Module 18.3: Local Independencies in a Markov Network

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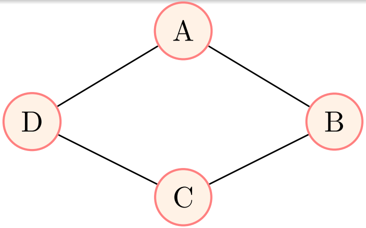
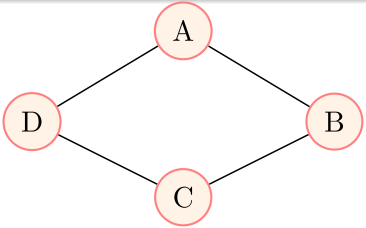
- Let us see this in the context of our original example



- In this example

$$P(A,B,C,D) = \frac{1}{Z}[\phi_1(A,B)\phi_2(B,C)\phi_3(C,D)\phi_4(D,A)]$$



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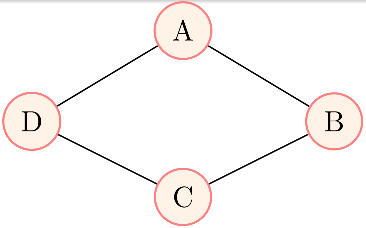

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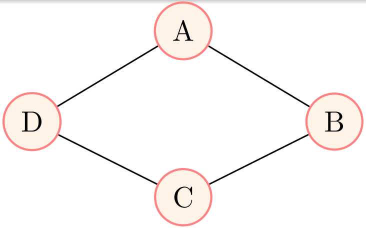
- We can say that $B \perp D | \{A,C\}$ which is indeed true



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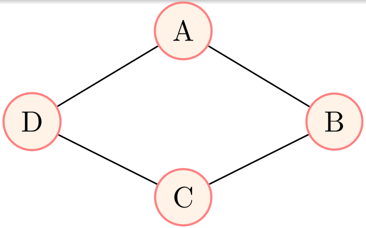

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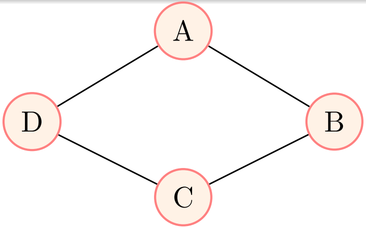

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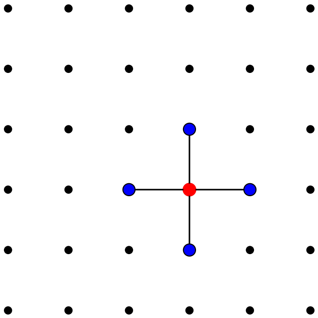
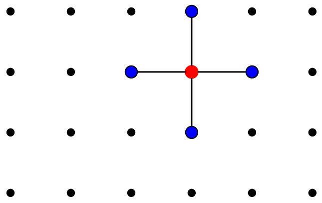
- We can say that $A \perp C | \{B,D\}$ which is indeed true



- For a given Markov network H we define Markov Blanket of a RV X to be the neighbors of X in H



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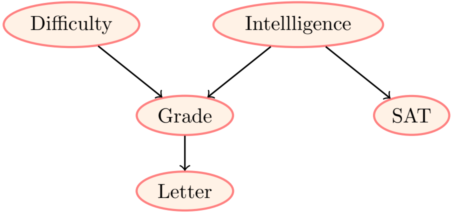
- Analogous to the case of Bayesian Networks we can define the local independences associated with H to be

$$X \perp (U - \{X\} - MB_H) | MB_H(X)$$

Bayesian network

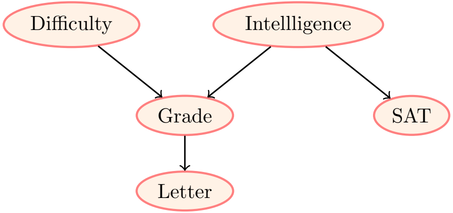


Markov network

Local Independencies

$$X_i \perp NonDescendents_{X_i} | Parent^G_{X_i}$$

Bayesian network



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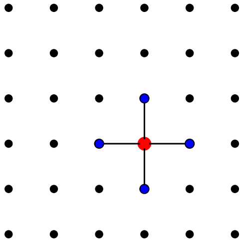

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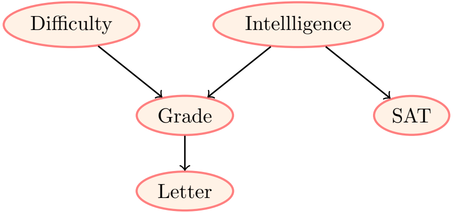

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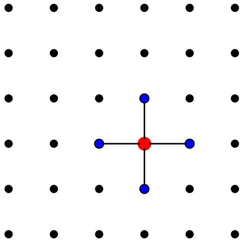

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