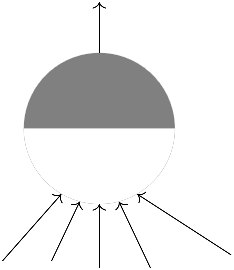
Module 2.2: McCulloch Pitts Neuron



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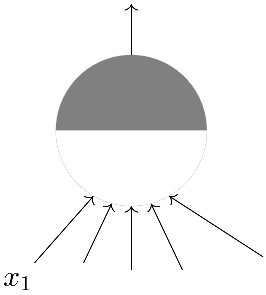

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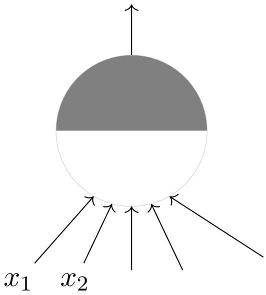

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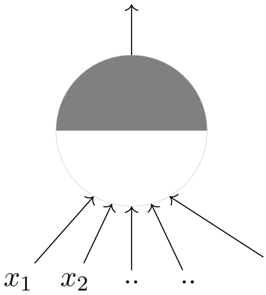

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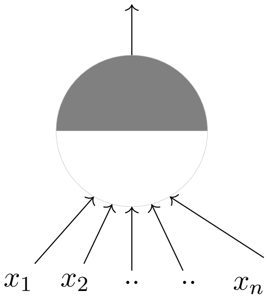

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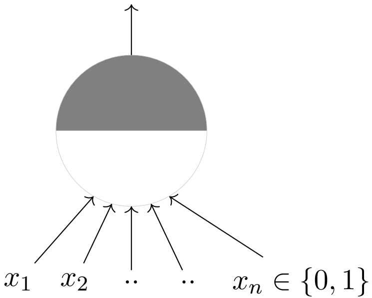

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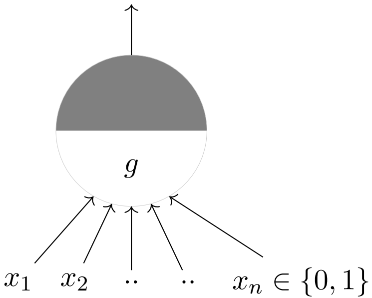

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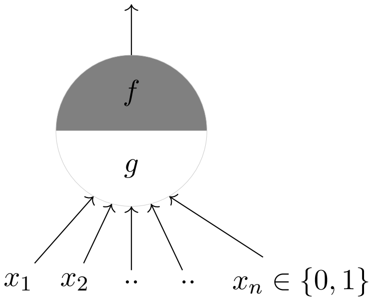

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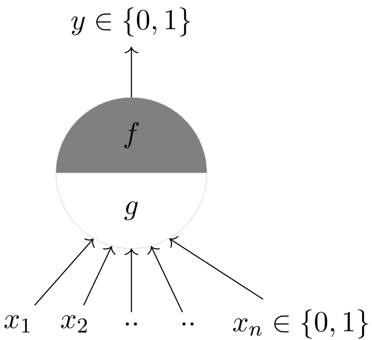

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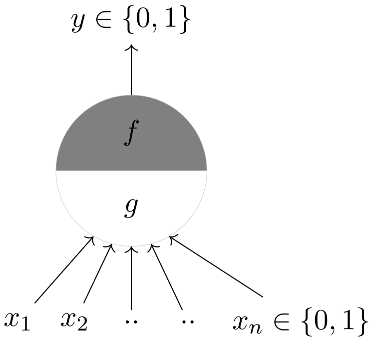

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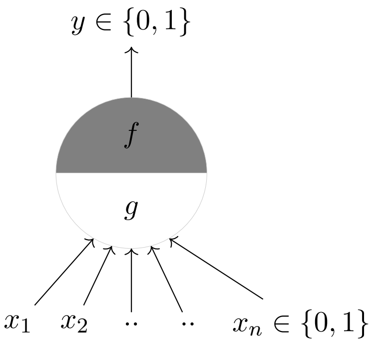

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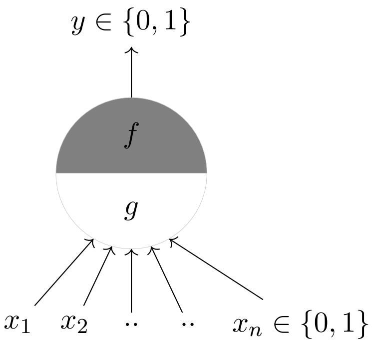

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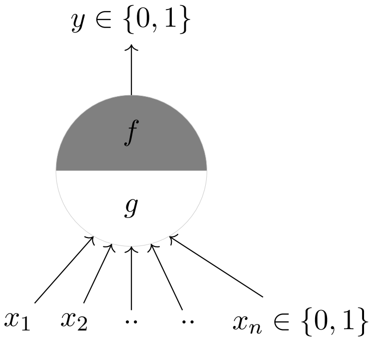

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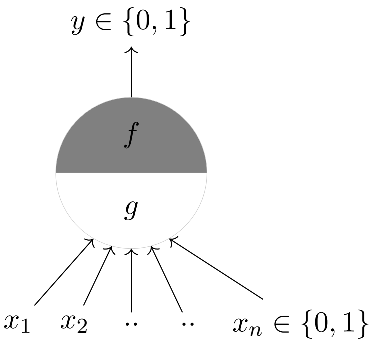

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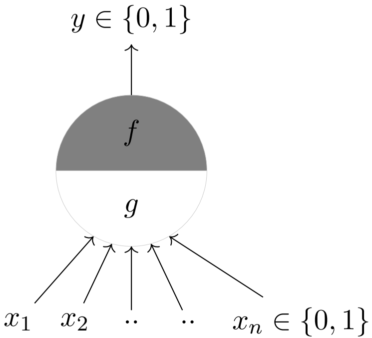

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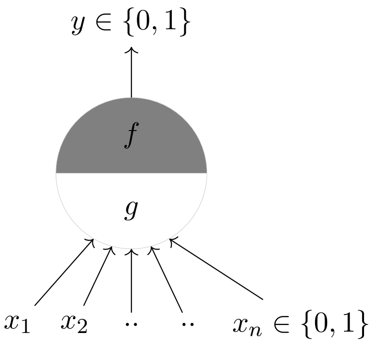

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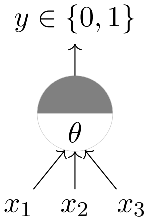
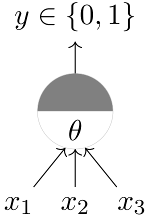
- This is called Thresholding Logic

Let us implement some boolean functions using this McCulloch Pitts (MP) neuron ...



A McCulloch Pitts unit



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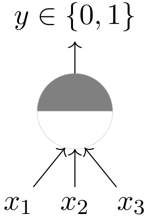
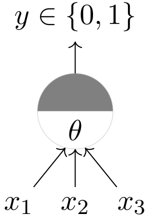


AND function



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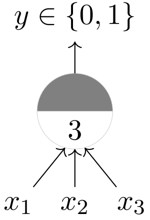

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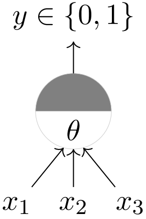

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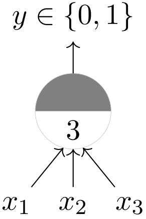

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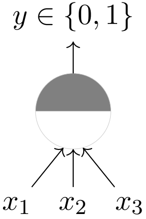

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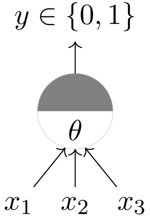

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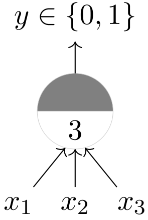

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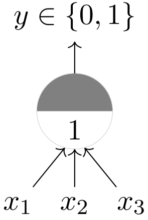

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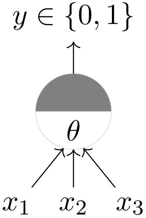

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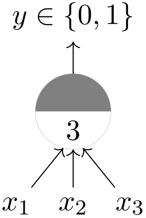

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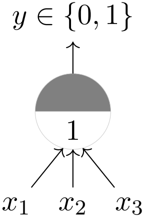

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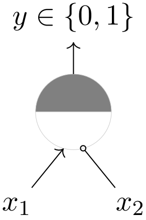

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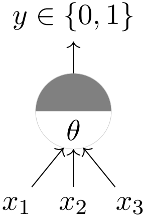

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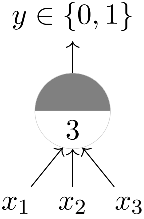

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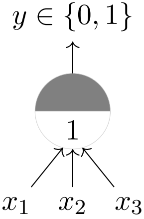

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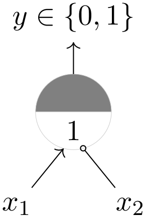

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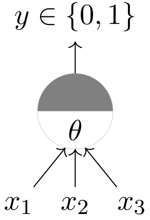

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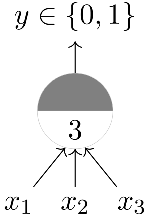

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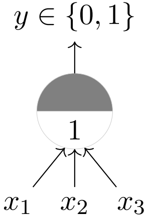

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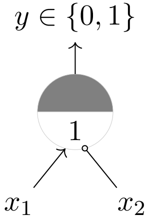

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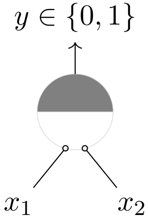

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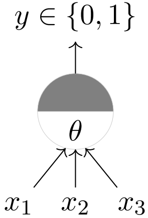

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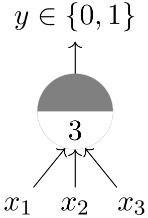

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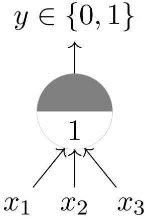

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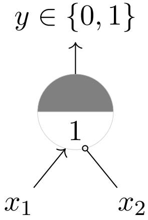

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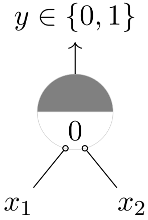

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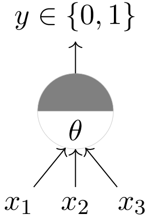

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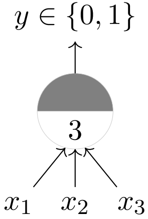

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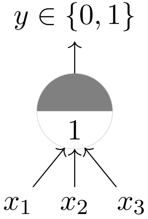

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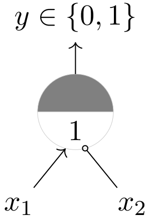

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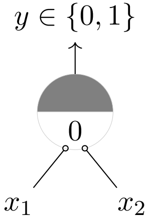

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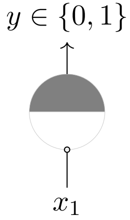

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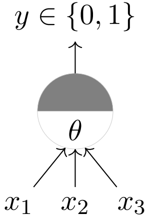

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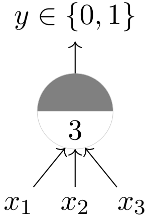

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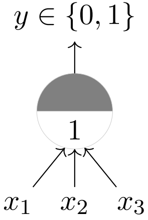

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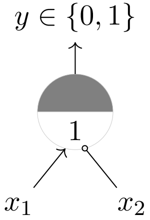

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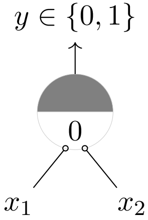

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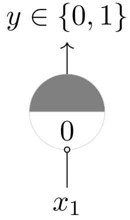

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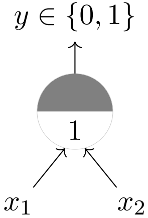
- Before answering this question let us first see the geometric interpretation of a MP unit ...



OR function

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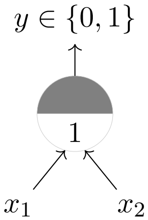

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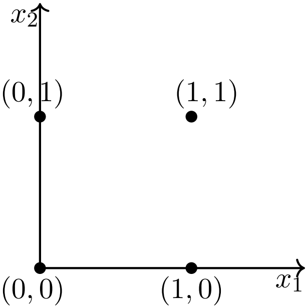

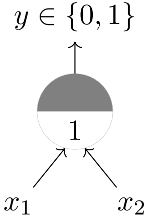

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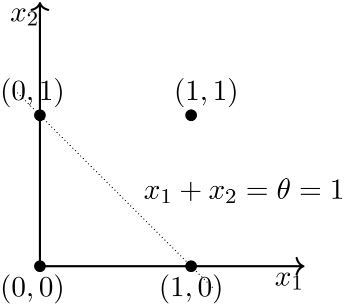

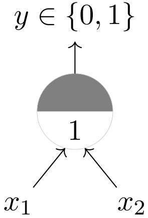

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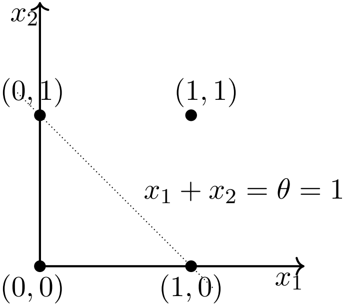
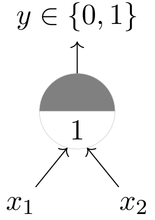


- A single MP neuron splits the input points (4 points for 2 binary inputs) into two halves



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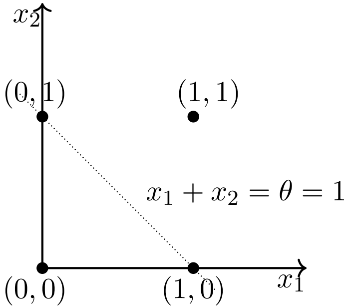
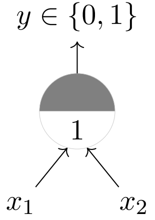


- A single MP neuron splits the input points (4 points for 2 binary inputs) into two halves
- Points lying on or above the line $\sum_{i=1}^{n} x_i - \theta = 0$ and points lying below this line



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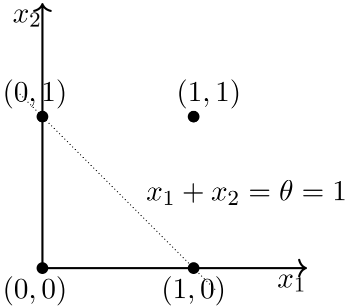

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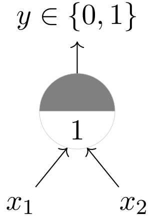

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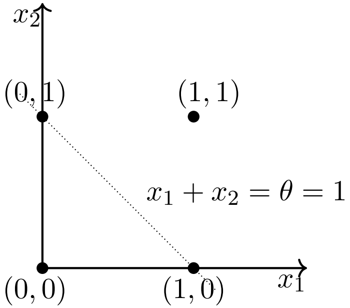

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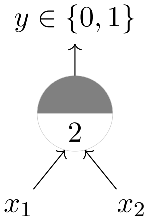
- Let us convince ourselves about this with a few more examples (if it is not already clear from the math)



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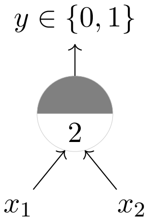

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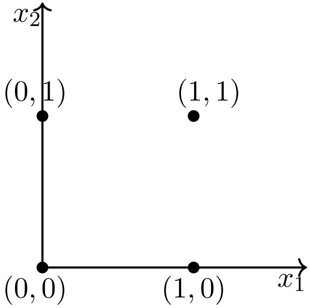

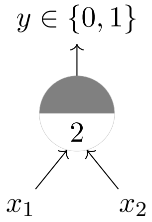

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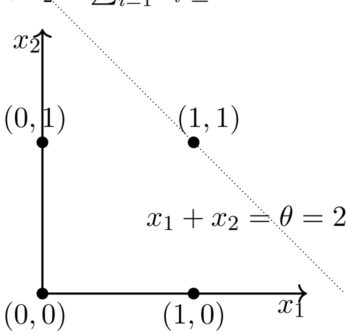

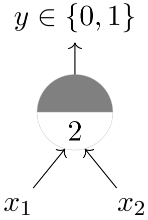

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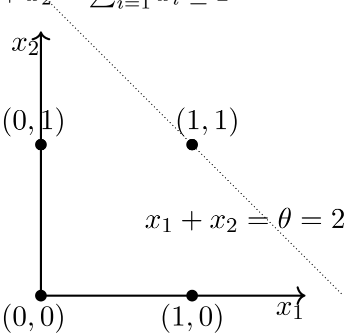
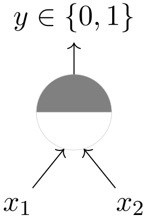
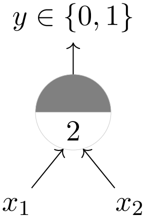




Tautology (always ON)



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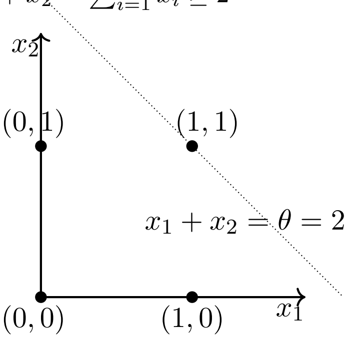

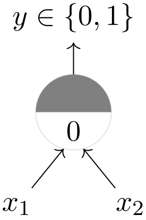

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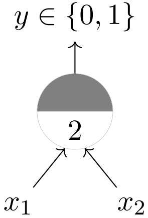

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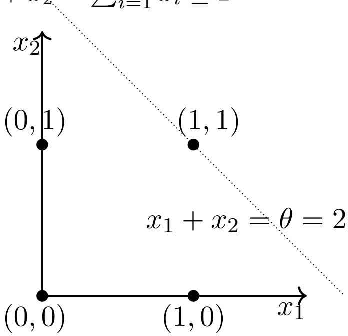

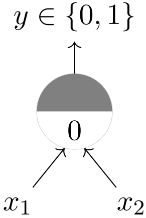

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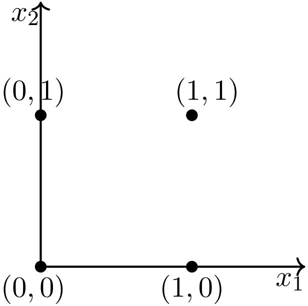

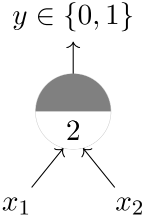

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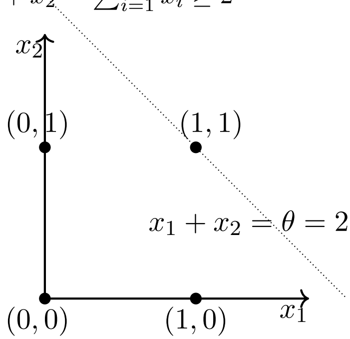

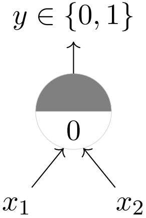

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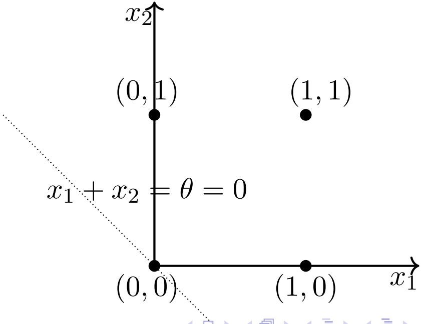

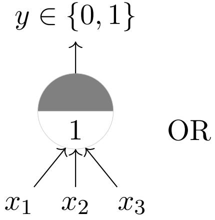

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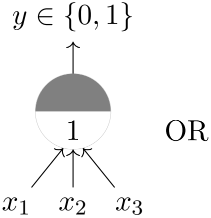

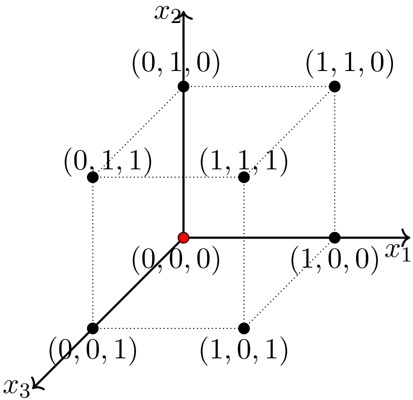

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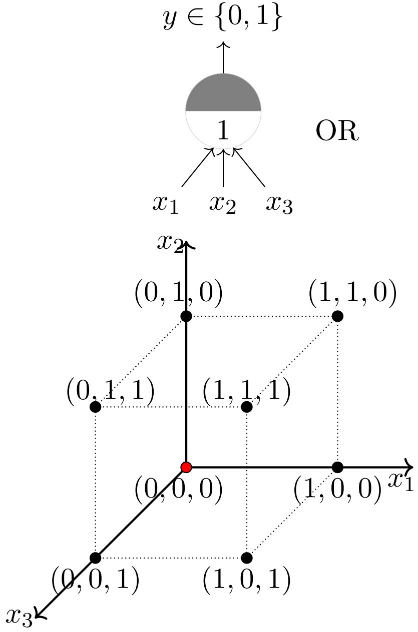

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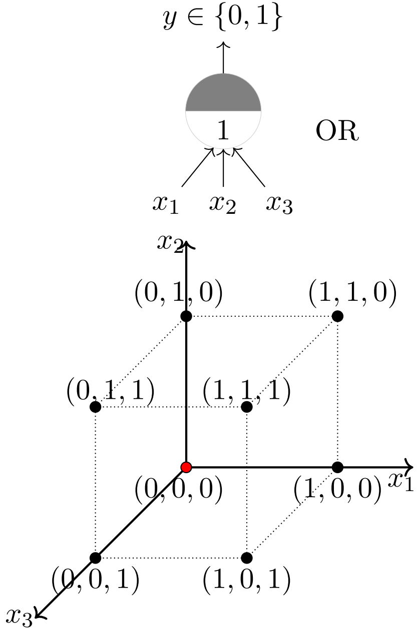
- Well, instead of a line we will have a plane



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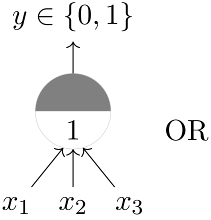
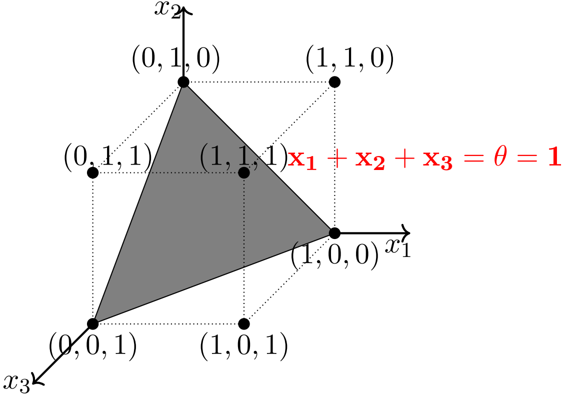
- For the OR function, we want a plane such that the point (0,0,0) lies on one side and the remaining 7 points lie on the other side of the plane





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- Linear separability (for boolean functions) : There exists a line (plane) such that all inputs which produce a 1 lie on one side of the line (plane) and all inputs which produce a 0 lie on other side of the line (plane)