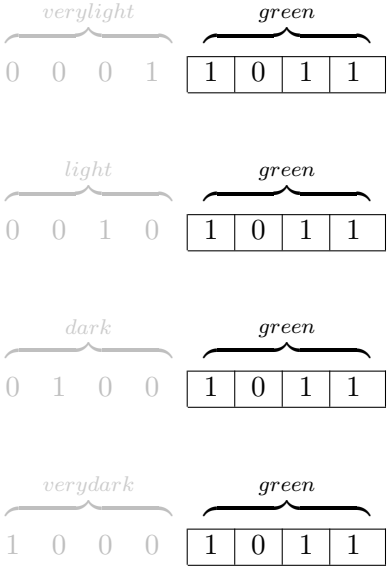
- We will retain the most important 4 bits and now the previously (slightly) latent similarity between the colors now becomes very obvious



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- We will retain the most important 4 bits and now the previously (slightly) latent similarity between the colors now becomes very obvious
- Something similar is guaranteed by SVD (retain the most important information and discover the latent similarities between words)

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	**0**
machine	2.944	0	0	2.25	...	0
system	0	**0**	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

Co-occurrence Matrix (X)

	human	machine	system	for	...	user
human	2.01	2.01	0.23	2.14	...	**0.43**
machine	2.01	2.01	0.23	2.14	...	0.43
system	0.23	**0.23**	1.17	0.96	...	1.29
for	2.14	2.14	0.96	1.87	...	-0.13
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0.43	0.43	1.29	-0.13	...	1.71

Low rank $X \rightarrow$ Low rank $\hat{X}$

- Notice that after low rank reconstruction with SVD, the latent co-occurrence between $\{system, machine\}$ and $\{human, user\}$ has become visible

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

- Recall that earlier each row of the original matrix X served as the representation of a word

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

$XX^T =$

	human	machine	system	for	...	user
human	32.5	23.9	7.78	20.25	...	7.01
machine	23.9	32.5	7.78	20.25	...	7.01
system	7.78	7.78	0	17.65	...	21.84
for	20.25	20.25	17.65	36.3	...	11.8
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	7.01	7.01	21.84	11.8	...	28.3

$cosine\_sim(human, user) = 0.21$

- Recall that earlier each row of the original matrix X served as the representation of a word
- Then XX^T is a matrix whose ij-th entry is the dot product between the representation of word i ($X[i:]$) and word j ($X[j:]$)

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

$XX^T =$

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human	32.5	23.9	7.78	20.25	...	7.01
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.	.	.	.	.	.	.
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$X[i:]$

$X[j:]$

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
user	0	0	1.84	0	...	0

$XX^T =$

	human	machine	system	for	...	user
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.	.	.	.	.	.	.
.	.	.	.	.	.	.
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$$X[i:] \quad \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 1 & 3 & 5 \end{bmatrix}}_{X} \quad X[j:]$$

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
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user	0	0	1.84	0	...	0

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$$X[i:] \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ X[j:] \; 1 & 3 & 5 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 5 \end{bmatrix}}_{X^T}$$

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
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system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
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.	.	.	.	.	.	.
.	.	.	.	.	.	.
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- Recall that earlier each row of the original matrix X served as the representation of a word
- Then XX^T is a matrix whose ij-th entry is the dot product between the representation of word i ($X[i:]$) and word j ($X[j:]$)

$$\underbrace{\begin{matrix} X[i:] \\ \\ X[j:] \end{matrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 1 & 3 & 5 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 5 \end{bmatrix}}_{X^T}$$

$$= \underbrace{\begin{bmatrix} . & . & 22 \\ . & . & . \\ . & . & . \end{bmatrix}}_{XX^T}$$

$X =$

	human	machine	system	for	...	user
human	0	2.944	0	2.25	...	0
machine	2.944	0	0	2.25	...	0
system	0	0	0	1.15	...	1.84
for	2.25	2.25	1.15	0	...	0
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
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.	.	.	.	.	.	.
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- Then XX^T is a matrix whose ij-th entry is the dot product between the representation of word i ($X[i:]$) and word j ($X[j:]$)

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 1 & 3 & 5 \end{bmatrix}}_{X} \underbrace{\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 3 & 0 & 5 \end{bmatrix}}_{X^T} = \underbrace{\begin{bmatrix} . & . & 22 \\ . & . & . \\ . & . & . \end{bmatrix}}_{XX^T}$$

- The ij-th entry of XX^T thus (roughly) captures the cosine similarity between $word_i, word_j$

- Once we do an SVD what is a good choice for the representation of $word_i$?

$$\hat{X} =$$

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.	.	.	.	.	.	.
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	human	machine	system	for	...	user
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machine	2.01	2.01	0.23	2.14	...	0.43
system	0.23	0.23	1.17	0.96	...	1.29
for	2.14	2.14	0.96	1.87	...	-0.13
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
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$\hat{X}\hat{X}^T =$

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machine	25.4	25.4	7.6	21.9	...	6.84
system	7.6	7.6	24.8	18.03	...	20.6
for	21.9	21.9	0.96	24.6	...	15.32
.	.	.	.	.	.	.
.	.	.	.	.	.	.
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- **Wishlist:** We would want representations of words (i, j) to be of smaller dimensions but still have the same similarity (dot product) as the corresponding rows of $\hat{X}$

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- Conventionally,

$$W_{word} = U\Sigma \in \mathbb{R}^{m\times k}$$

is taken as the representation of the m words in the vocabulary and

$$W_{context} = V$$

is taken as the representation of the context words

Module 10.4: Continuous bag of words model

- The methods that we have seen so far are called **count based models** because they use the co-occurrence counts of words

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- We will now see methods which directly **learn** word representations (these are called **(direct) prediction based models**)

The story ahead ...

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- Skip gram model with negative sampling (the famous word2vec)

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- Good old SVD does just fine!!

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- **Example:** he sat on a chair

Sometime in the 21st century, Joseph Cooper, a widowed former engineer and former NASA pilot, runs a farm with his father-in-law Donald, son Tom, and daughter Murphy, It is post-truth society (Cooper is reprimanded for telling Murphy that the Apollo missions did indeed happen) and a series of crop blights threatens humanity's survival. Murphy believes her bedroom is haunted by a poltergeist. When a pattern is created out of dust on the floor, Cooper realizes that gravity is behind its formation, not a "ghost". He interprets the pattern as a set of geographic coordinates formed into binary code. Cooper and Murphy follow the coordinates to a secret NASA facility, where they are met by Cooper's former professor, Dr. Brand.

Some sample 4 word windows from a corpus

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- Training data for this task is easily available (take all n word windows from the whole of wikipedia)
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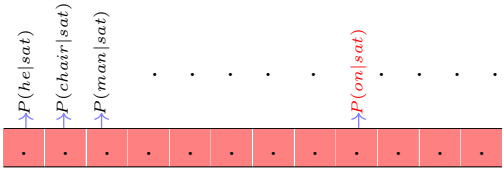
- How do you model this task?
- What is the connection between this task and learning word representations?

- We will model this problem using a feedforward neural network



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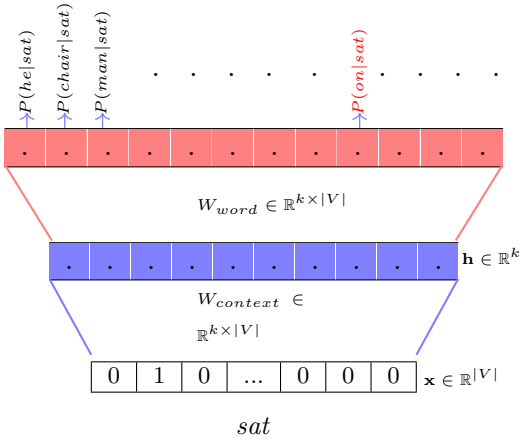
- **Input:** One-hot representation of the **context word**



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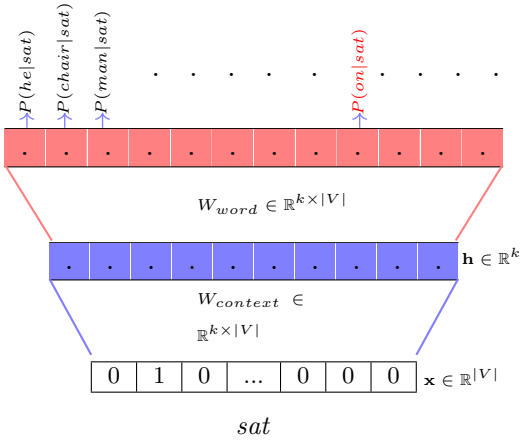

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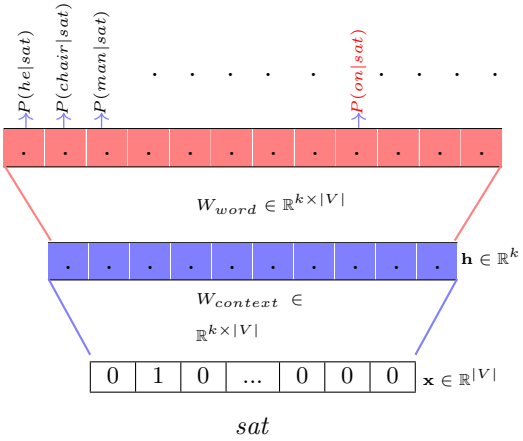
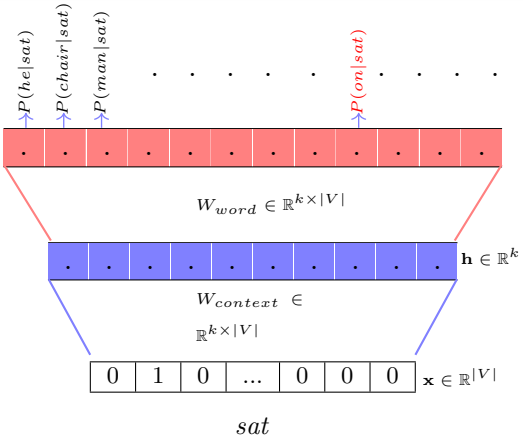
- **Input:** One-hot representation of the **context word**
- **Output:** There are $|V|$ words (classes) possible and we want to predict a probability distribution over these $|V|$ classes (multi-class classification problem)
- **Parameters:** $\mathbf{W}_{context} \in \mathbb{R}^{k \times |V|}$ and $\mathbf{W}_{word} \in \mathbb{R}^{k \times |V|}$
  (we are assuming that the set of **words** and **context** words is the same: each of size $|V|$)

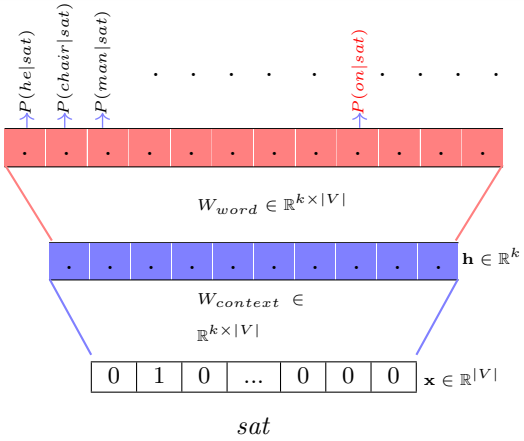


- What is the product $\mathbf{W}_{context}\mathbf{x}$ given that $\mathbf{x}$ is a one hot vector



- What is the product $\mathbf{W}_{context}\mathbf{x}$ given that $\mathbf{x}$ is a one hot vector
- It is simply the i-th column of $\mathbf{W}_{context}$

$$\begin{bmatrix} -1 & 0.5 & 2 \\ 3 & -1 & -2 \\ -2 & 1.7 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \\ 1.7 \end{bmatrix}$$



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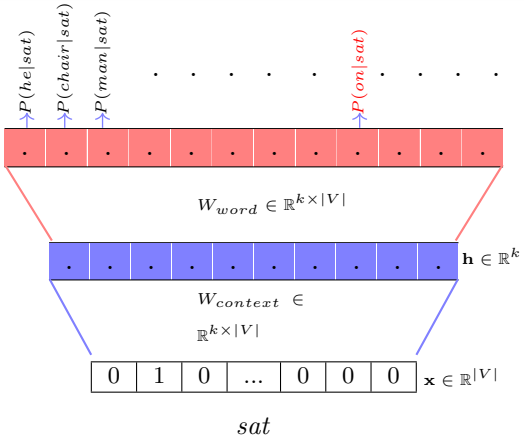

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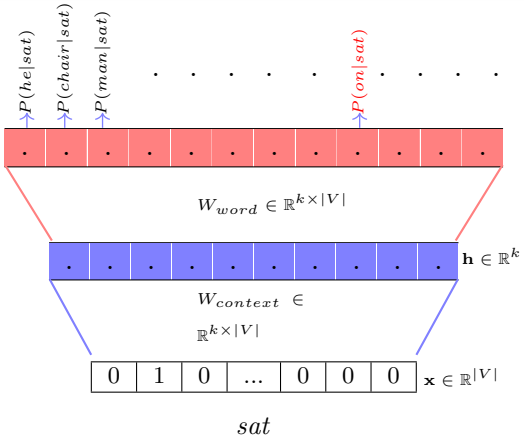
- So when the i^{th} word is present the i^{th} element in the one hot vector is ON and the i^{th} column of $\mathbf{W}_{context}$ gets selected
- In other words, there is a one-to-one correspondence between the words and the column of $\mathbf{W}_{context}$



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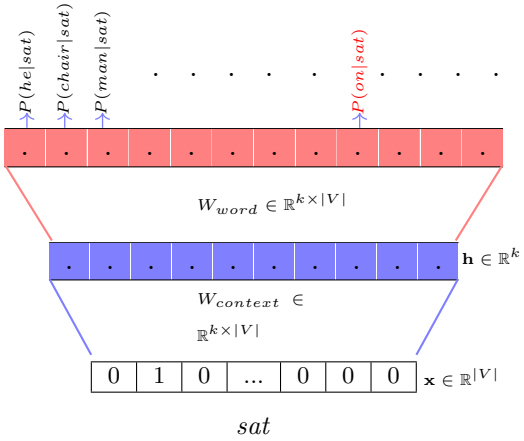
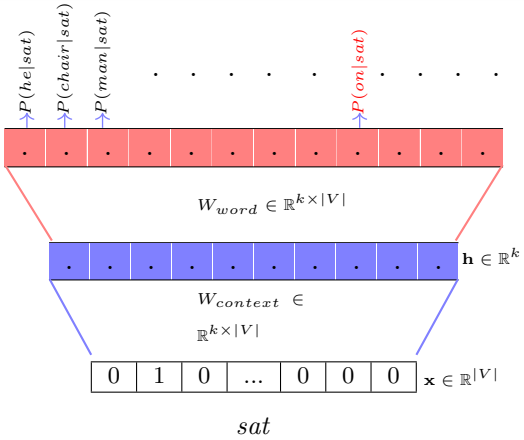
- More specifically, we can treat the i-th column of $\mathbf{W}_{context}$ as the representation of context i



- How do we obtain $P(on|sat)$? For this multi-class classification problem what is an appropriate output function?



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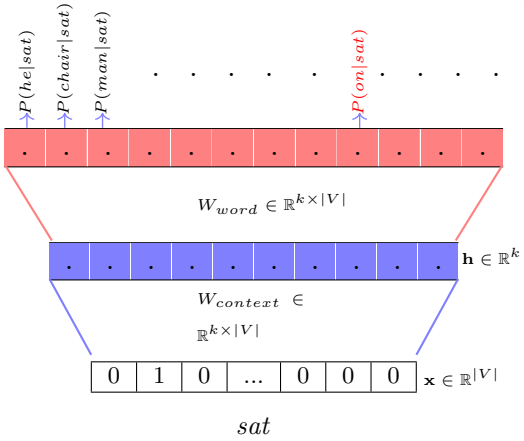

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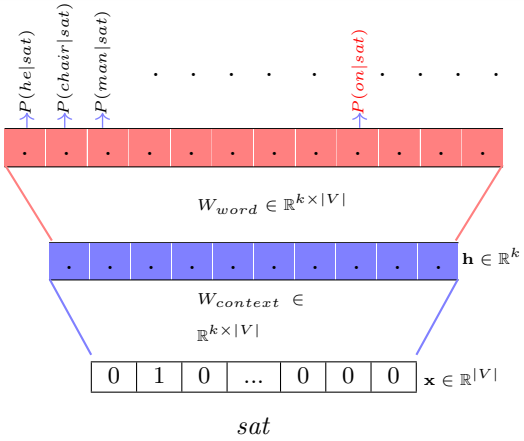

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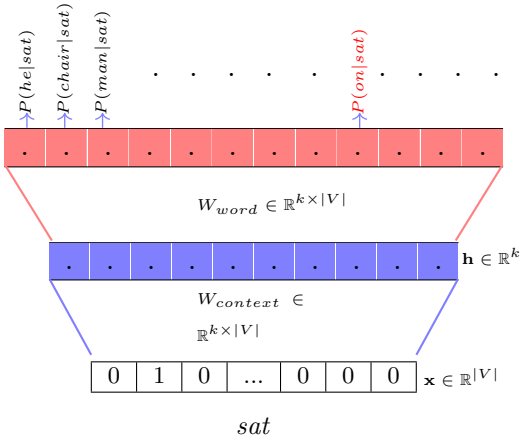

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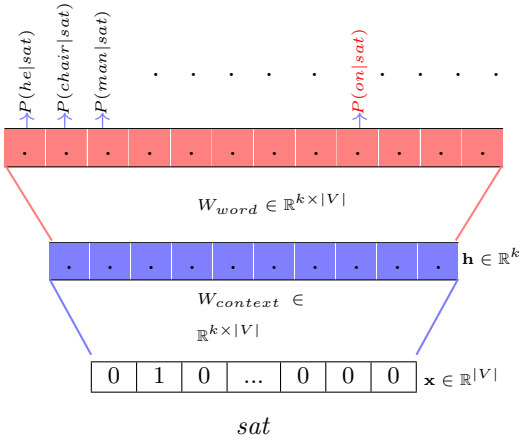
- Therefore, $P(on|sat)$ is proportional to the dot product between j^{th} column of $\mathbf{W}_{context}$ and i^{th} column of $\mathbf{W}_{word}$
- $P(word = i|sat)$ thus depends on the i^{th} column of $\mathbf{W}_{word}$



$$P(on|sat) = \frac{e^{(\mathbf{W}_{word}h)[i]}}{\sum_j e^{(\mathbf{W}_{word}h)[j]}}$$

- How do we obtain $P(on|sat)$? For this multi-class classification problem what is an appropriate output function? (**softmax**)
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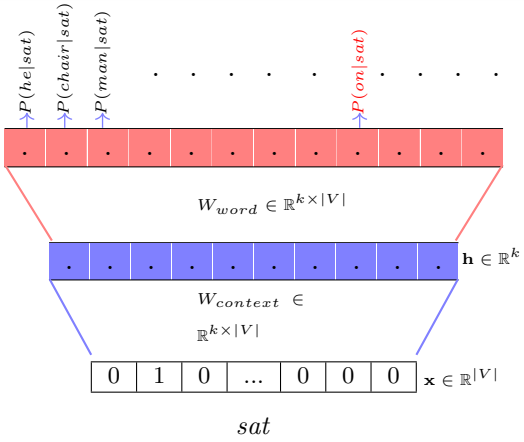

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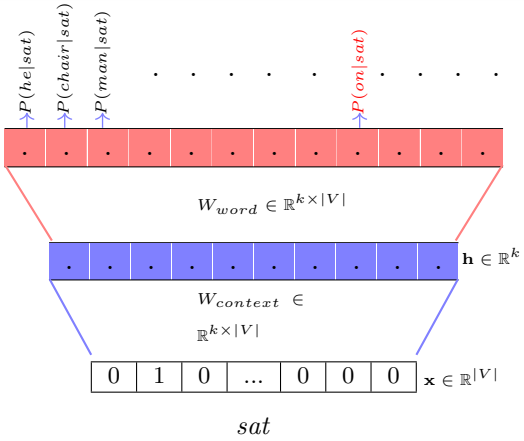
- Hope you see an analogy with SVD! (there we had a different way of learning $\mathbf{W}_{context}$ and $\mathbf{W}_{word}$ but we saw that the i^{th} column of $\mathbf{W}_{word}$ corresponded to the representation of the i^{th} word)



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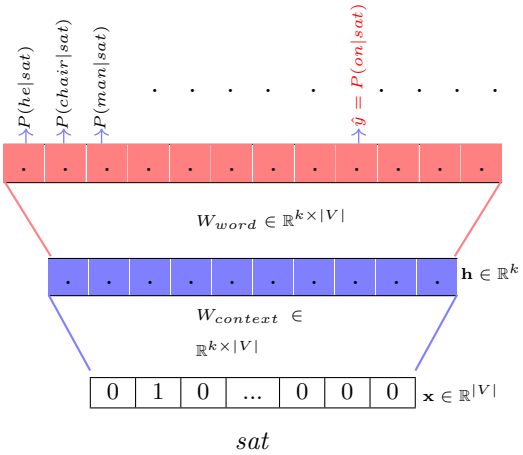
- Now that we understood the interpretation of $\mathbf{W}_{context}$ and $\mathbf{W}_{word}$, our aim now is to learn these parameters



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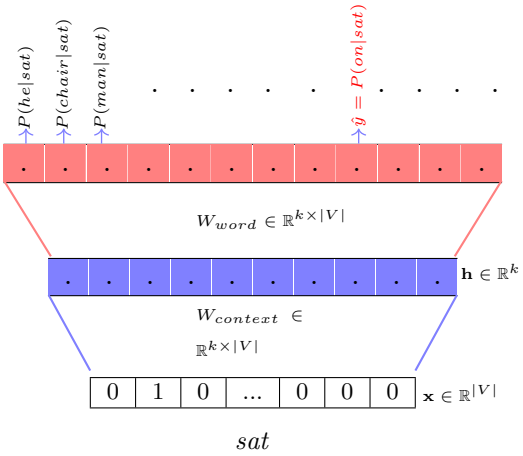

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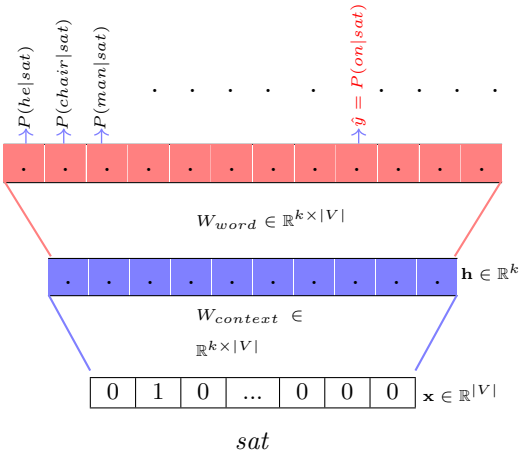
- For this multiclass classification problem what is an appropriate output function ($\hat{y} = f(x)$) ?



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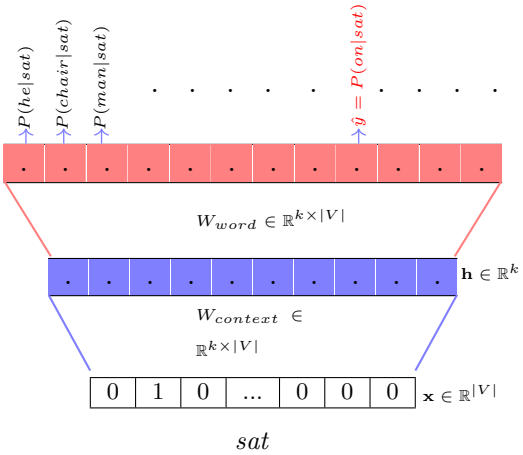

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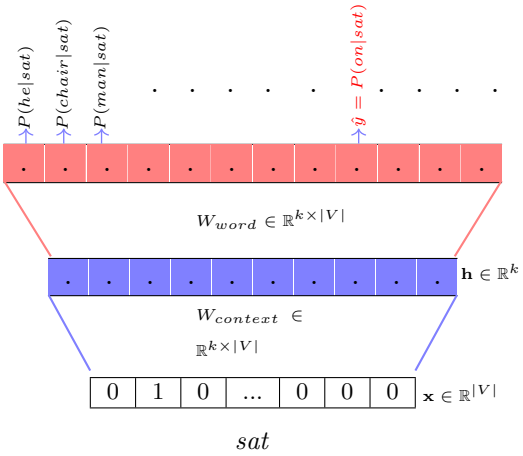

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- What is an appropriate loss function? **cross entropy**

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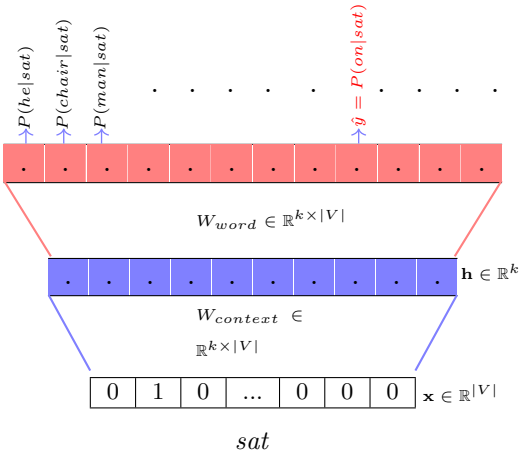

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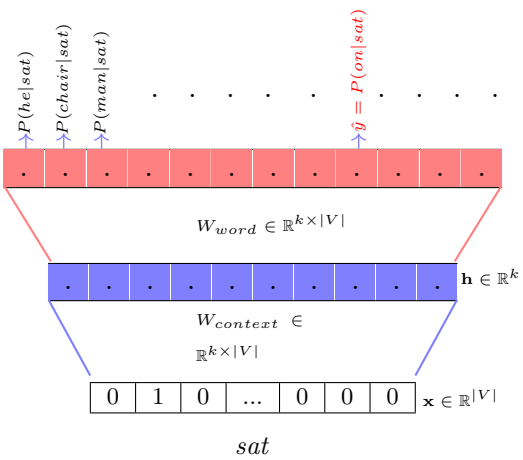

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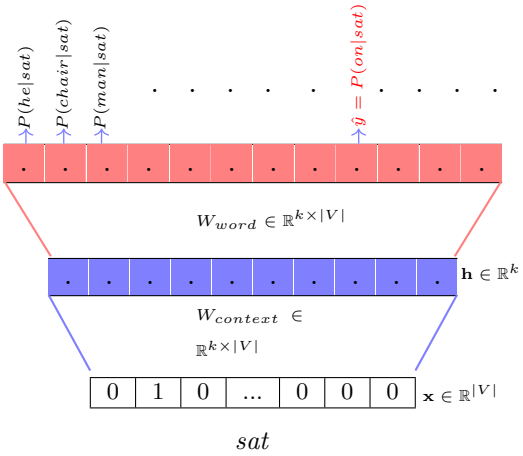

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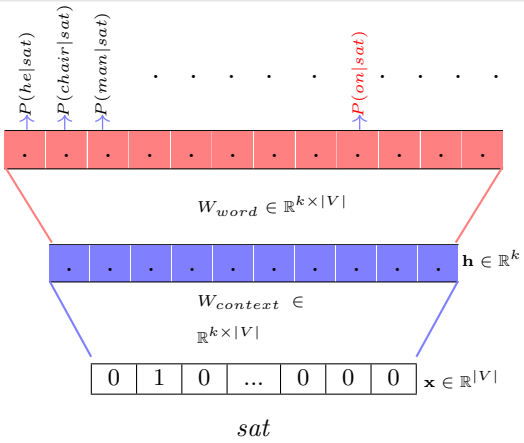
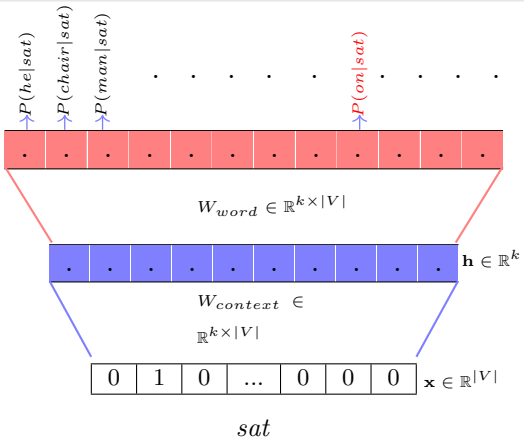
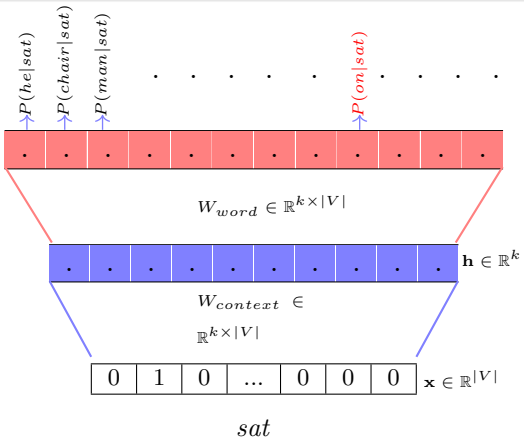
u_c is the column of $W_{context}$ corresponding to context c and v_w is the column of W_{word} corresponding to context w



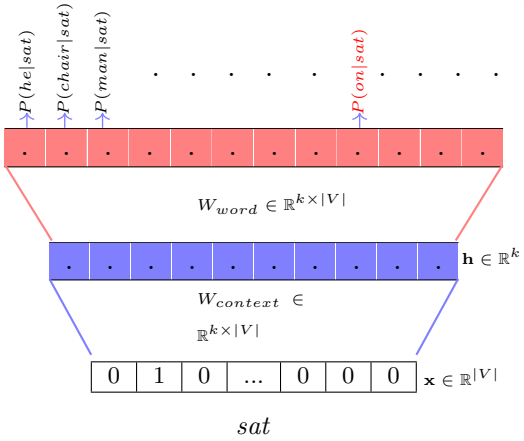
- How do we train this simple feed forward neural network?



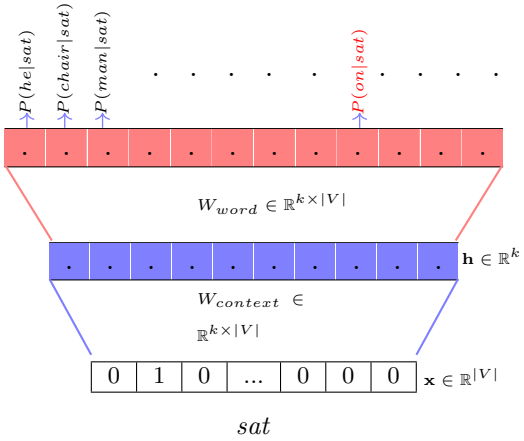
- How do we train this simple feed forward neural network? backpropagation



- How do we train this simple feed forward neural network? backpropagation
- Let us consider one input-output pair (c, w) and see the update rule for v_w

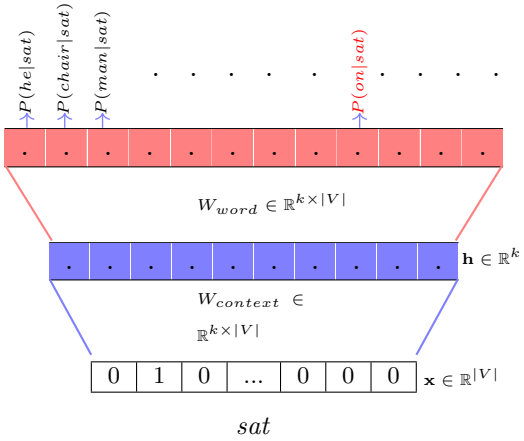


$$\mathscr{L}(\theta) = -\log \hat{y}_w$$

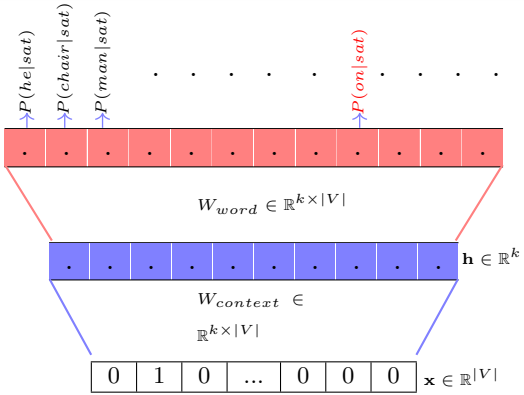


$$\mathscr{L}(\theta) = -\log \hat{y}_w$$

$$= -\log \frac{exp(u_c \cdot v_w)}{\sum_{w' \in V} exp(u_c \cdot v_{w'})}$$



$$\begin{aligned}\mathscr{L}(\theta) &= -\log \hat{y}_w \\ &= -\log \frac{exp(u_c \cdot v_w)}{\sum_{w' \in V} exp(u_c \cdot v_{w'})} \\ &= -(u_c \cdot v_w - \log \sum_{w' \in V} exp(u_c \cdot v_{w'}))\end{aligned}$$



$$\nabla_{v_w} = -\frac{\partial}{\partial v_w}\mathscr{L}(\theta)$$

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\mathscr{L}(\theta) &= -\log \hat{y}_w \\
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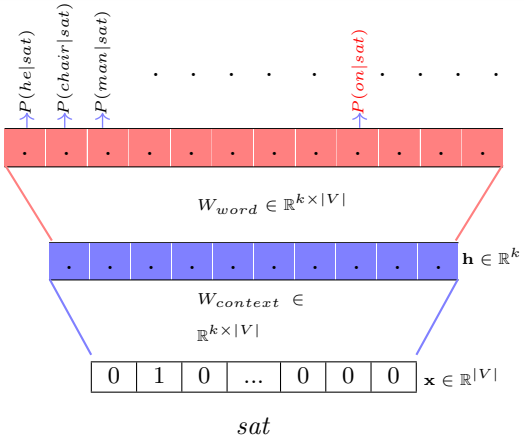

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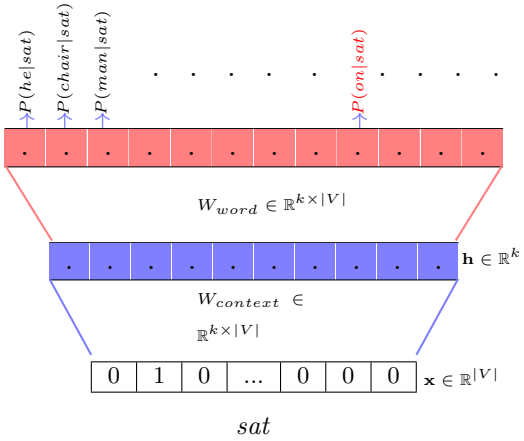
&= -u_c(1 - \hat{y}_w)
\end{aligned}$$



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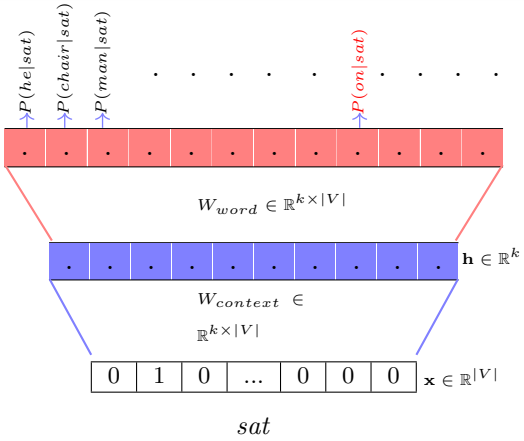

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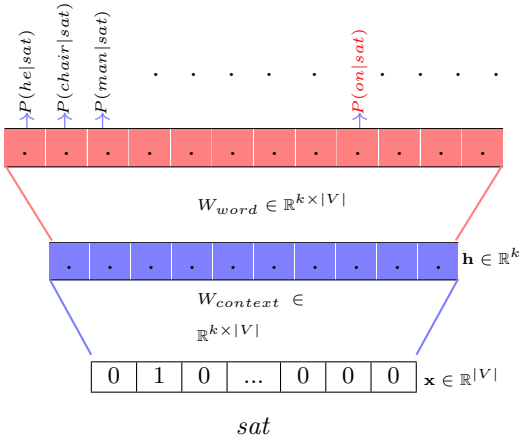
$$v_w = v_w - \eta \nabla_{v_w}$$



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\begin{aligned}
v_w &= v_w - \eta \nabla_{v_w} \\
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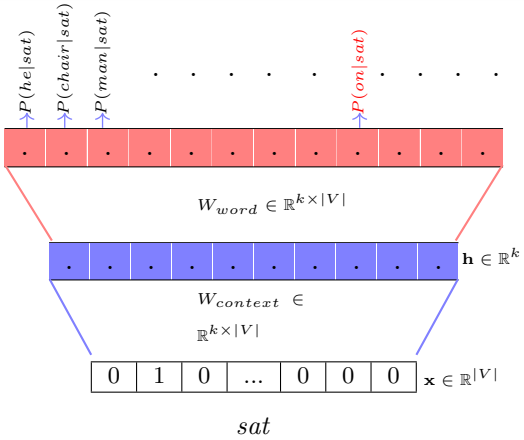
$$



- This update rule has a nice interpretation

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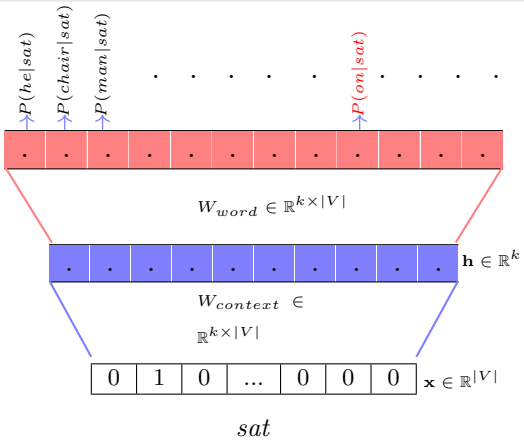

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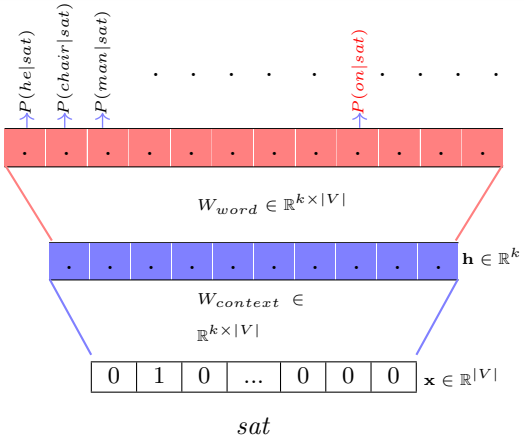

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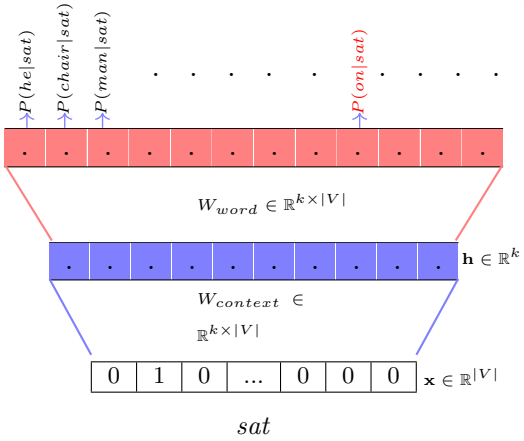

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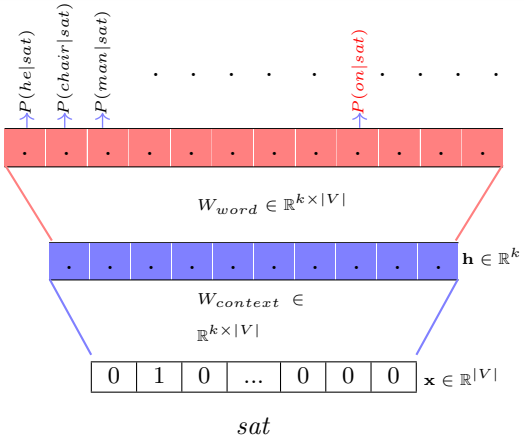

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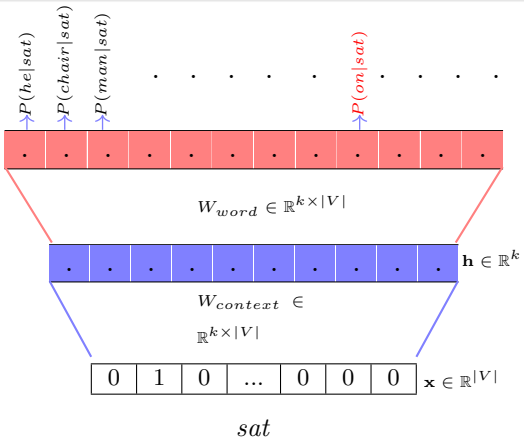
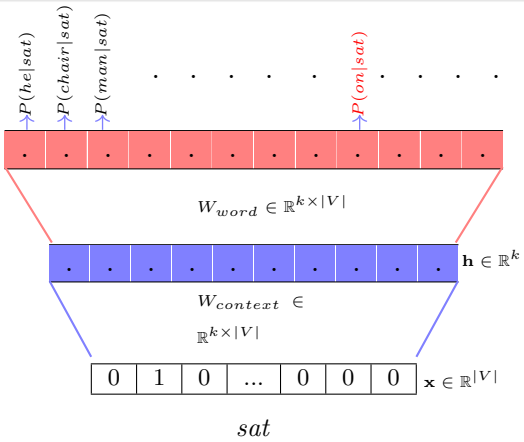
- The training objective ensures that the cosine similarity between word (v_w) and context word (u_c) is maximized



- What happens to the representations of two words w and w' which tend to appear in similar context (c)



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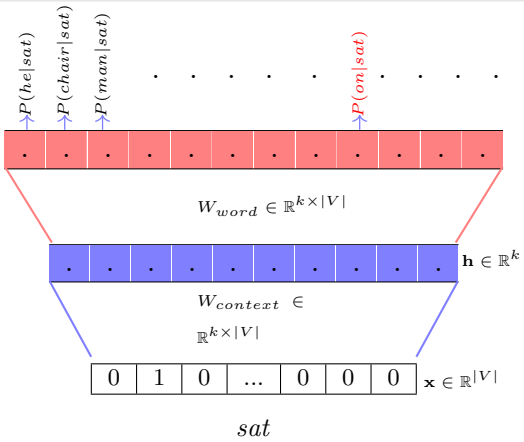

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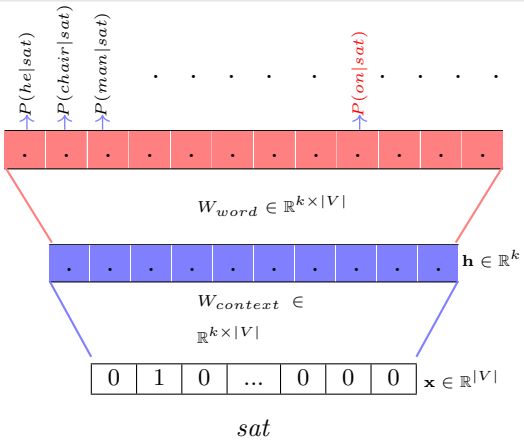

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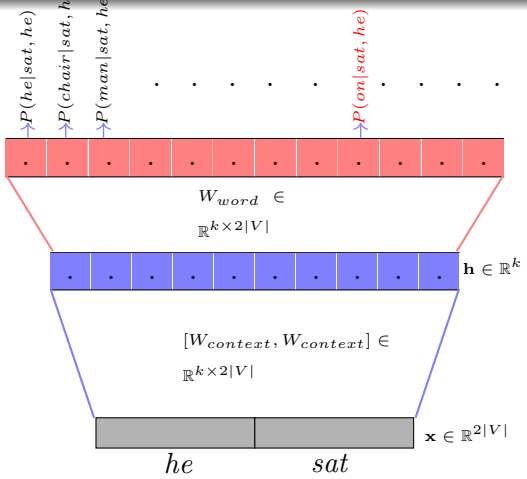
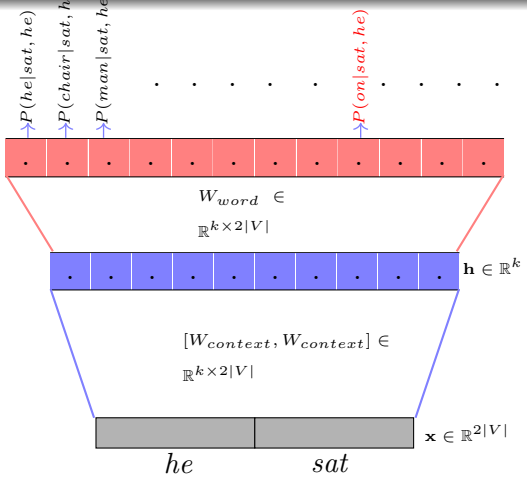
- This is only an intuition (reasonable)
- Haven't come across a formal proof for this!



- In practice, instead of window size of 1 it is common to use a window size of d



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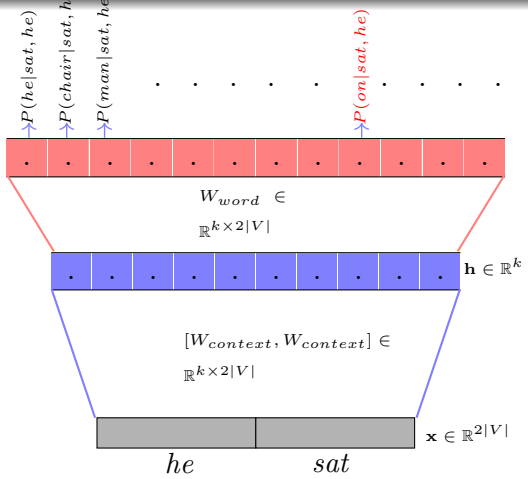

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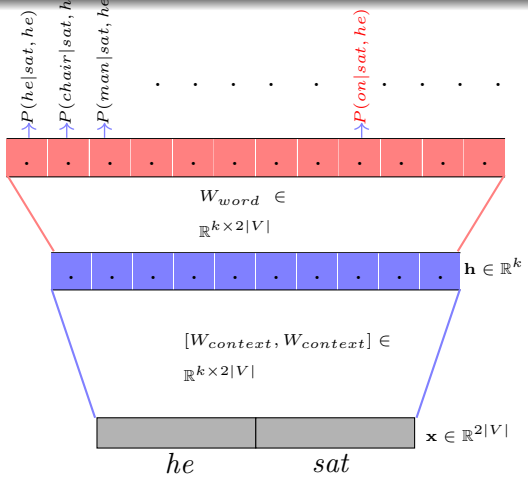

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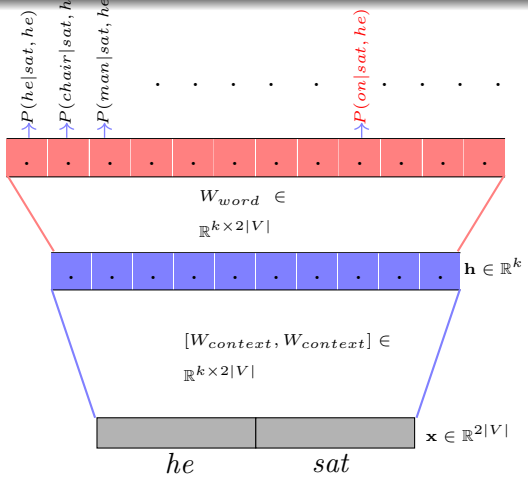

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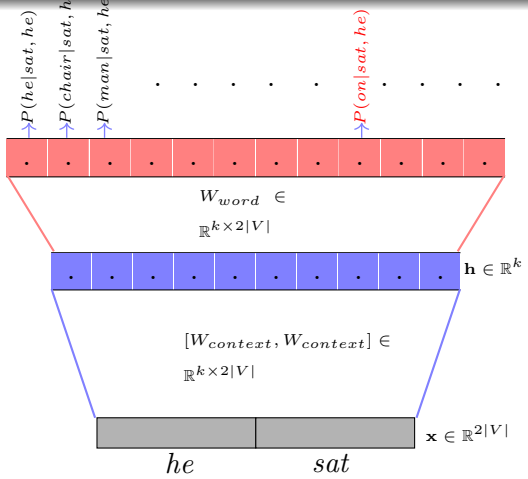

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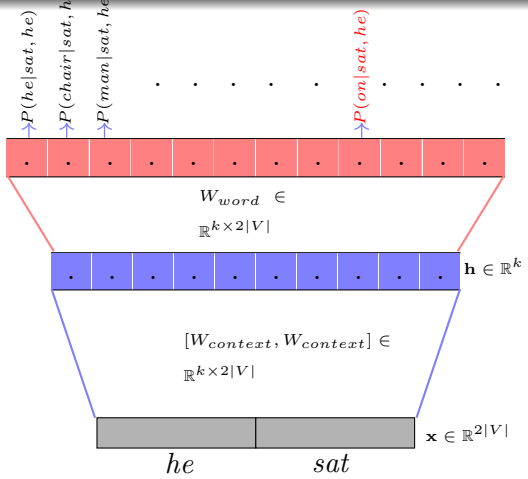

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- The resultant product would simply be the sum of the columns corresponding to 'sat' and 'he'

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- If 'he' is i^{th} word in the vocabulary and *sat* is the j^{th} word then we will simply access columns $\mathbf{W}[i:]$ and $\mathbf{W}[j:]$ and add them

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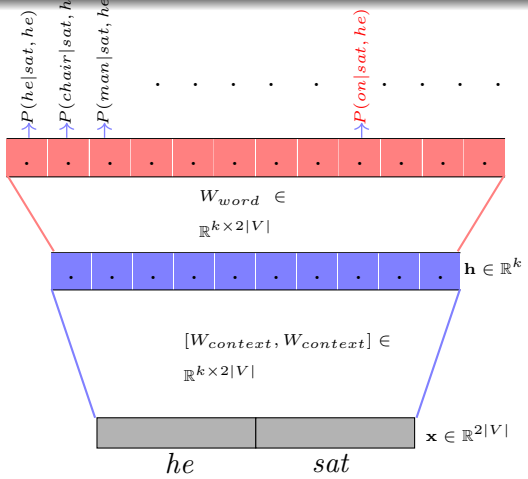
- Try deriving the update rule for v_w now and see how it differs from the one we derived before



Some problems:

- Notice that the softmax function at the output is computationally very expensive

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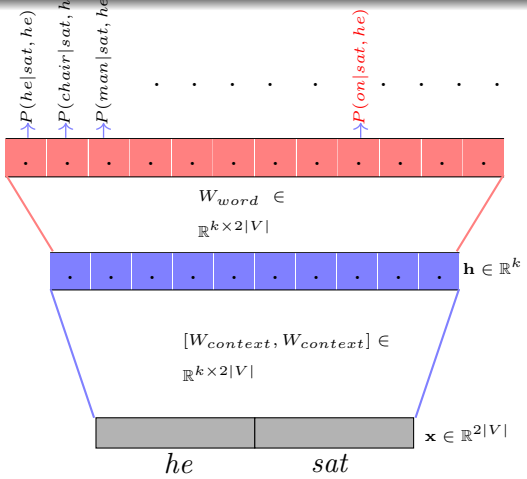

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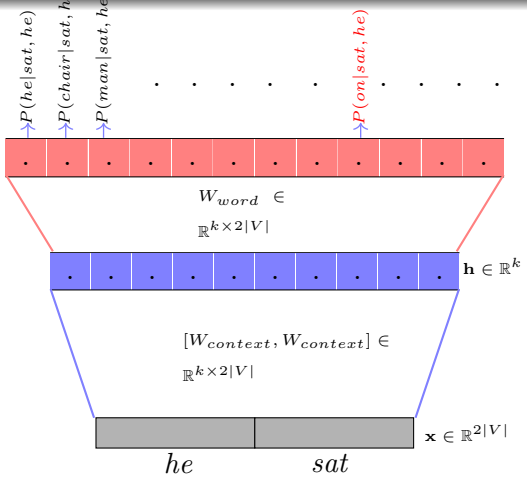

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- We will revisit this issue soon

Module 10.5: Skip-gram model

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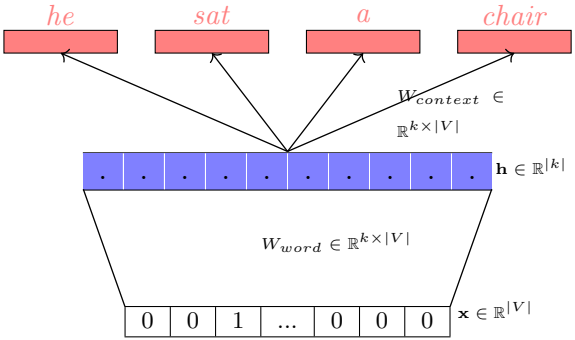
- We will now see the skip gram model (which predicts context words given an input word)



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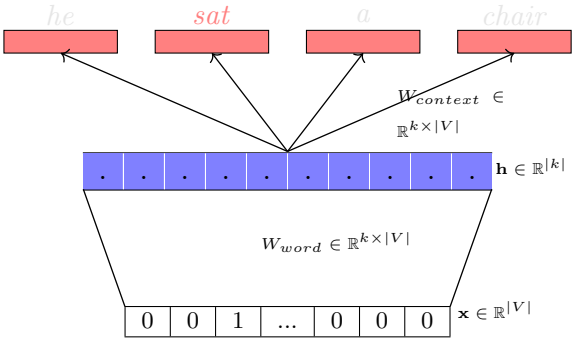

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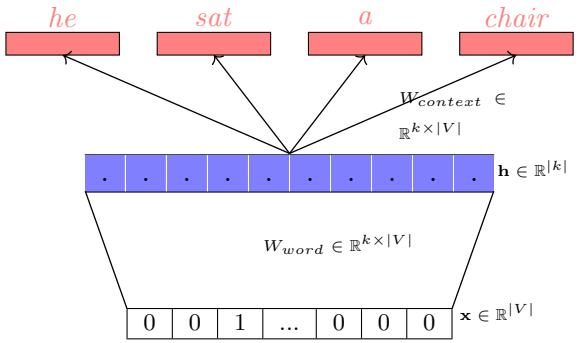

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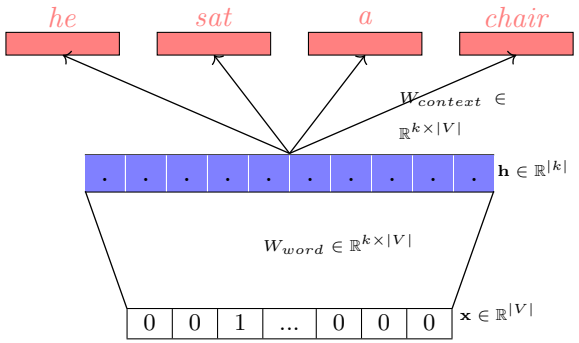

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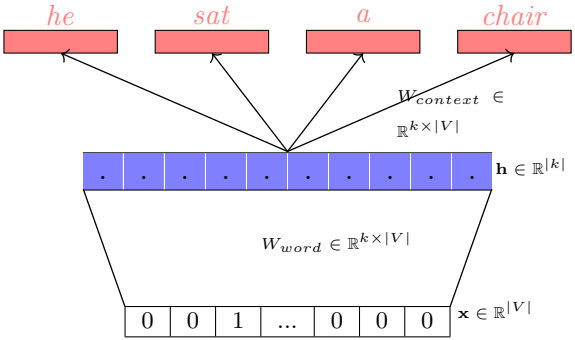
- Typically, we predict context words on both sides of the given word



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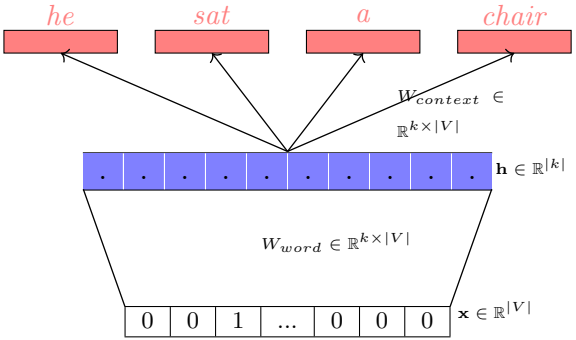

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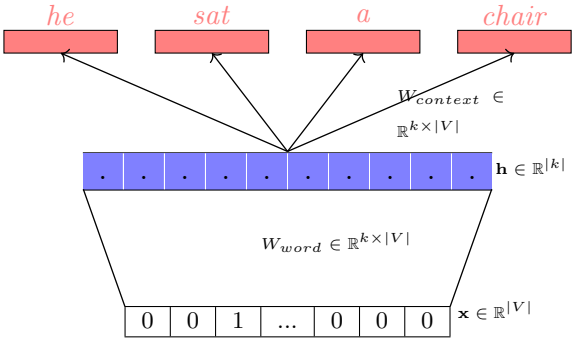

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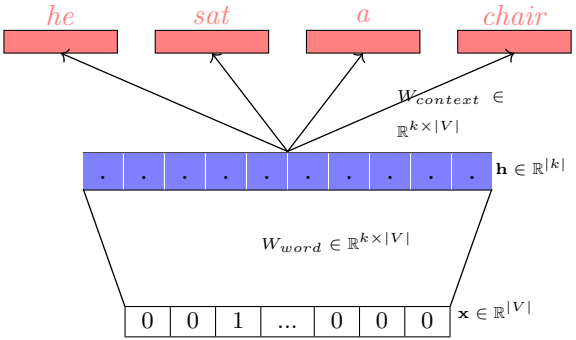

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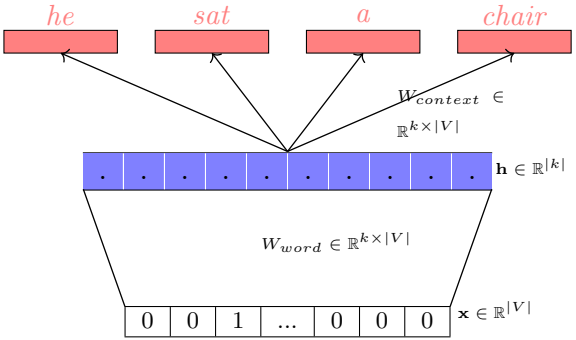

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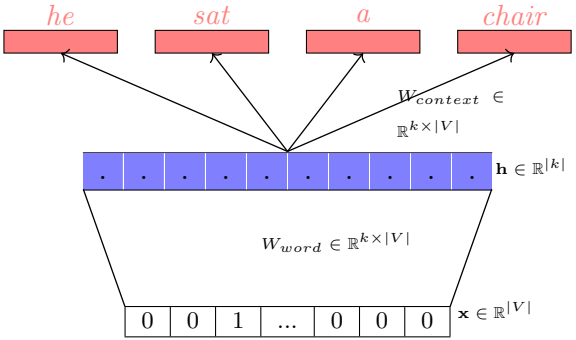

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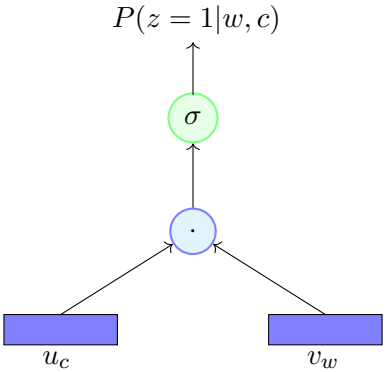
- As before let v_w be the representation of the word w and u_c be the representation of the context word c



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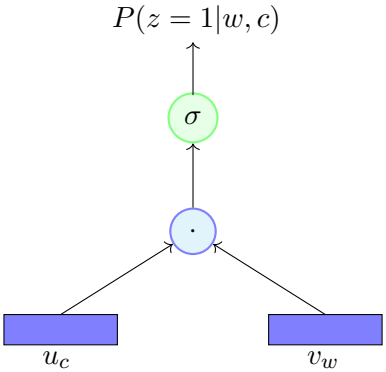

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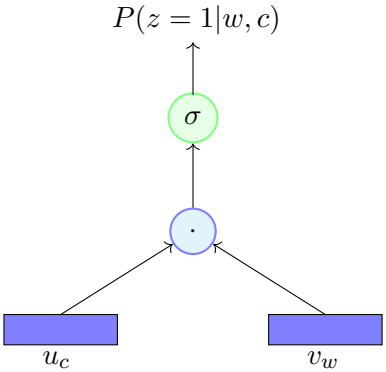

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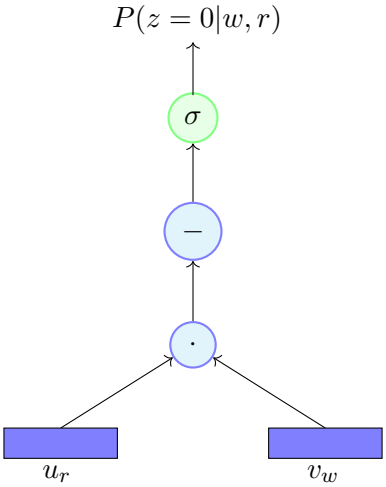

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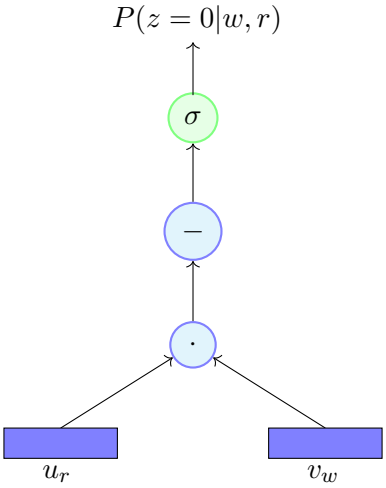

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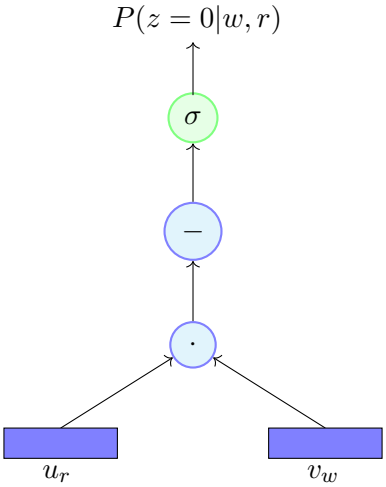

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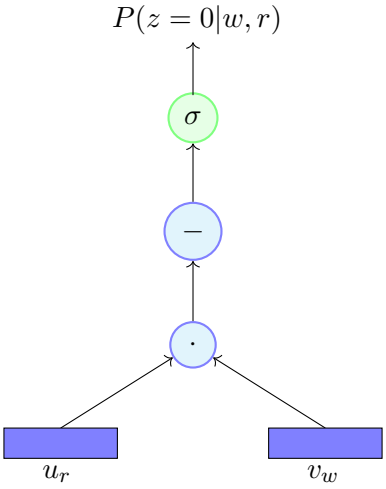

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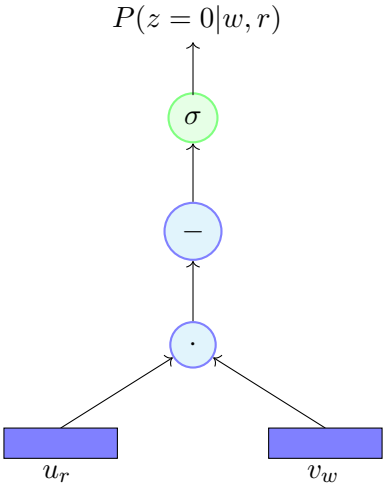

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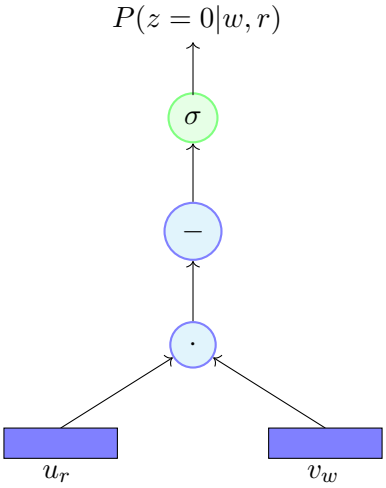

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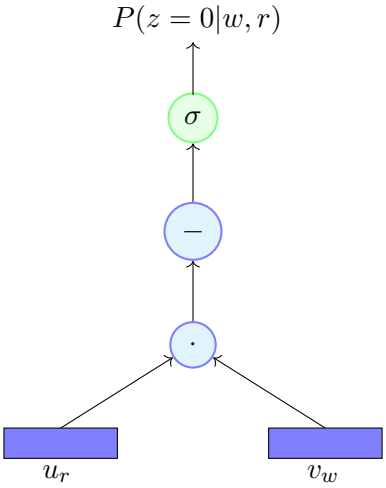
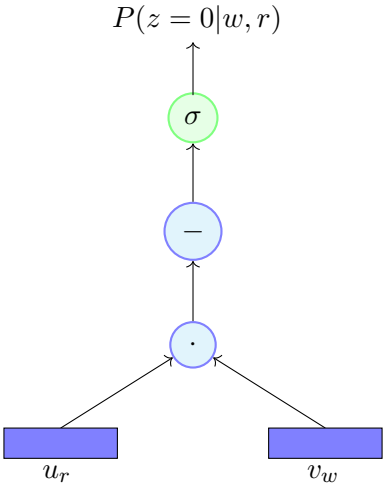


- Combining the two we get:

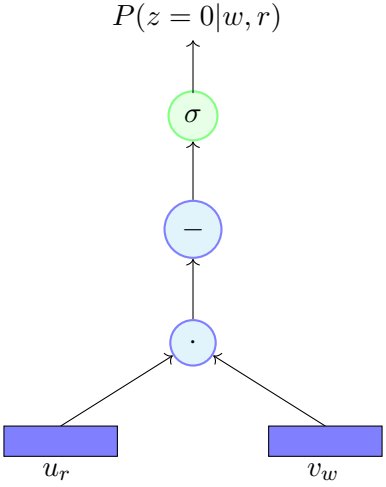
$$\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z = 1|w, c) \prod_{(w,r)\in D'} p(z = 0|w, r)$$



- Combining the two we get:

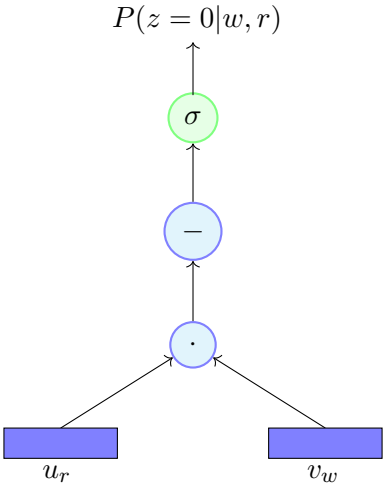
$$\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} p(z=0|w,r)$$

$$=\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} (1-p(z=1|w,r))$$



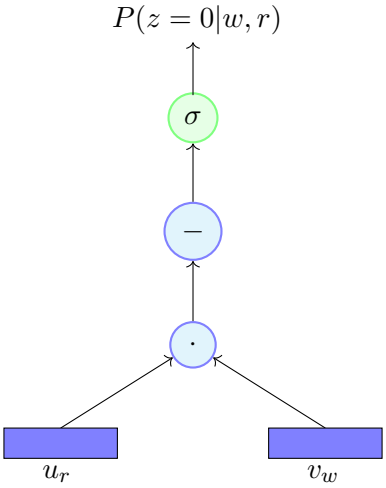
- Combining the two we get:

$$\begin{aligned}
&\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} p(z=0|w,r) \\
=&\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} (1-p(z=1|w,r)) \\
=&\underset{\theta}{maximize} \sum_{(w,c)\in D} \log p(z=1|w,c) \\
&+ \sum_{(w,r)\in D'} \log(1-p(z=1|w,r))
\end{aligned}$$



- Combining the two we get:

$$\begin{aligned}
&\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} p(z=0|w,r) \\
=&\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} (1-p(z=1|w,r)) \\
=&\underset{\theta}{maximize} \sum_{(w,c)\in D} \log p(z=1|w,c) \\
&\quad + \sum_{(w,r)\in D'} \log(1-p(z=1|w,r)) \\
=&\underset{\theta}{maximize} \sum_{(w,c)\in D} \log \frac{1}{1+e^{-v_c^T v_w}} + \sum_{(w,r)\in D'} \log \frac{1}{1+e^{v_r^T v_w}}
\end{aligned}$$



- Combining the two we get:

$$\underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} p(z=0|w,r)$$

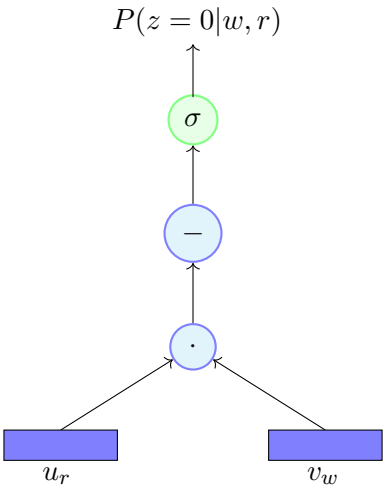
$$= \underset{\theta}{maximize} \prod_{(w,c)\in D} p(z=1|w,c) \prod_{(w,r)\in D'} (1-p(z=1|w,r))$$

$$= \underset{\theta}{maximize} \sum_{(w,c)\in D} \log p(z=1|w,c) + \sum_{(w,r)\in D'} \log(1-p(z=1|w,r))$$

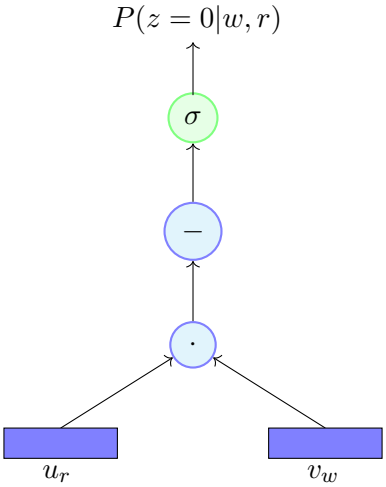
$$= \underset{\theta}{maximize} \sum_{(w,c)\in D} \log \frac{1}{1+e^{-v_c^T v_w}} + \sum_{(w,r)\in D'} \log \frac{1}{1+e^{v_r^T v_w}}$$

$$= \underset{\theta}{maximize} \sum_{(w,c)\in D} \log \sigma(v_c^T v_w) + \sum_{(w,r)\in D'} \log \sigma(-v_r^T v_w)$$

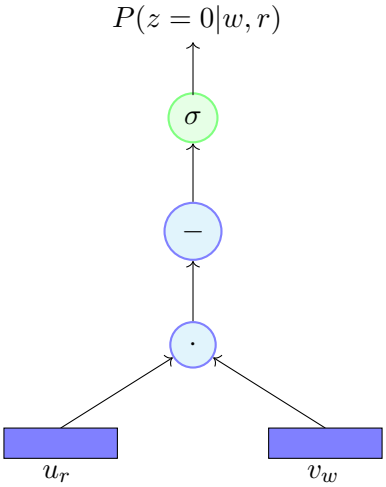
where $\sigma(x) = \frac{1}{1+e^{-x}}$



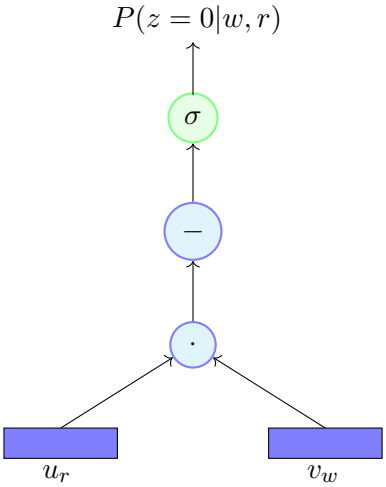
- In the original paper, *Mikolov et. al.* sample k negative (w, r) pairs for every positive (w, c) pairs



- In the original paper, *Mikolov et. al.* sample k negative (w, r) pairs for every positive (w, c) pairs
- The size of $D^{'}$ is thus k times the size of D



- In the original paper, *Mikolov et. al.* sample k negative (w, r) pairs for every positive (w, c) pairs
- The size of $D^{'}$ is thus k times the size of D
- The random context word is drawn from a modified unigram distribution



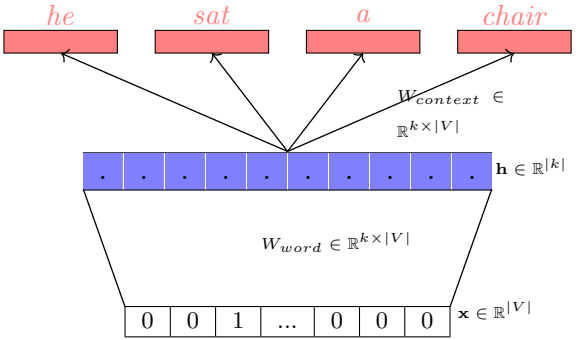
- In the original paper, *Mikolov et. al.* sample k negative (w, r) pairs for every positive (w, c) pairs
- The size of $D^{'}$ is thus k times the size of D
- The random context word is drawn from a modified unigram distribution

$$r \sim p(r)^{\frac{3}{4}}$$

$$r \sim \frac{count(r)^{\frac{3}{4}}}{N}$$

N = total number of words in the corpus

Module 10.6: Contrastive estimation



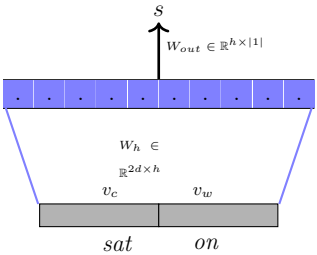
Some problems

- Same as bag of words
- The softmax function at the output is computationally expensive
- Solution 1: Use negative sampling
- Solution 2: Use contrastive estimation
- Solution 3: Use hierarchical softmax

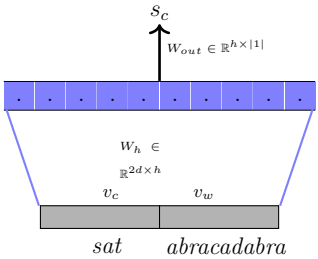
Positive: *He sat on a chair*

Negative: *He sat abracadabra a chair*

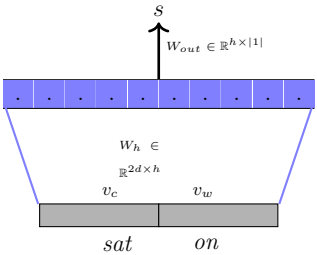
Positive: *He sat on a chair*



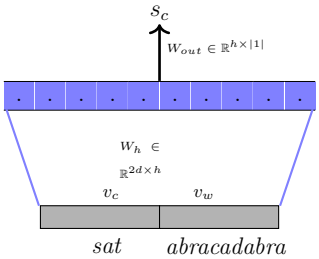
Negative: *He sat abracadabra a chair*



Positive: *He sat on a chair*

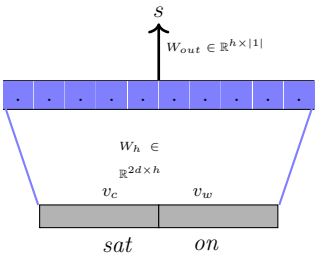


Negative: *He sat abracadabra a chair*

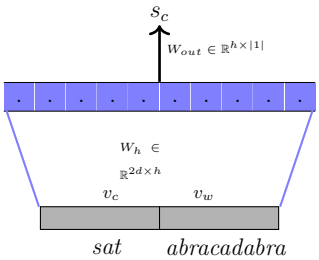


- We would like s to be greater than s_c

Positive: *He sat on a chair*

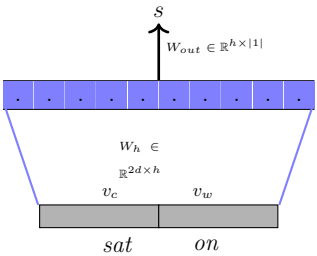


Negative: *He sat abracadabra a chair*

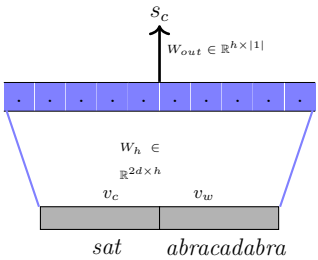


- We would like s to be greater than s_c
- Okay, so let us try to maximize $s - s_c$

Positive: *He sat on a chair*

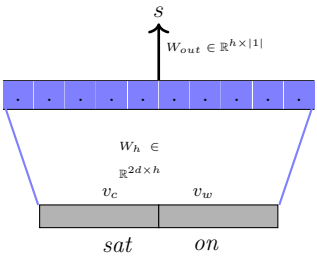


Negative: *He sat abracadabra a chair*

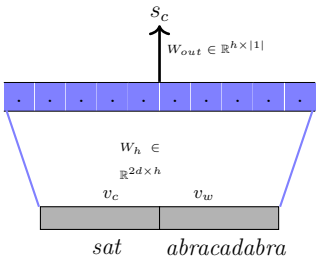


- We would like s to be greater than s_c
- Okay, so let us try to maximize $s - s_c$
- But we would like the difference to be at least m

Positive: *He sat on a chair*



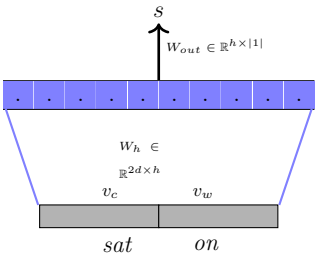
Negative: *He sat abracadabra a chair*



- We would like s to be greater than s_c
- Okay, so let us try to maximize $s - s_c$
- But we would like the difference to be at least m

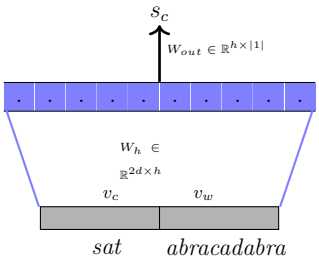
- So we can maximize $s - (s_c + m)$

Positive: *He sat on a chair*



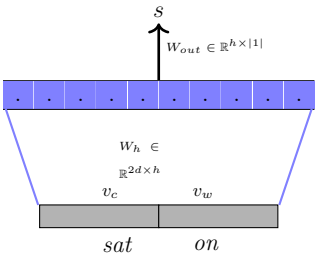
- We would like s to be greater than s_c
- Okay, so let us try to maximize $s - s_c$
- But we would like the difference to be at least m

Negative: *He sat abracadabra a chair*



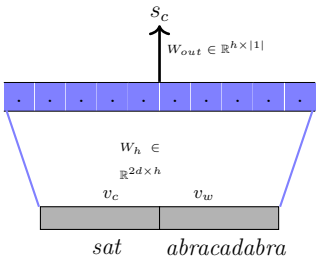
- So we can maximize $s - (s_c + m)$
- What if $s > s_c + m$

Positive: *He sat on a chair*



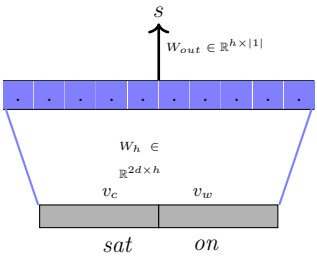
- We would like s to be greater than s_c
- Okay, so let us try to maximize $s - s_c$
- But we would like the difference to be at least m

Negative: *He sat abracadabra a chair*



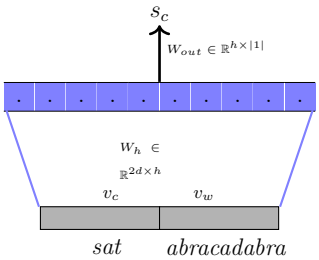
- So we can maximize $s - (s_c + m)$
- What if $s > s_c + m$ *(don't do any thing)*

Positive: *He sat on a chair*



- We would like s to be greater than s_c
- Okay, so let us try to maximize $s - s_c$
- But we would like the difference to be at least m

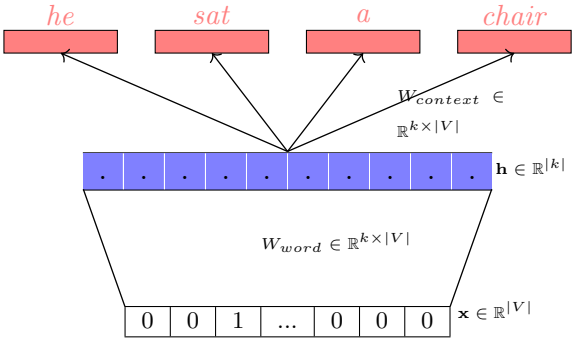
Negative: *He sat abracadabra a chair*



- So we can maximize $s - (s_c + m)$
- What if $s > s_c + m$ *(don't do any thing)*

maximize $max(0, s - (s_c + m))$

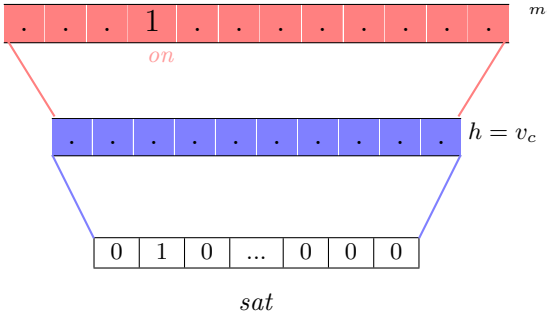
Module 10.7: Hierarchical softmax

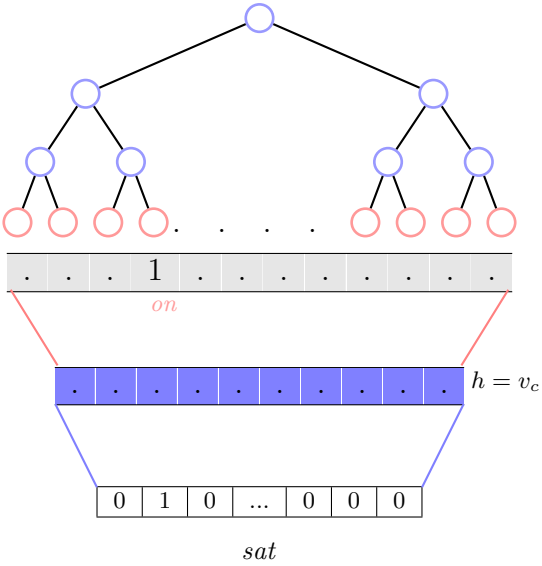


Some problems

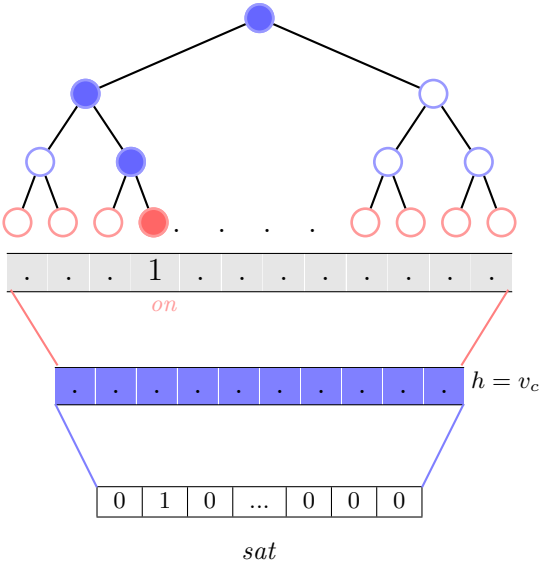
- Same as bag of words
- The softmax function at the output is computationally expensive
- Solution 1: Use negative sampling
- Solution 2: Use contrastive estimation
- Solution 3: Use hierarchical softmax

- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary

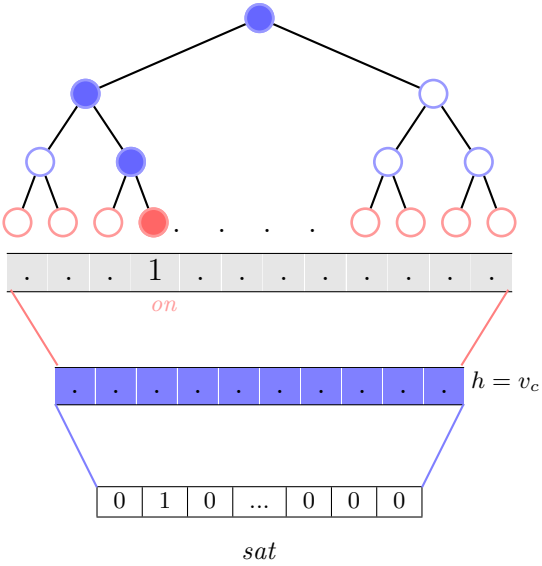




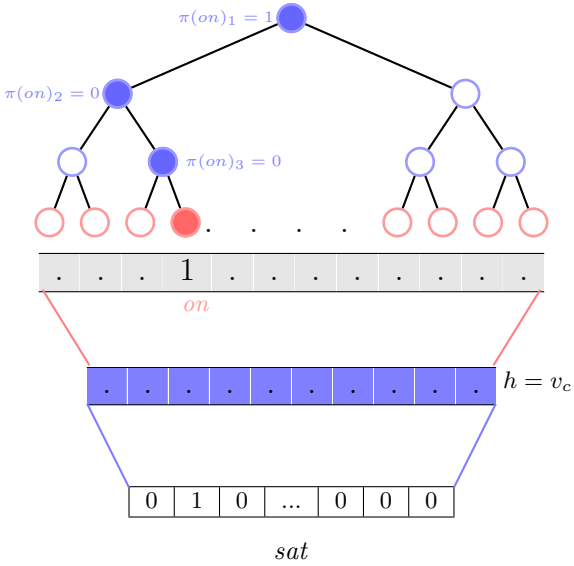
- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary



- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary
- There exists a unique path from the root node to a leaf node.

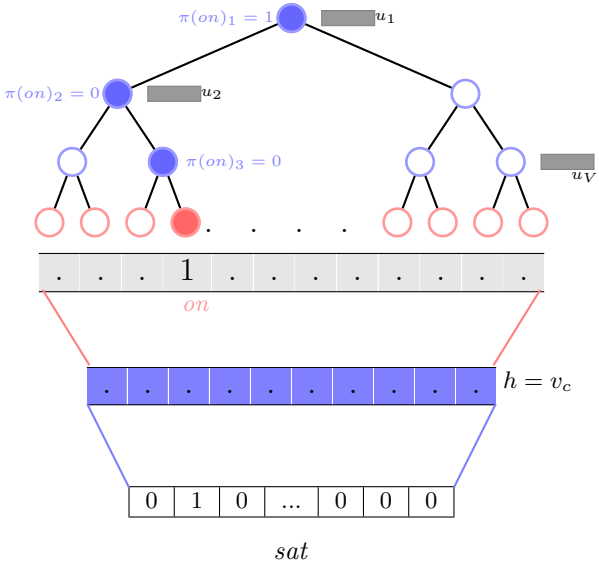


- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary
- There exists a unique path from the root node to a leaf node.
- Let $l(w_1)$, $l(w_2)$, ..., $l(w_p)$ be the nodes on the path from root to w



- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary
- There exists a unique path from the root node to a leaf node.
- Let $l(w_1)$, $l(w_2)$, ..., $l(w_p)$ be the nodes on the path from root to w
- Let $\pi(w)$ be a binary vector such that:

$$\pi(w)_k = 1 \quad \text{path branches left at node } l(w_k)$$
$$= 0 \quad otherwise$$



- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary
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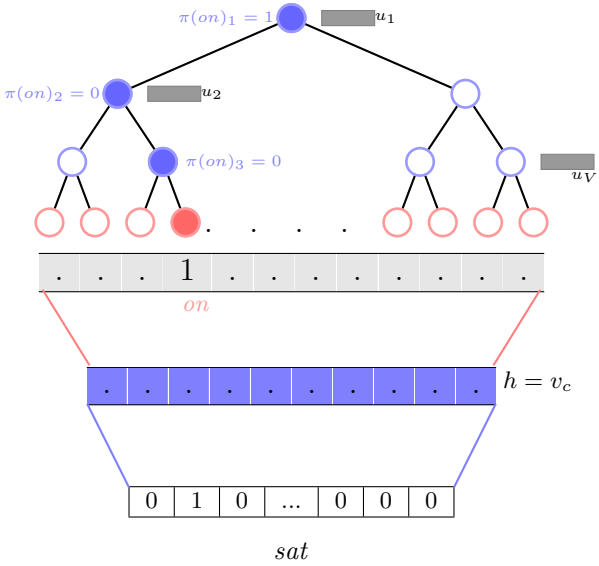
- Finally each internal node is associated with a vector u_i



- Construct a binary tree such that there are $|V|$ leaf nodes each corresponding to one word in the vocabulary
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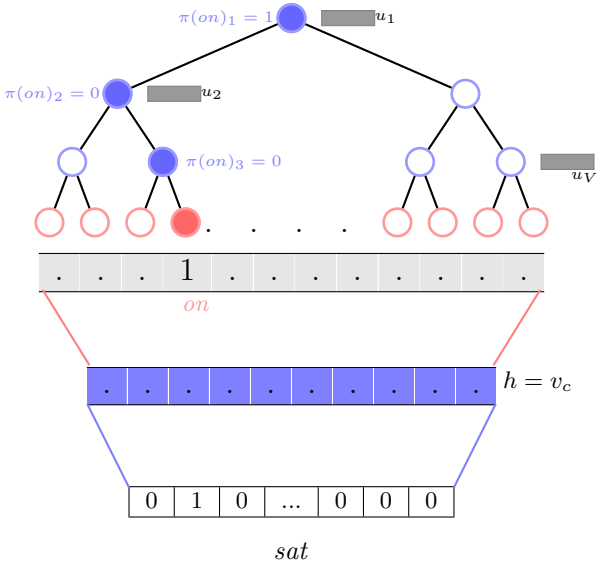
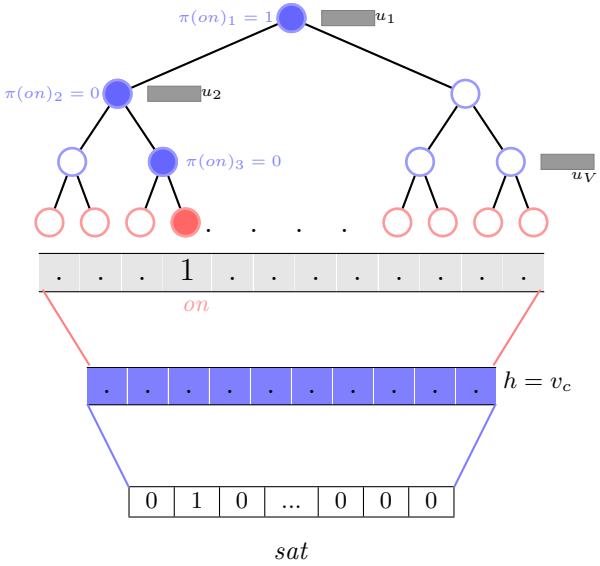
- Let $\pi(w)$ be a binary vector such that:

$$\pi(w)_k = 1 \quad \text{path branches left at node } l(w_k)$$
$$= 0 \quad otherwise$$

- Finally each internal node is associated with a vector u_i
- So the parameters of the module are $\mathbf{W}_{context}$ and $u_1, u_2, \ldots, u_v$ (in effect, we have the same number of parameters as before)

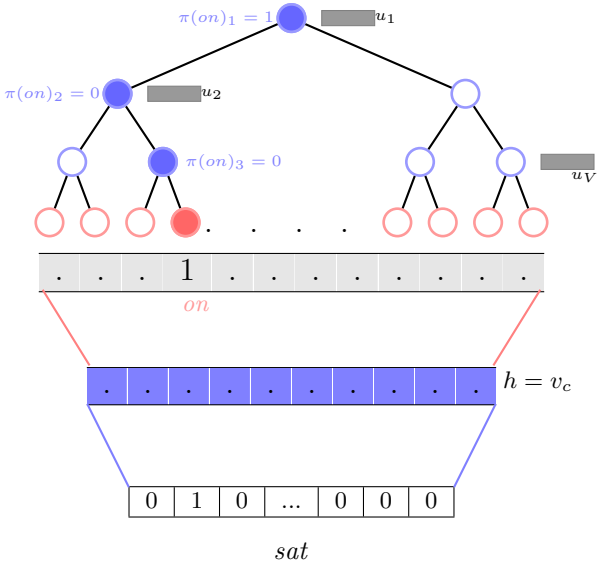


- For a given pair (w, c) we are interested in the probability $p(w|v_c)$



- For a given pair (w, c) we are interested in the probability $p(w|v_c)$
- We model this probability as

$$p(w|v_c) = \prod_k (\pi(w_k)|v_c)$$



- For a given pair (w, c) we are interested in the probability $p(w|v_c)$
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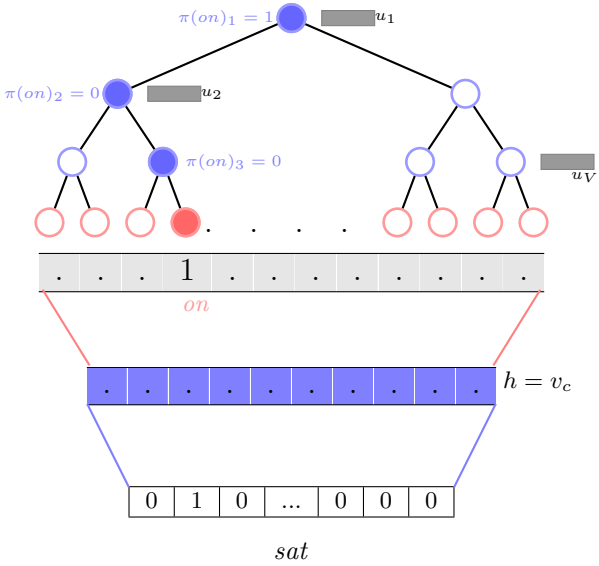
$$p(w|v_c) = \prod_k (\pi(w_k)|v_c)$$

- For example

$$P(on|v_{sat}) = P(\pi(on)_1 = 1|v_{sat}) \\ *P(\pi(on)_2 = 0|v_{sat}) \\ *P(\pi(on)_3 = 0|v_{sat})$$



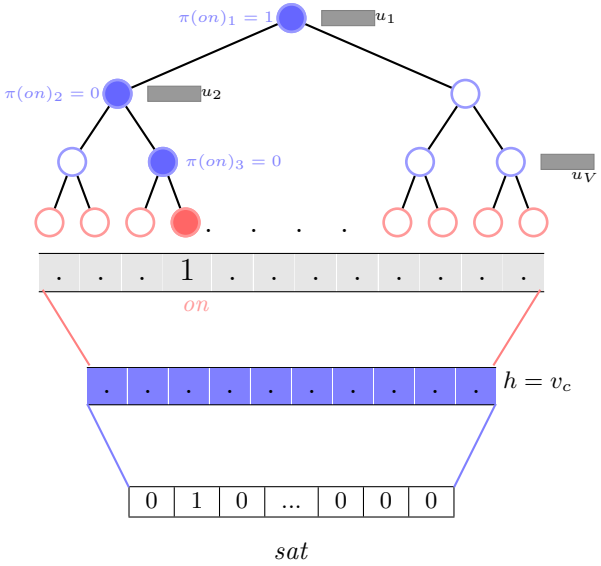
- For a given pair (w, c) we are interested in the probability $p(w|v_c)$
- We model this probability as

$$p(w|v_c) = \prod_k (\pi(w_k)|v_c)$$

- For example

$$P(on|v_{sat}) = P(\pi(on)_1 = 1|v_{sat}) \\ *P(\pi(on)_2 = 0|v_{sat}) \\ *P(\pi(on)_3 = 0|v_{sat})$$

- In effect, we are saying that the probability of predicting a word is the same as predicting the correct unique path from the root node to that word

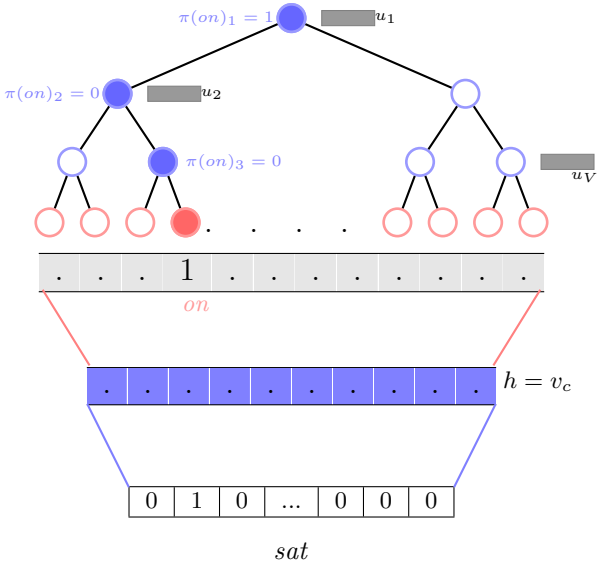


- We model

$$P(\pi(on)_i = 1) = \frac{1}{1 + e^{-v_c^T u_i}}$$

$$P(\pi(on)_i = 0) = 1 - P(\pi(on)_i = 1)$$

$$P(\pi(on)_i = 0) = \frac{1}{1 + e^{v_c^T u_i}}$$



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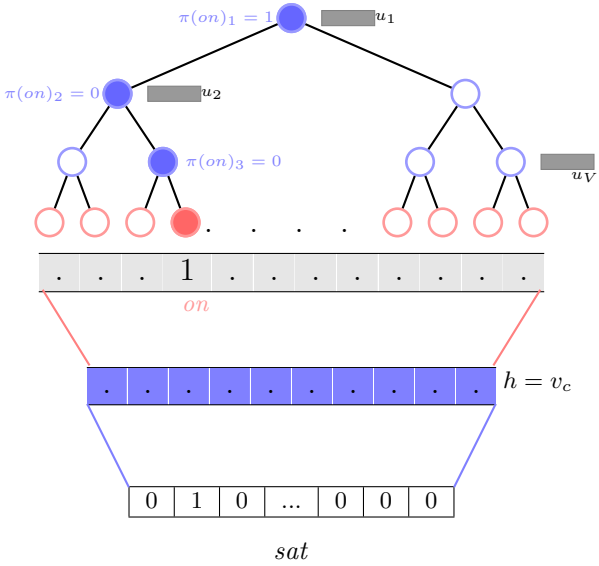
- The above model ensures that the representation of a context word v_c will have a high(low) similarity with the representation of the node u_i if u_i appears and the path branches to the left(right) at u_i



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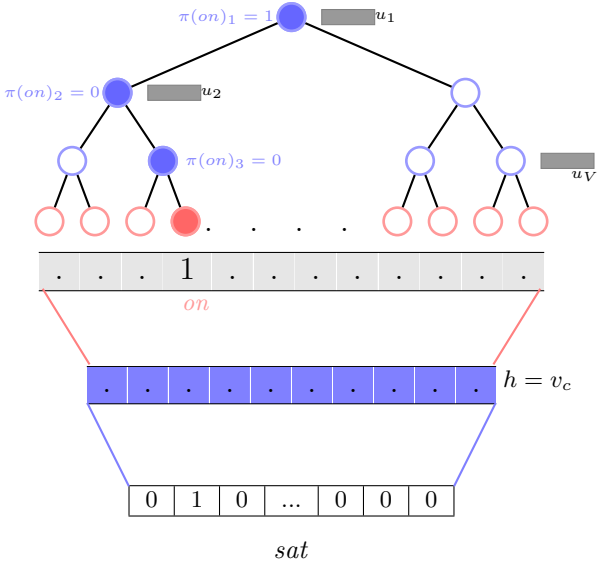
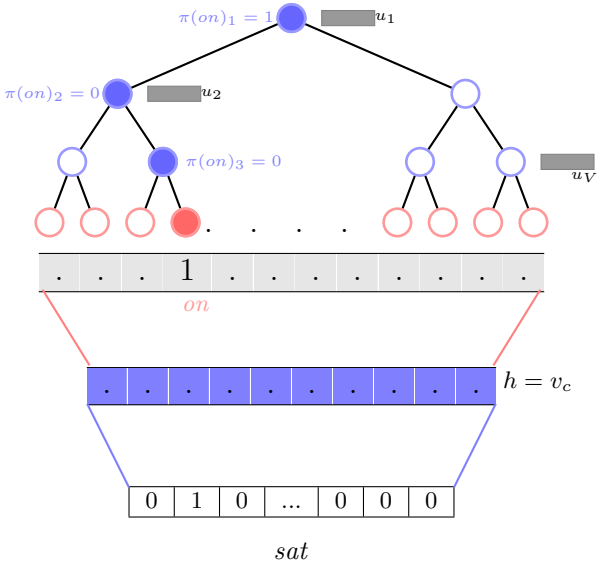
- The above model ensures that the representation of a context word v_c will have a high(low) similarity with the representation of the node u_i if u_i appears and the path branches to the left(right) at u_i
- Again, transitively the representations of contexts which appear with the same words will have high similarity



$$P(w|v_c) = \prod_{k=1}^{|\pi(w)|} P(\pi(w_k)|v_c)$$



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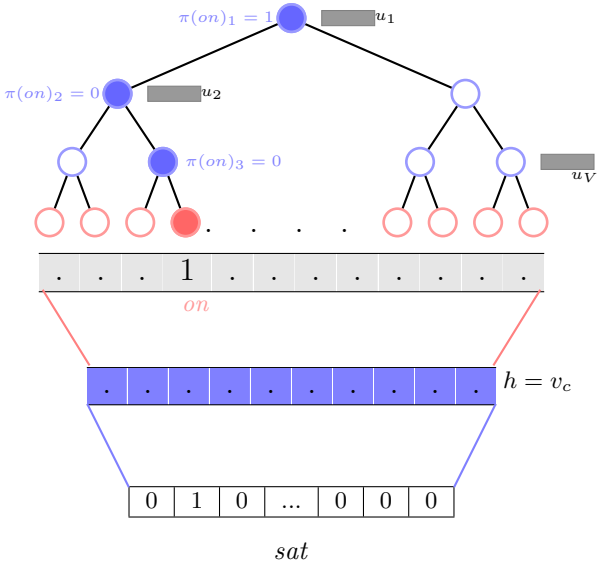

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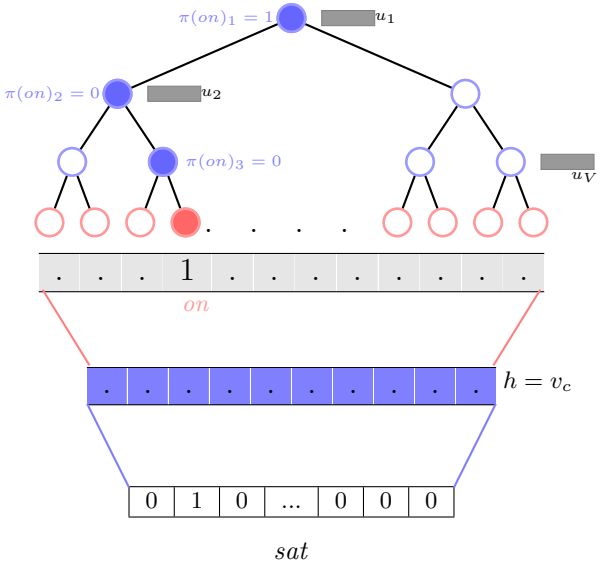

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- Note that $p(w|v_c)$ can now be computed using $|\pi(w)|$ computations instead of $|V|$ required by softmax
- How do we construct the binary tree?
- Turns out that even a random arrangement of the words on leaf nodes does well in practice

Module 10.8: GloVe representations

- **Count** based methods (SVD) rely on global co-occurrence counts from the corpus for computing word representations

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- Predict based methods **learn** word representations using co-occurrence information
- Why not combine the two (**count** and **learn**) ?

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- User opinion of computer system response time
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$$X =$$

	human	machine	system	for	...	user
human	2.01	2.01	0.23	2.14	...	0.43
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$$P(j|i) = \frac{X_{ij}}{\sum X_{ij}} = \frac{X_{ij}}{X_i}$$

$$X_{ij} = X_{ji}$$

- X_{ij} encodes important global information about the co-occurrence between i and j (global: because it is computed for the entire corpus)

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- Essentially we are saying that we want word vectors v_i and v_j such that $v_i^T v_j$ is faithful to the globally computed $P(j|i)$

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- Adding the two equations we get

$$2v_i^T v_j = 2\log X_{ij} - \log X_i - \log X_j$$

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- Note that $\log X_i$ and $\log X_j$ depend only on the words i & j and we can think of them as word specific biases which will be learned

$$v_i^T v_j = \log X_{ij} - b_i - b_j$$

$$v_i^T v_j + b_i + b_j = \log X_{ij}$$

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$$v_i^T v_j = \log X_{ij} - b_i - b_j$$

$$v_i^T v_j + b_i + b_j = \log X_{ij}$$

- We can then formulate this as the following optimization problem

$$\min_{v_i, v_j, b_i, b_j} \sum_{i,j} (\underbrace{v_i^T v_j + b_i + b_j}_{\text{predicted value using model parameters}} - \underbrace{\log X_{ij}}_{\text{actual value computed from the given corpus}})^2$$

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- **Drawback:** weighs all co-occurrences equally

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- **Drawback:** weighs all co-occurrences equally
- **Solution:** add a weighting function

$$\min_{v_i, v_j, b_i, b_j} \sum_{i,j} f(X_{ij})(v_i^T v_j + b_i + b_j - \log X_{ij})^2$$

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- **Wishlist:** $f(X_{ij})$ should be such that neither rare nor frequent words are over-weighted.

$$f(x) = \left\{ \begin{array}{ll} (\frac{x}{x_{max}})^\alpha, & \text{if } x < x_{max} \\ 1, & \text{otherwise} \end{array} \right\}$$

where α can be tuned for a given dataset

Module 10.9: Evaluating word representations

How do we evaluate the learned word representations ?

Semantic Relatedness

Semantic Relatedness

- Ask humans to judge the relatedness between a pair of words

$S_{human}(cat, dog) = 0.8$

Semantic Relatedness

- Ask humans to judge the relatedness between a pair of words
- Compute the cosine similarity between the corresponding word vectors learned by the model

$$S_{human}(cat, dog) = 0.8$$

$$S_{model}(cat, dog) = \frac{v_{cat}^T v_{dog}}{\| v_{cat} \| \| v_{dog} \|} = 0.7$$

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Semantic Relatedness

- Ask humans to judge the relatedness between a pair of words
- Compute the cosine similarity between the corresponding word vectors learned by the model
- Given a large number of such word pairs, compute the correlation between S_{model} & S_{human}, and compare different models

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Semantic Relatedness

- Ask humans to judge the relatedness between a pair of words
- Compute the cosine similarity between the corresponding word vectors learned by the model
- Given a large number of such word pairs, compute the correlation between S_{model} & S_{human}, and compare different models
- Model 1 is better than Model 2 if

$$correlation(S_{model1}, S_{human}) > correlation(S_{model2}, S_{human})$$

Synonym Detection

Synonym Detection

- Given: a term and four candidate synonyms

Term : levied

Candidates : {unposed,
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- Given: a term and four candidate synonyms
- Pick the candidate which has the largest cosine similarity with the term

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$$\textbf{Synonym} := \underset{c \in C}{argmax}\ cosine(v_{term}, v_c)$$

Synonym Detection

- Given: a term and four candidate synonyms
- Pick the candidate which has the largest cosine similarity with the term
- Compute the accuracy of different models and compare

Term : levied

Candidates : {unposed, believed, requested, correlated}

$$\textbf{Synonym} := \underset{c \in C}{argmax}\ cosine(v_{term}, v_c)$$

Analogy

Analogy

- Semantic Analogy: Find nearest neighbour of $v_{sister} - v_{brother} + v_{grandson}$

brother : sister :: grandson : ?

Analogy

- Semantic Analogy: Find nearest neighbour of $v_{sister} - v_{brother} + v_{grandson}$
- Syntactic Analogy: Find nearest neighbour of $v_{works} - v_{work} + v_{speak}$

brother : sister :: grandson : ?
work : works :: speak : ?

- So which algorithm gives the best result ?

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- Boroni et.al [2014] showed that predict models consistently outperform count models in all tasks.

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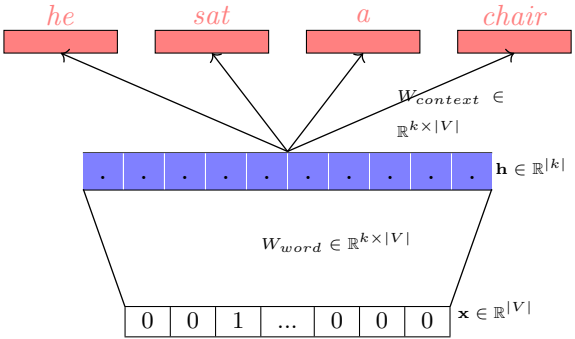
- Boroni et.al [2014] showed that predict models consistently outperform count models in all tasks.
- Levy et.al [2015] do a much more through analysis (IMO) and show that good old SVD does better than prediction based models on similarity tasks but not on analogy tasks.

Module 10.10: Relation between SVD & word2Vec

The story ahead ...

- Continuous bag of words model
- Skip gram model with negative sampling (the famous word2vec)
- GloVe word embeddings
- Evaluating word embeddings
- Good old SVD does just fine!!



- Recall that SVD does a matrix factorization of the co-occurrence matrix

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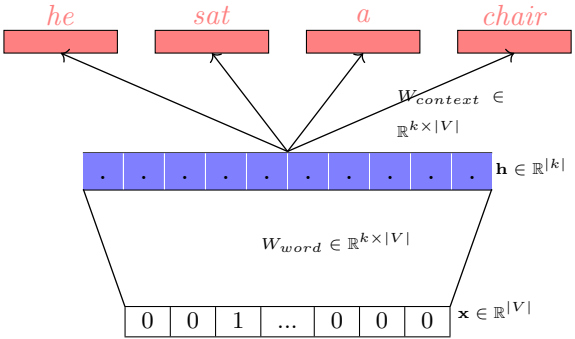
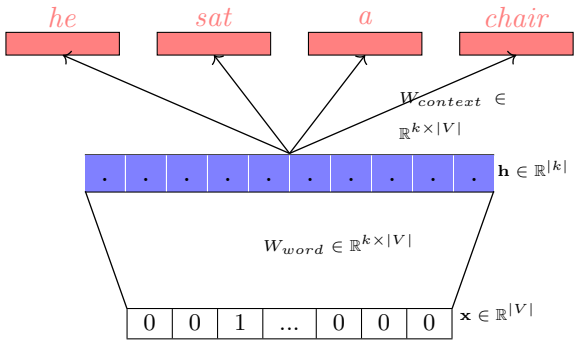


- Recall that SVD does a matrix factorization of the co-occurrence matrix
- Levy et.al [2015] show that word2vec also implicitly does a matrix factorization

.



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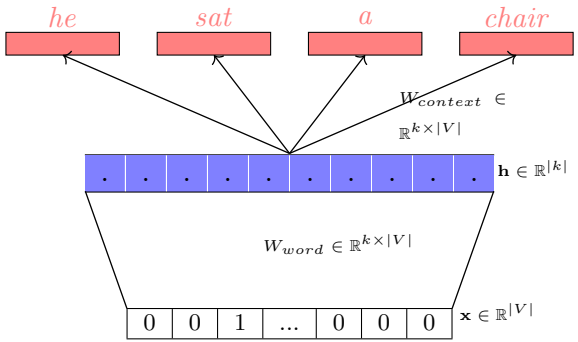

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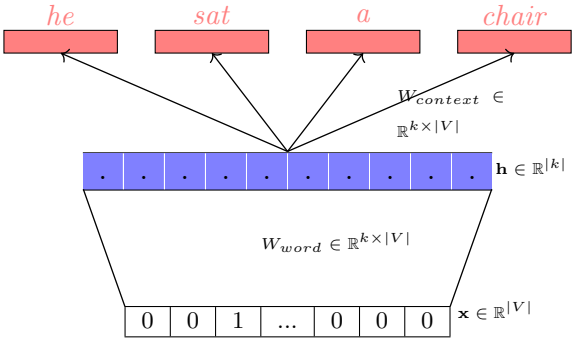

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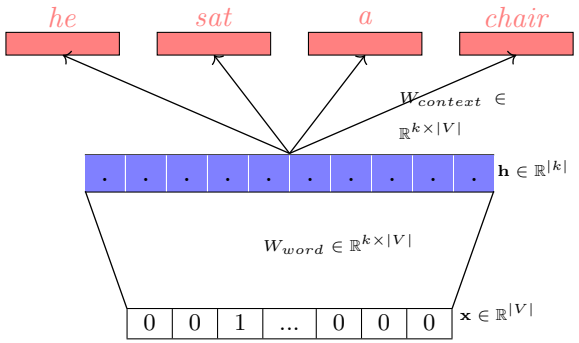

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- So essentially, word2vec factorizes a matrix M which is related to the PMI based co-occurrence matrix (very similar to what SVD does)