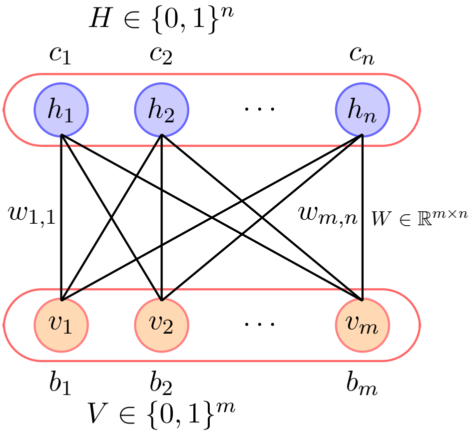
Module 19.4: RBMs as Stochastic Neural Networks

- But what is the connection between this and deep neural networks?
- We will get to it over the next few slides!



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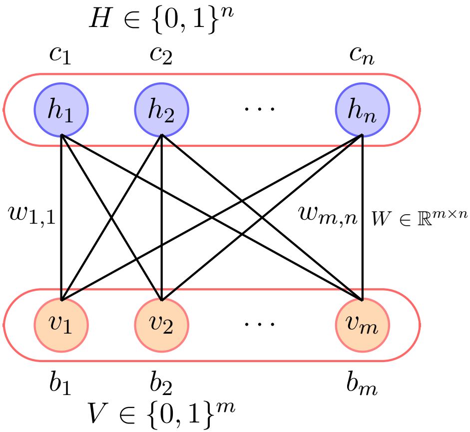

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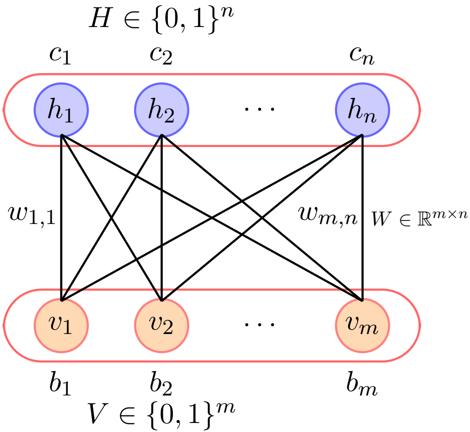

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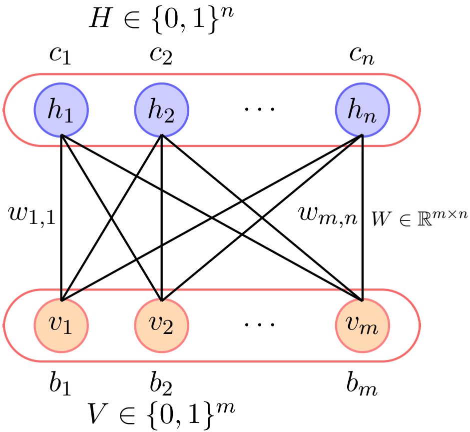

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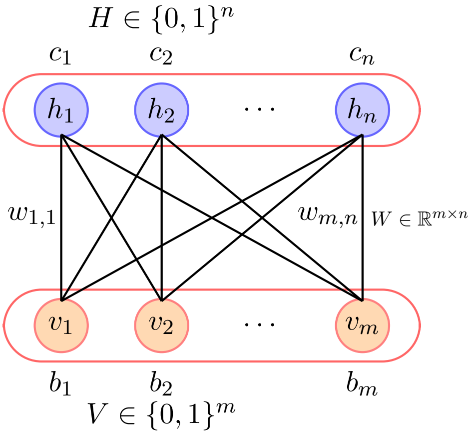
- We now define the following quantities

$$\alpha_l(H) = -\sum_{i=1}^{n} w_{il} h_i - b_l$$

$$\beta(V_{-l}, H) = -\sum_{i=1}^{n} \sum_{j=1, j \neq l}^{m} w_{ij} h_i v_j - \sum_{j=1, j \neq l}^{m} b_i v_i - \sum_{i=1}^{n} c_i h_i$$



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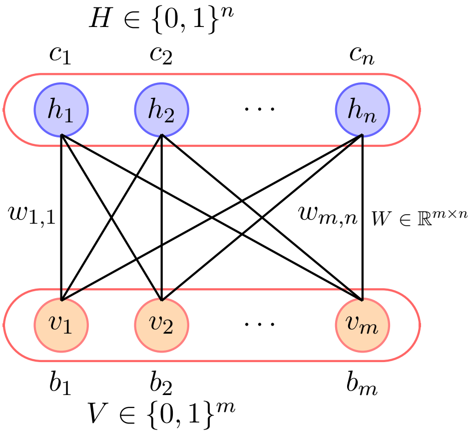
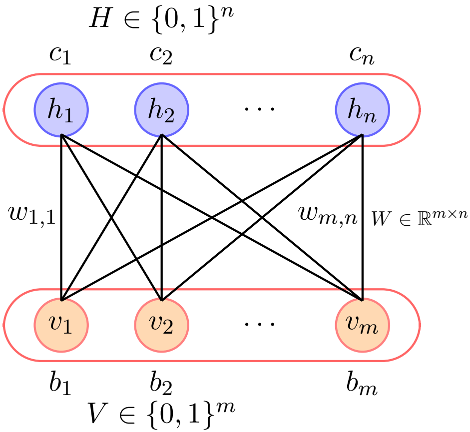
- Notice that

$$E(V, H) = v_l \alpha(H) + \beta(V_{-l}, H)$$

- We can now write $P(v_l = 1|H)$ as





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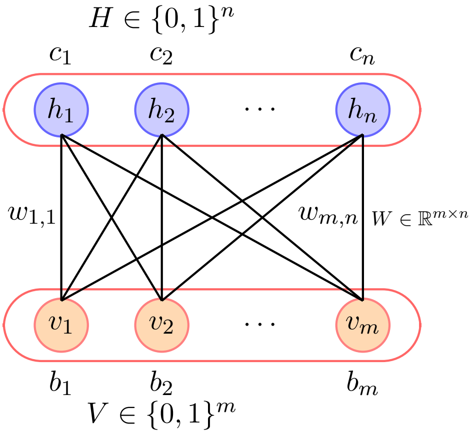

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$$p(v_l = 1|H) = P(v_l = 1|V_{-l}, H)$$
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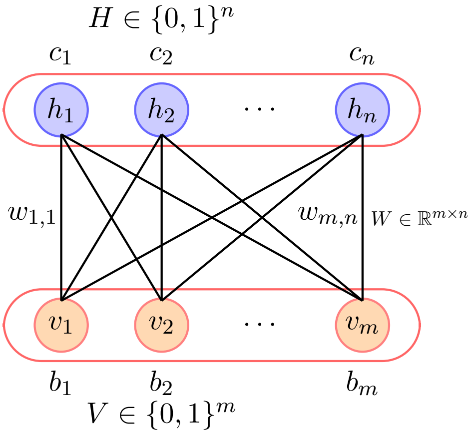

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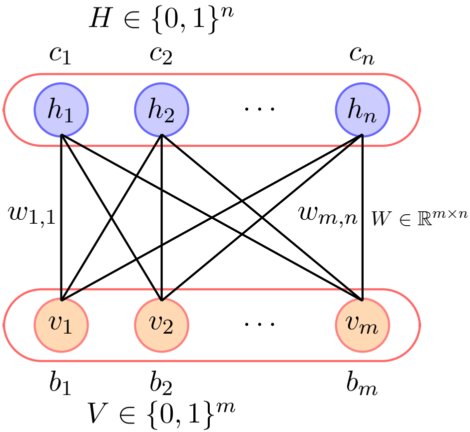
$$\begin{aligned} p(v_l = 1|H) &= P(v_l = 1|V_{-l}, H) \\ &= \frac{p(v_l = 1, V_{-l}, H)}{p(V_{-l}, H)} \\ &= \frac{e^{-E(v_l=1,V_{-l},H)}}{e^{-E(v_l=1,V_{-l},H)} + e^{-E(v_l=0,V_{-l},H)}} \end{aligned}$$



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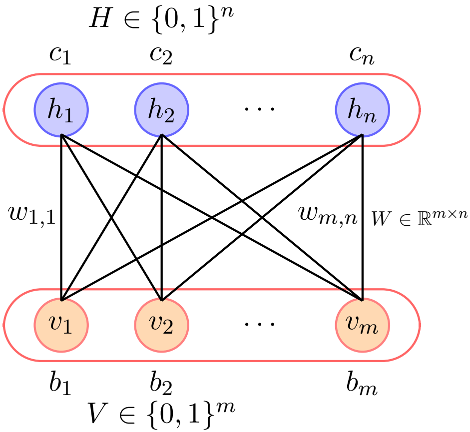

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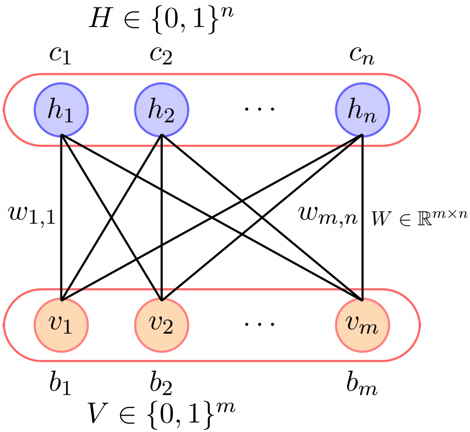

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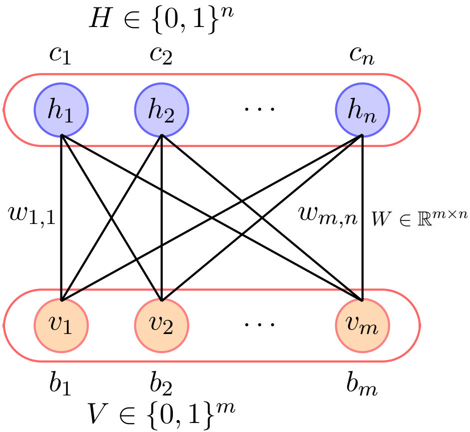

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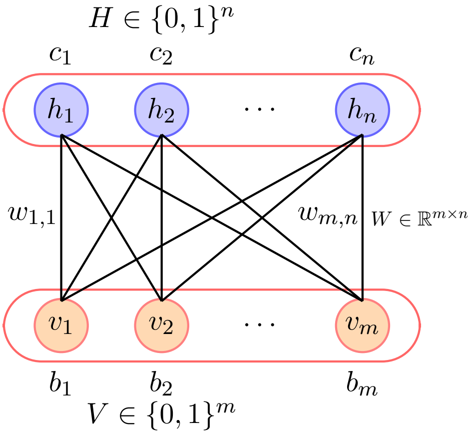
&= \frac{e^{-\alpha_l(H)}}{e^{-\alpha_l(H)} + 1} = \frac{1}{1 + e^{\alpha_l(H)}} \\
&= \sigma(-\alpha_l(H)) = \sigma(\sum_{i=1}^{n} w_{il} h_i + b_l)
\end{aligned}
$$



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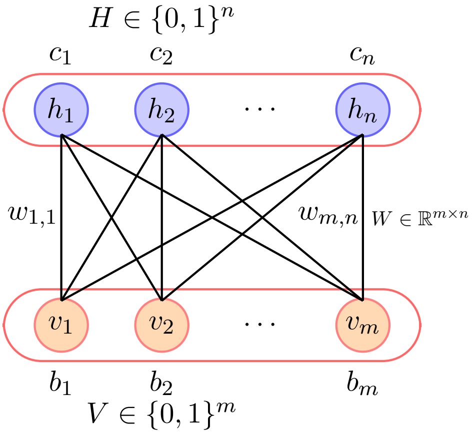

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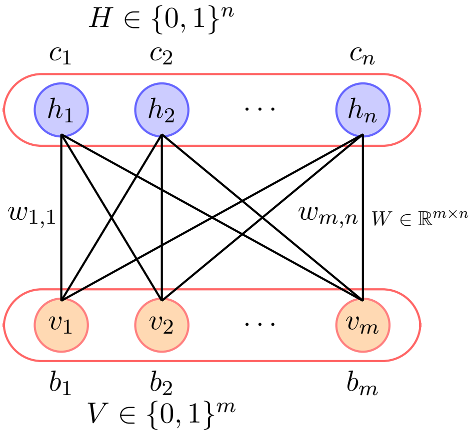

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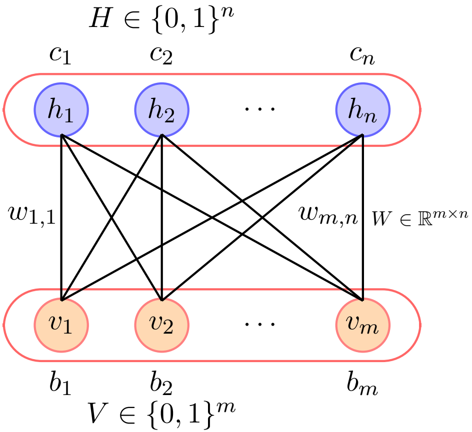
- The RBM can thus be interpreted as a stochastic neural network, where the nodes and edges correspond to neurons and synaptic connections, respectively.



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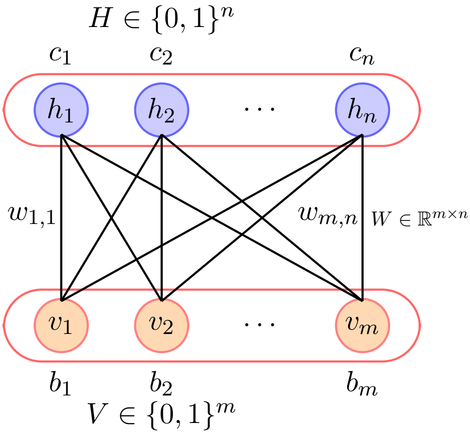
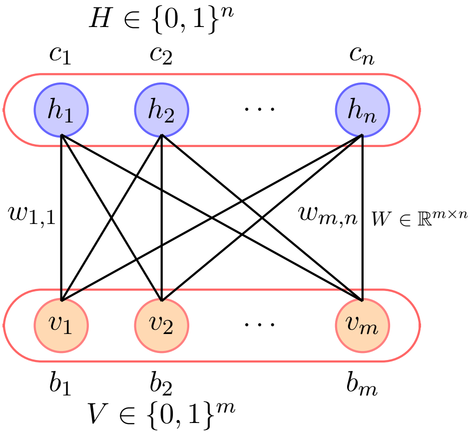
- The RBM can thus be interpreted as a stochastic neural network, where the nodes and edges correspond to neurons and synaptic connections, respectively.
- The conditional probability of a single (hidden or visible) variable being 1 can be interpreted as the firing rate of a (stochastic) neuron with sigmoid activation function



- Given this neural network view of RBMs, can you say something about what h is trying to learn?



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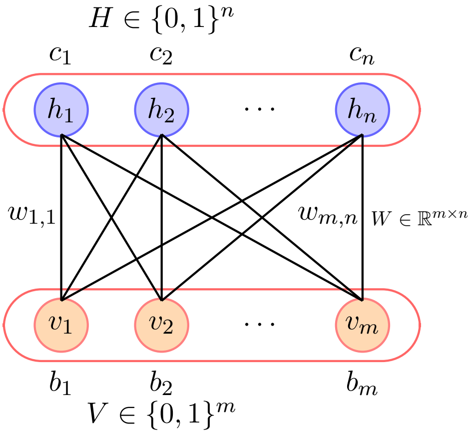

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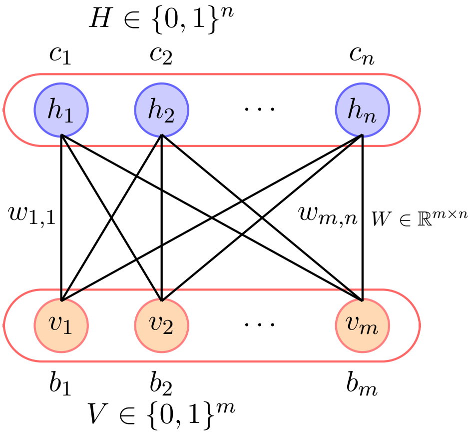

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- We will see this in the next lecture!