Module 2.5: Perceptron Learning Algorithm

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- Apart from implementing boolean functions (which does not look very interesting) what can a perceptron be used for ?
- Our interest lies in the use of perceptron as a binary classifier. Let us see what this means...

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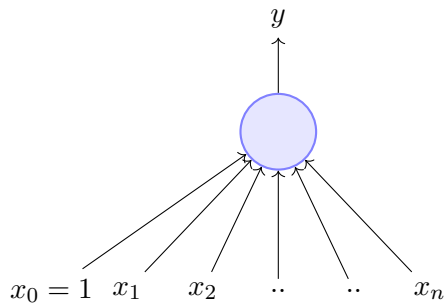
$x_1 = isActorDamon$

$x_2 = isGenreThriller$

$x_3 = isDirectorNolan$

$x_4 = imdbRating(scaled\ to\ 0\ to\ 1)$

...   ...

$x_n = criticsRating(scaled\ to\ 0\ to\ 1)$

y

$x_0 = 1 \quad x_1 \quad x_2 \quad .. \quad .. \quad x_n$

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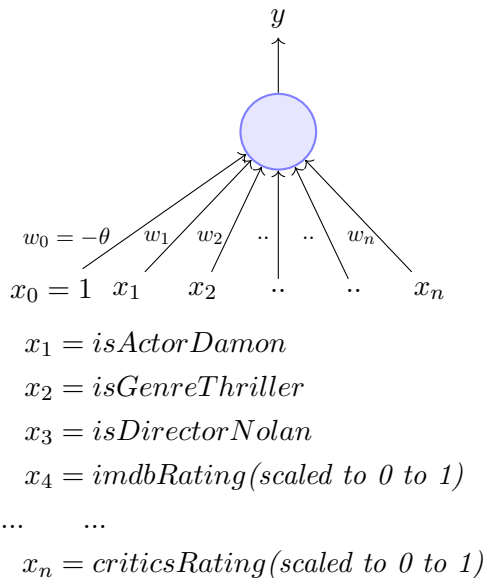
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- In other words, we want the perceptron to find the equation of this separating plane (or find the values of $w_0, w_1, w_2, .., w_m$)

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- To understand why this works we will have to get into a bit of Linear Algebra and a bit of geometry...

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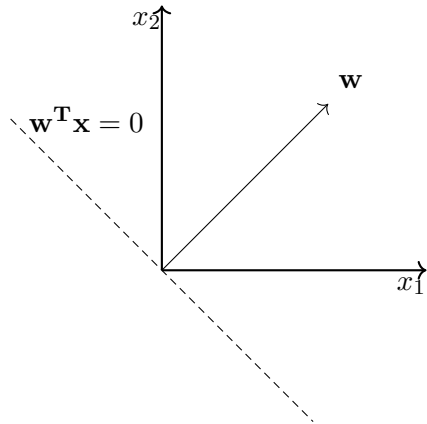$$\mathbf{w} \cdot \mathbf{x} = \mathbf{w^T x} = \sum_{i=0}^{n} w_i * x_i$$

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- Since the vector $\mathbf{w}$ is perpendicular to every point on the line it is actually perpendicular to the line itself

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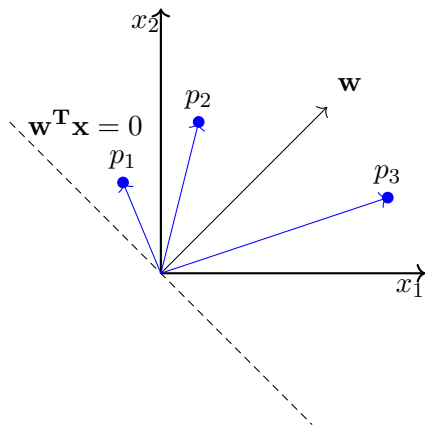
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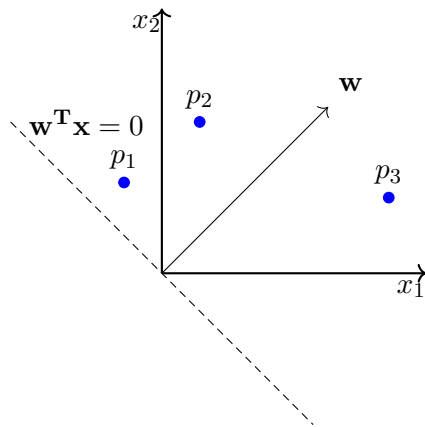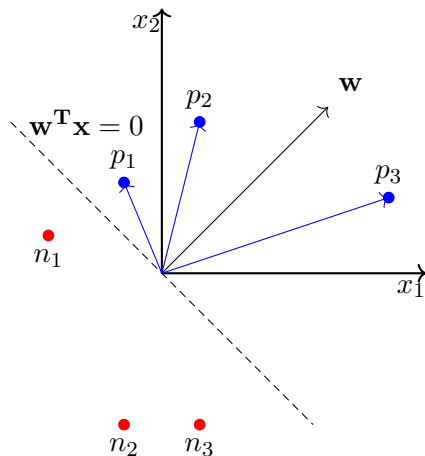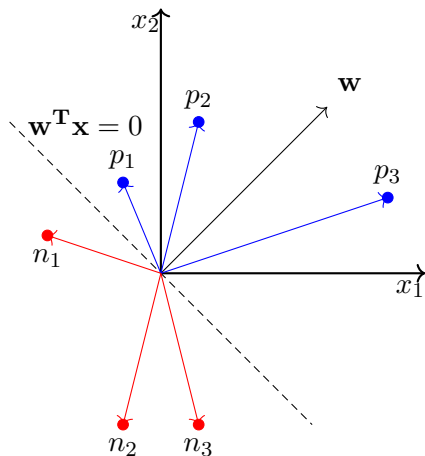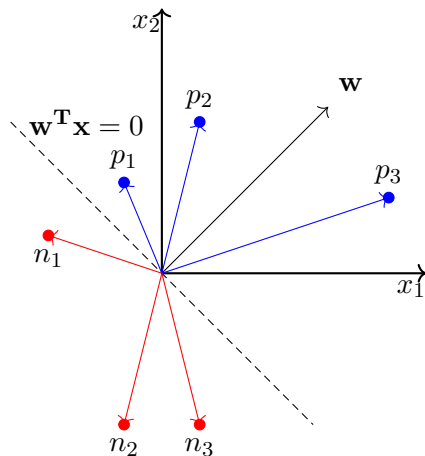
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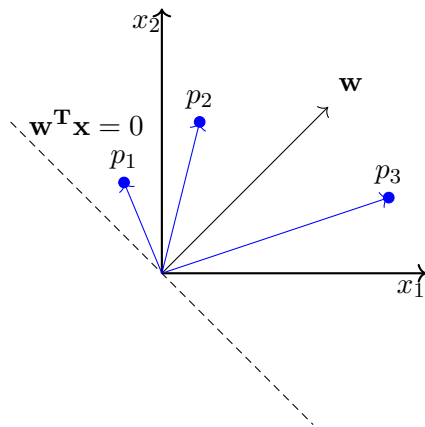
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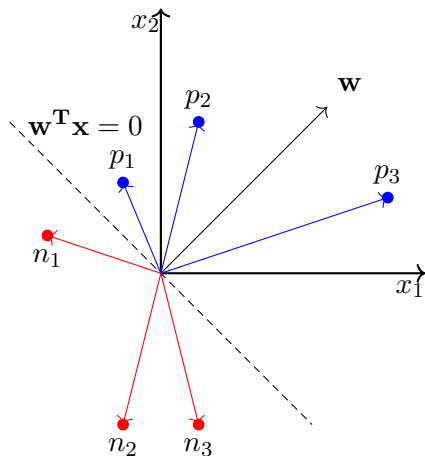- What about points (vectors) which lie in the negative half space of this line (*i.e.*, $\mathbf{w^T x} < 0$)

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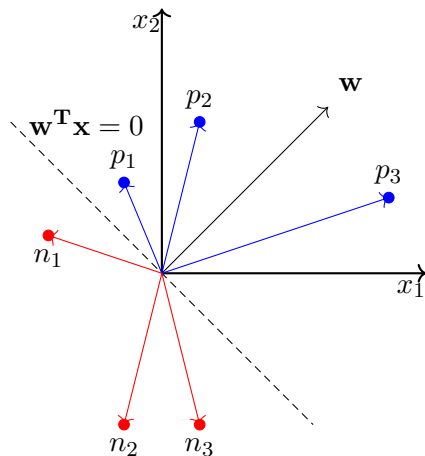- What will be the angle between any such vector and $\mathbf{w}$?

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- Keeping this picture in mind let us revisit the algorithm

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    if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**

        | $\mathbf{w} = \mathbf{w} + \mathbf{x}$;
    end
    if $\mathbf{x} \in N \quad and \quad \mathbf{w}.\mathbf{x} \geq 0$ **then**

        | $\mathbf{w} = \mathbf{w} - \mathbf{x}$;
    end
end

//the algorithm converges when all the
 inputs are classified correctly

$$cos\alpha = \frac{\mathbf{w}^T \mathbf{x}}{||\mathbf{w}||||\mathbf{x}||}$$

- For $\mathbf{x} \in P$ if $\mathbf{w}.\mathbf{x} < 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is greater than $90°$ (but we want α to be less than $90°$)

- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} + \mathbf{x}$

$$cos(\alpha_{new}) \propto \mathbf{w_{new}}^T \mathbf{x}$$
$$\propto (\mathbf{w} + \mathbf{x})^T \mathbf{x}$$
$$\propto \mathbf{w}^T \mathbf{x} + \mathbf{x}^T \mathbf{x}$$

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1;$
$N \leftarrow inputs \quad with \quad label \quad 0;$
Initialize **w** randomly;
while !*convergence* **do**

    Pick random $\mathbf{x} \in P \cup N$;
    if $\mathbf{x} \in P \quad and \quad \mathbf{w}.\mathbf{x} < 0$ **then**
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Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 2

Algorithm: Perceptron Learning Algorithm

$P \leftarrow inputs \quad with \quad label \quad 1$;
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Initialize **w** randomly;
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$$\propto cos\alpha + \mathbf{x}^T\mathbf{x}$$
$$cos(\alpha_{new}) > cos\alpha$$

Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 2

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$$\propto \mathbf{w}^T\mathbf{x} + \mathbf{x}^T\mathbf{x}$$
$$\propto cos\alpha + \mathbf{x}^T\mathbf{x}$$

$$cos(\alpha_{new}) > cos\alpha$$

- Thus α_{new} will be less than α and this is exactly what we want

Algorithm: Perceptron Learning Algorithm

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- For $\mathbf{x} \in N$ if $\mathbf{w}.\mathbf{x} \geq 0$ then it means that the angle (α) between this $\mathbf{x}$ and the current $\mathbf{w}$ is less than 90°

Algorithm: Perceptron Learning Algorithm

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Algorithm: Perceptron Learning Algorithm

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- What happens to the new angle (α_{new}) when $\mathbf{w_{new}} = \mathbf{w} - \mathbf{x}$

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Algorithm: Perceptron Learning Algorithm

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$$\propto cos\alpha - \mathbf{x}^T \mathbf{x}$$
$$cos(\alpha_{new}) < cos\alpha$$

Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 2

Algorithm: Perceptron Learning Algorithm

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$$cos(\alpha_{new}) < cos\alpha$$

- Thus α_{new} will be greater than α and this is exactly what we want

- We will now see this algorithm in action for a toy dataset

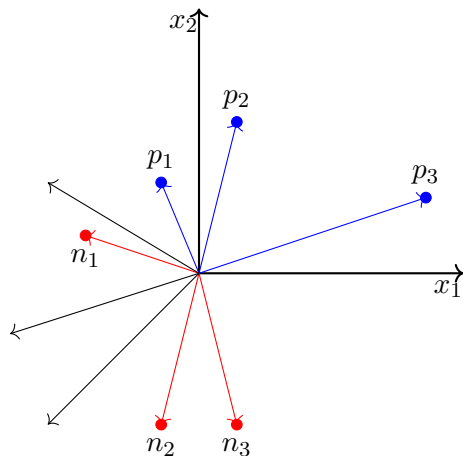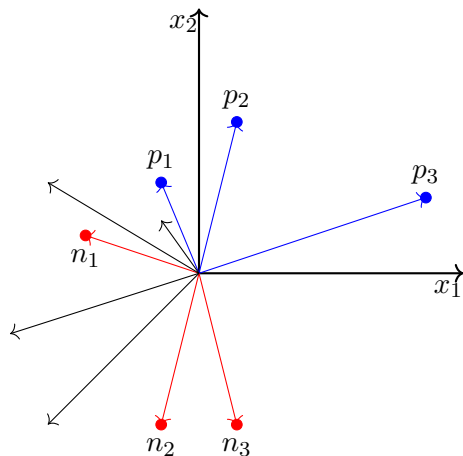- We initialized **w** to a random value
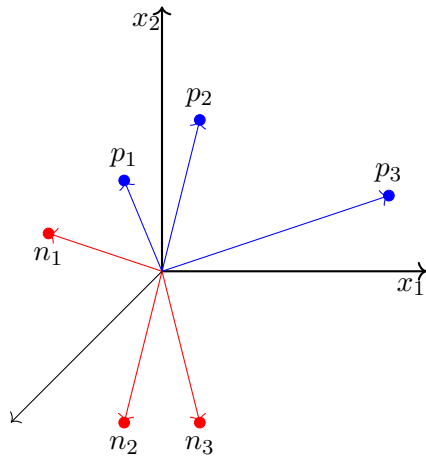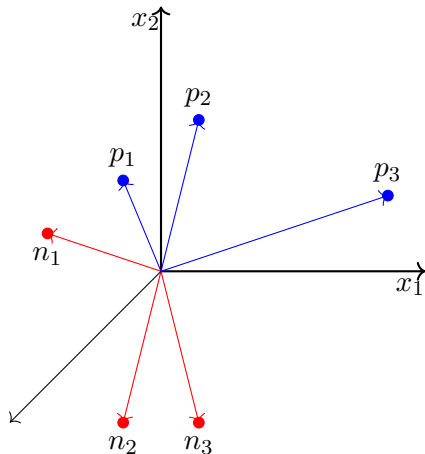
- We initialized $\mathbf{w}$ to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
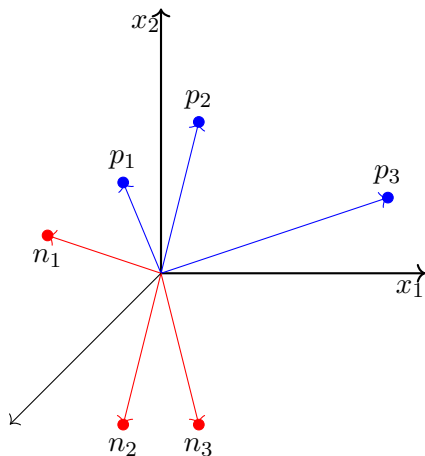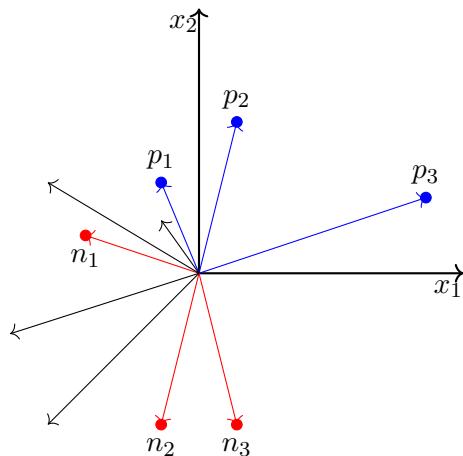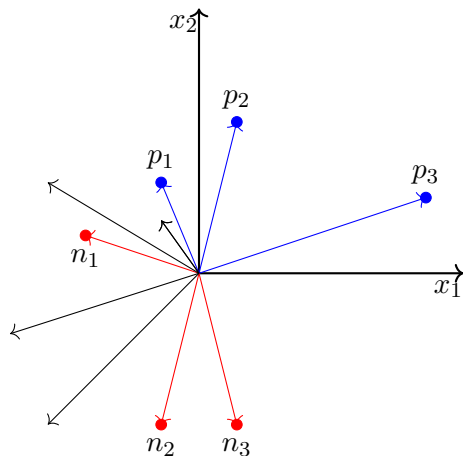
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- We now run the algorithm by randomly going over the points

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- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_1), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)

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Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 2
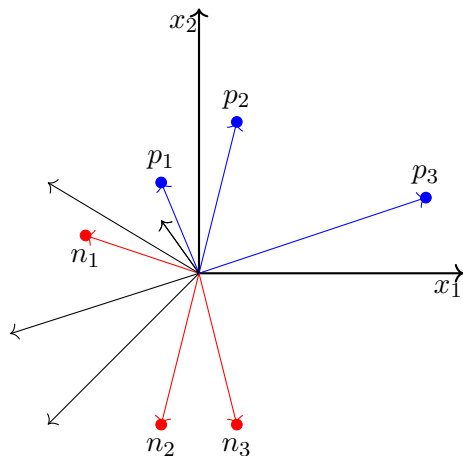
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- Randomly pick a point (say, p_2), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)
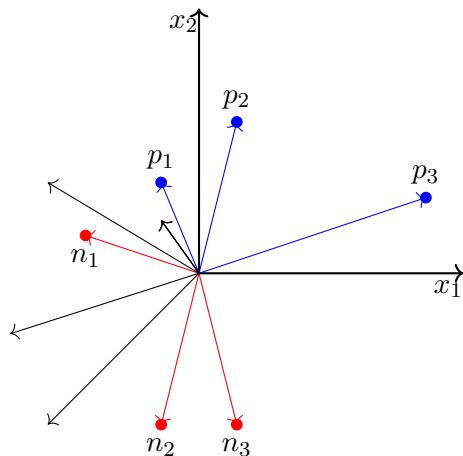
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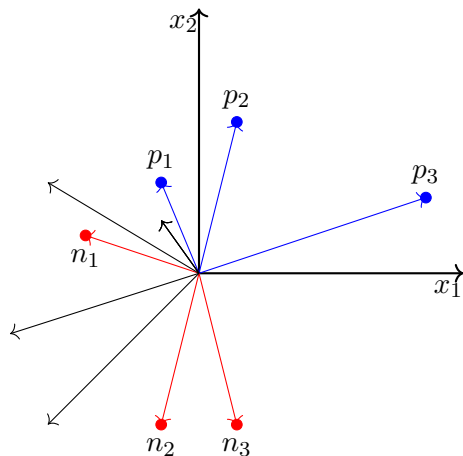
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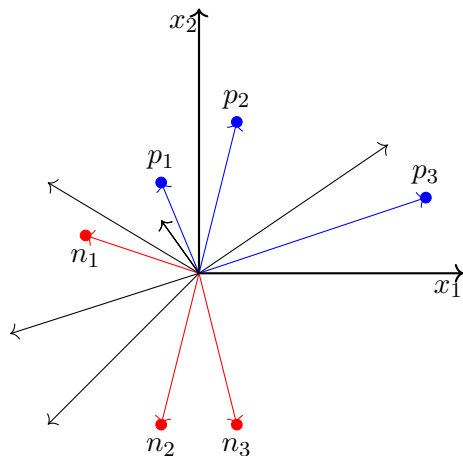
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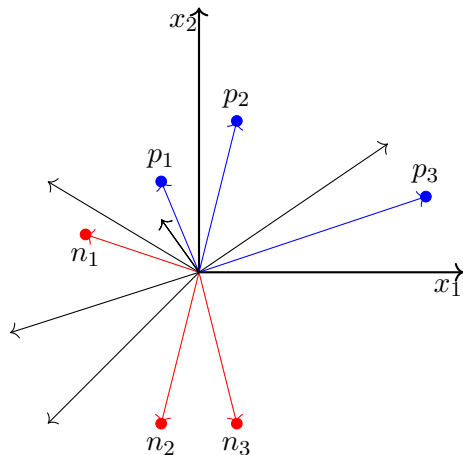
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- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_3), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)
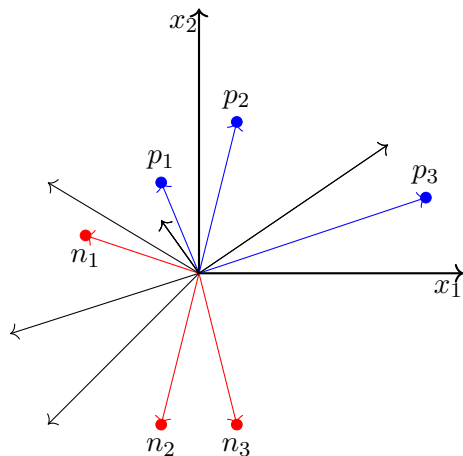
- We initialized **w** to a random value
- We observe that currently, $\mathbf{w} \cdot \mathbf{x} < 0$ ($\because$ angle $> 90°$) for all the positive points and $\mathbf{w} \cdot \mathbf{x} \geq 0$ ($\because$ angle $< 90°$) for all the negative points (the situation is exactly oppsite of what we actually want it to be)
- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, n_2), no correction needed $\because \mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)
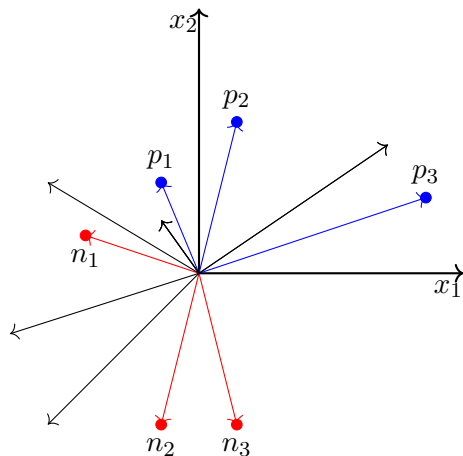
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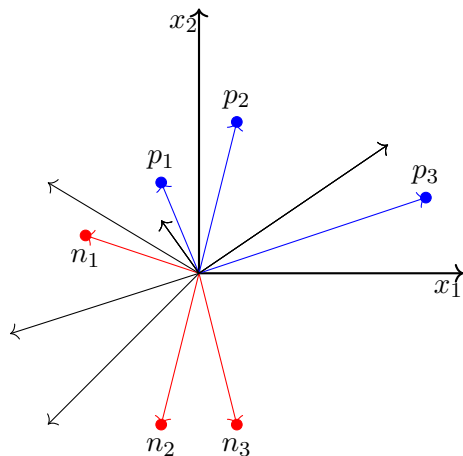
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- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_3), apply correction $\mathbf{w} = \mathbf{w} + \mathbf{x}$ $\because$ $\mathbf{w} \cdot \mathbf{x} < \mathbf{0}$ (you can check the angle visually)
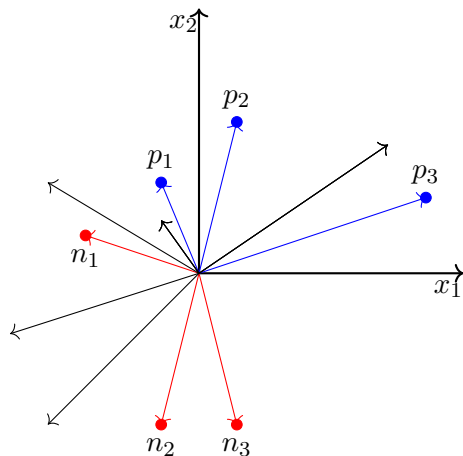
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- We now run the algorithm by randomly going over the points
- Randomly pick a point (say, p_1), no correction needed $\because$ $\mathbf{w} \cdot \mathbf{x} \geq \mathbf{0}$ (you can check the angle visually)
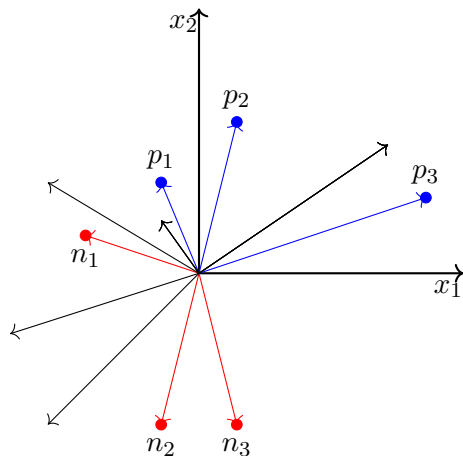
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Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 2

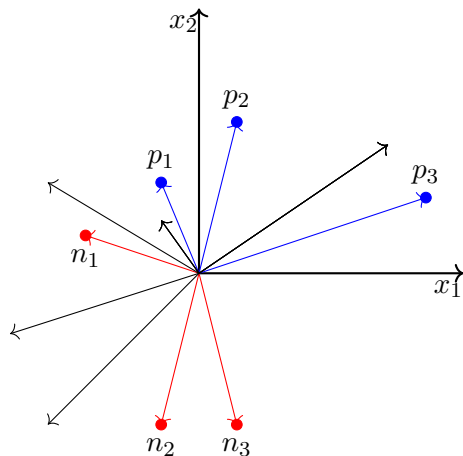- We initialized $\mathbf{w}$ to a random value
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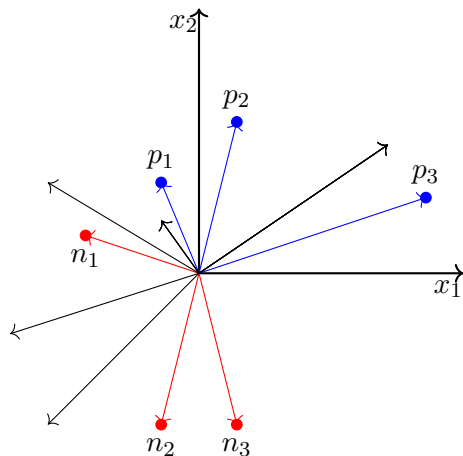
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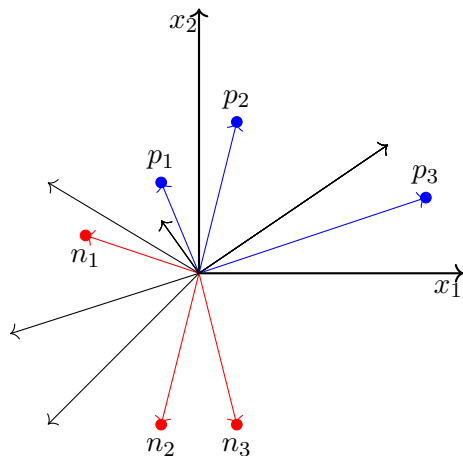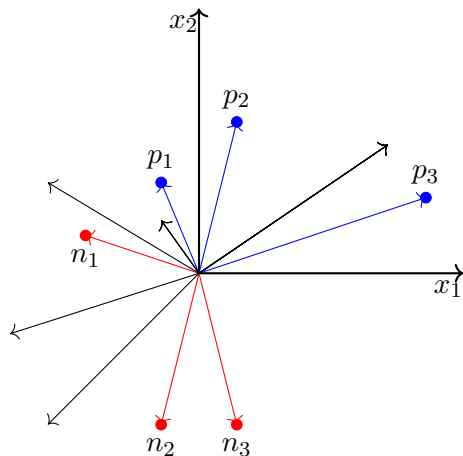
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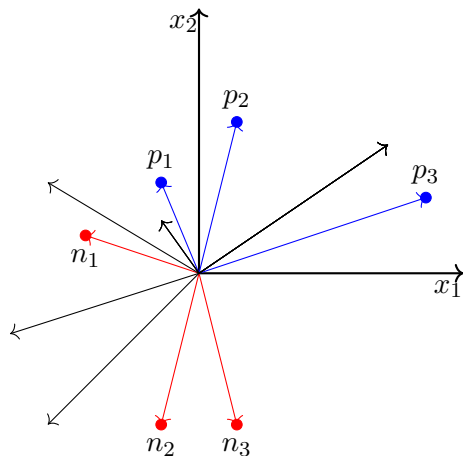
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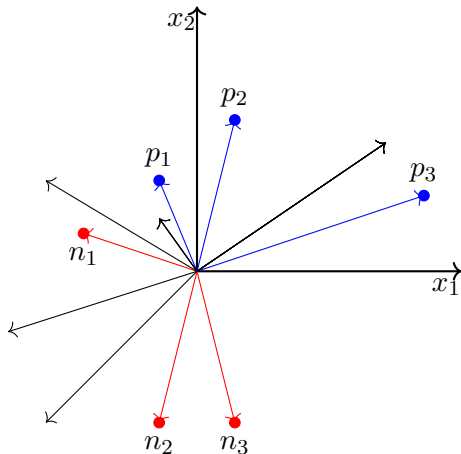
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- We now run the algorithm by randomly going over the points
- The algorithm has converged