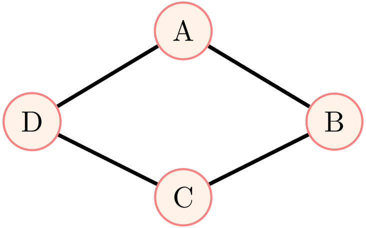
Module 18.1: Markov Networks: Motivation

- To motivate undirected graphical models let us consider a new example



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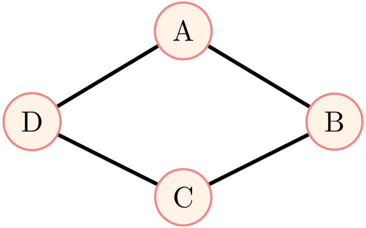
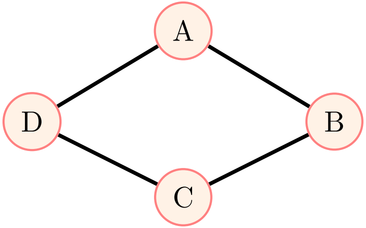


- To motivate undirected graphical models let us consider a new example
- A, B, C, D are four students



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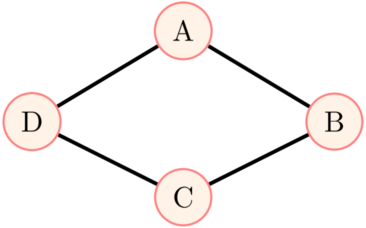
- A, B, C, D are four students
- A and B study together sometimes



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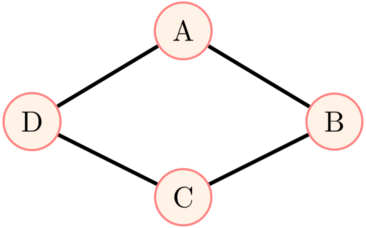

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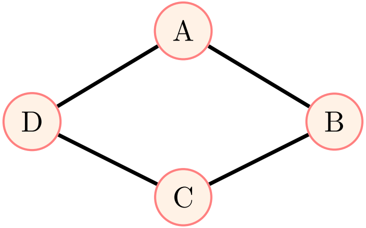

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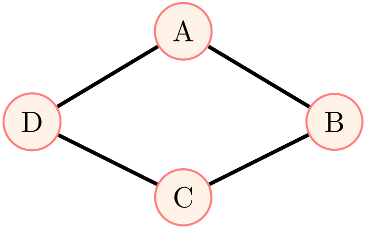

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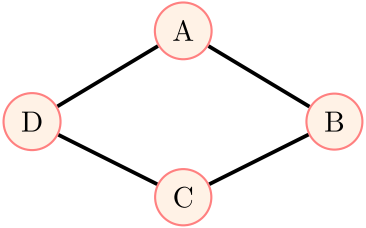

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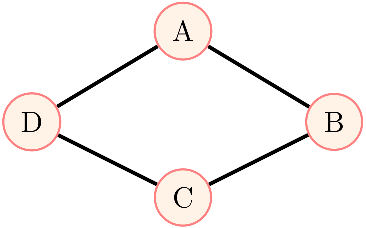

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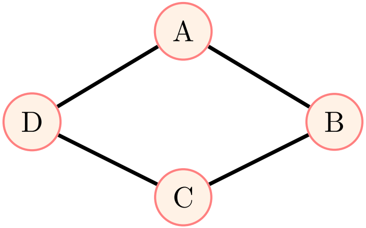

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- Now suppose there was some misconception in the lecture due to some error made by the teacher

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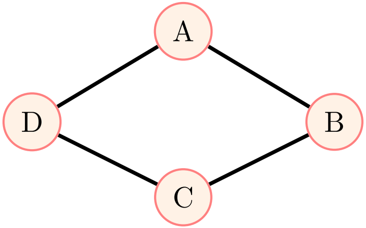

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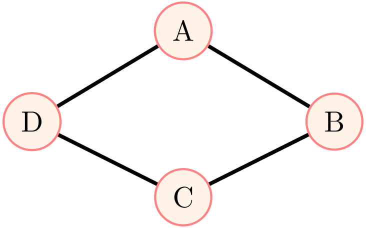
- Each one of A, B, C, D could have independently cleared this misconception by thinking about it after the lecture



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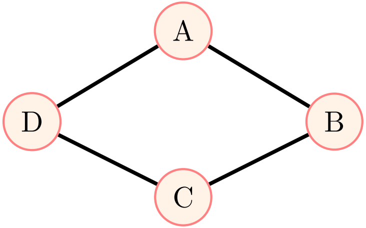
- To motivate undirected graphical models let us consider a new example
- Now suppose there was some misconception in the lecture due to some error made by the teacher
- Each one of A, B, C, D could have independently cleared this misconception by thinking about it after the lecture
- In subsequent study pairs, each student could then pass on this information to their partner



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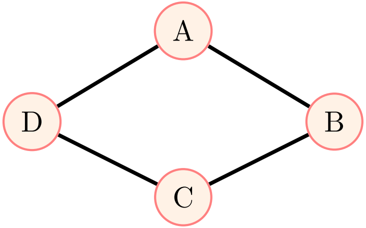
- We are now interested in knowing whether a student still has the misconception or not



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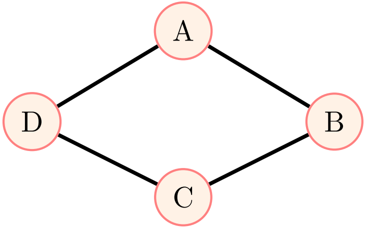
- Or we are interested in $P(A, B, C, D)$



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- where A, B, C, D can take values 0 (no misconception) or 1 (misconception)

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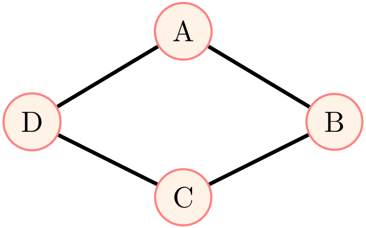

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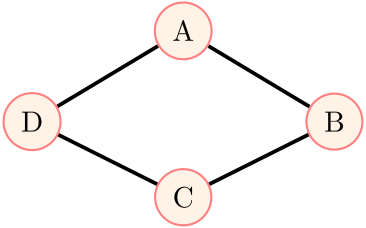
- How do we model this using a Bayesian Network ?



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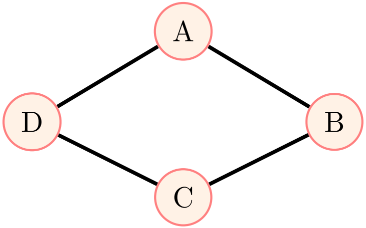
- First let us examine the conditional independencies in this problem



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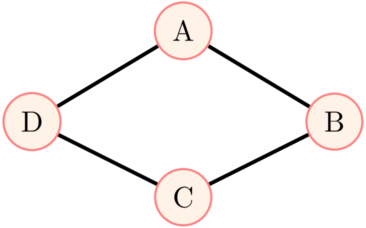
- $A \perp C|\{B, D\}$ (because A & C never interact)



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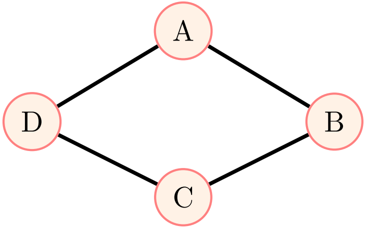

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- $A \perp C | \{B, D\}$ (because A & C never interact)
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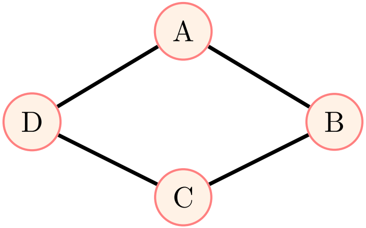
- There are no other conditional independencies in the problem



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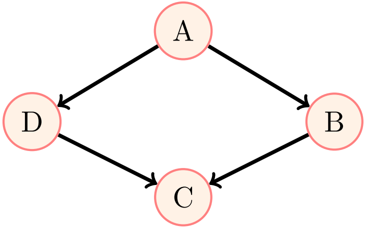
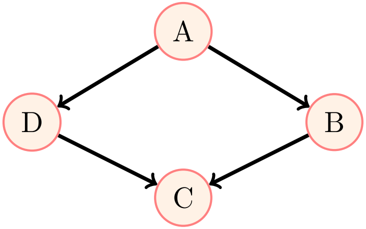
- Now let us try to represent this using a Bayesian Network



- How about this one?



- How about this one?
- Indeed, it captures the following independencies relation

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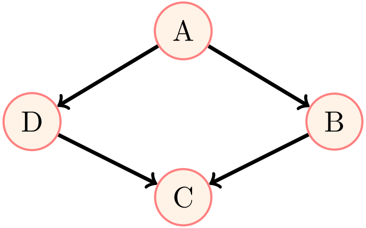


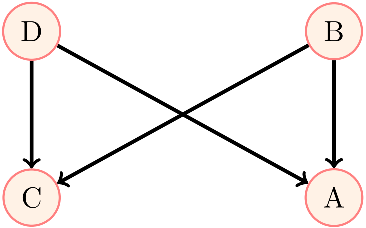
- How about this one?
- Indeed, it captures the following independencies relation

$$A \perp C | \{B, D\}$$

- But, it also implies that

$$B \not\perp D | \{A, C\}$$

- Let us try a different network



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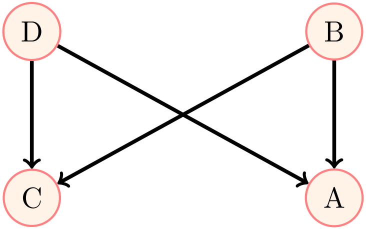

- Let us try a different network
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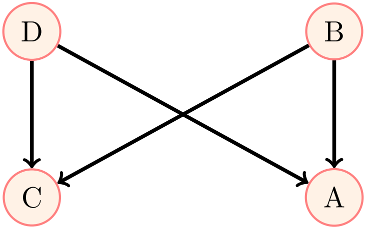

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$$A \perp C | \{B, D\}$$

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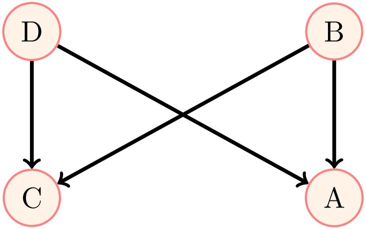
$$B \perp D (\text{unconditional})$$



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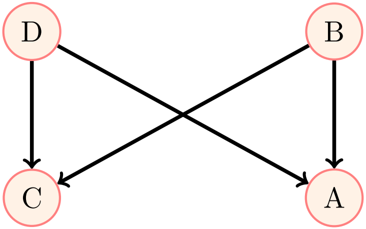

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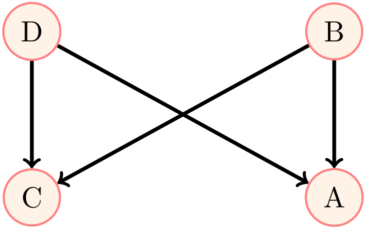

- **Perfect Map**: A graph G is a Perfect Map for a distribution P if the independance relations implied by the graph are exactly the same as those implied by the distribution

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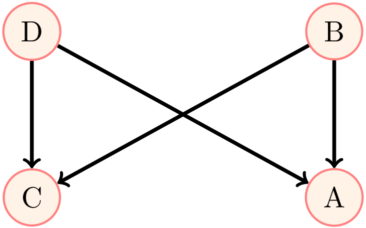

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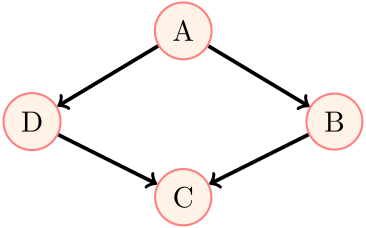
- There is no Perfect Map for the distribution



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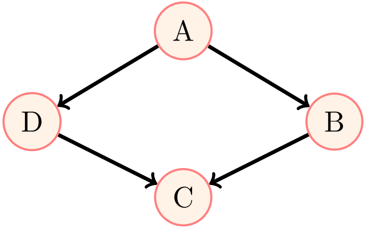

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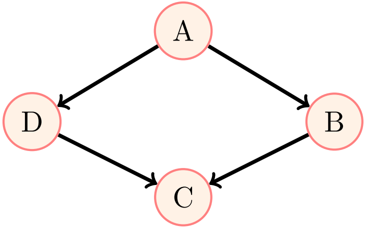

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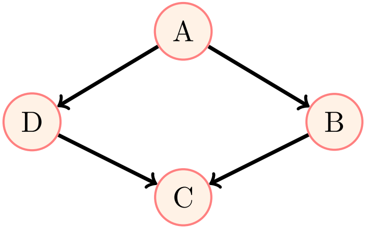

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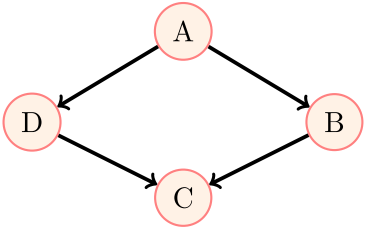

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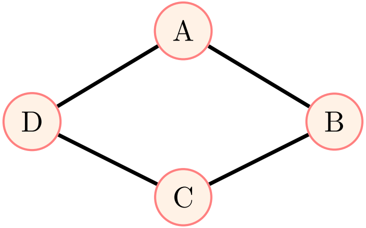
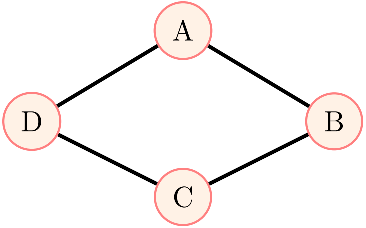
- But in our example $A\&B$ are equal partners (they both contribute to the study discussion)
- We want to capture the strength of this interaction (and there is no direction here)



- We move on from Directed Graphical Models to Undirected Graphical Models



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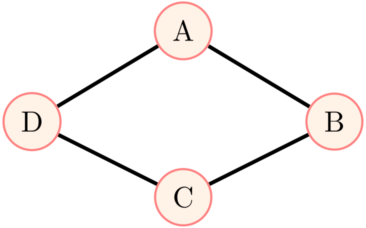
- Also known as **Markov Network**



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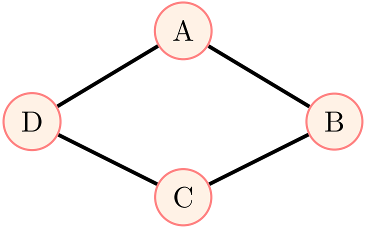

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- Also known as **Markov Network**
- The Markov Network on the left exactly captures the interactions inherent in the problem
- But how do we parameterize this graph?