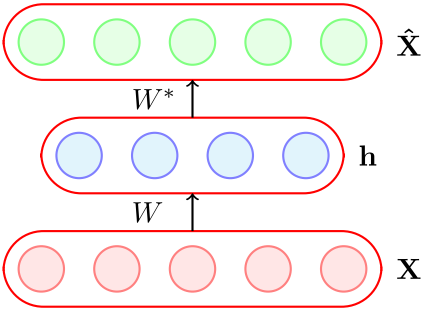
Module 21.1: Revisiting Autoencoders



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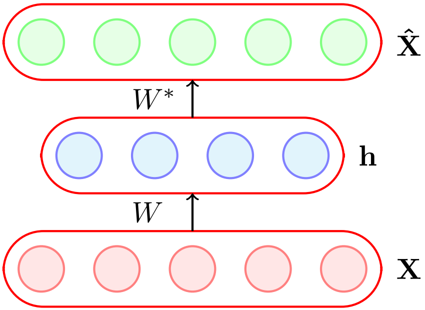

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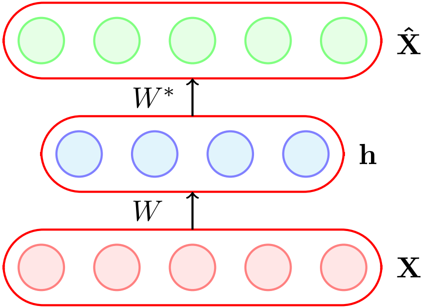

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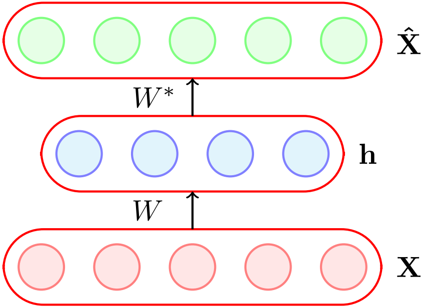
- The decoder then takes this hidden representation and tries to reconstruct the input from it as $\hat{X}$



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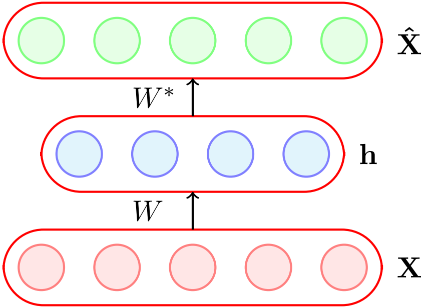

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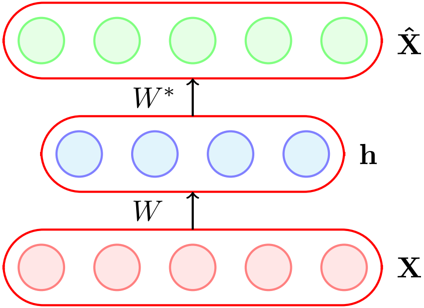
- where m is the number of training instances, $\{x_i\}_{i=1}^{m}$ and each $x_i \in R^n$ (x_{ij} is thus the j-th dimension of the i-th training instance)



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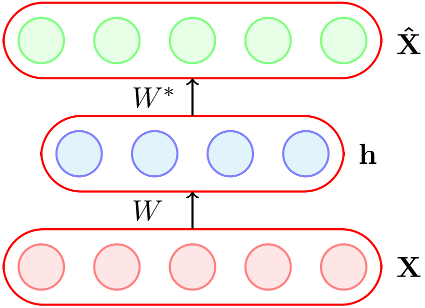

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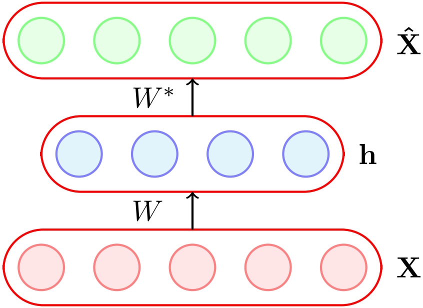

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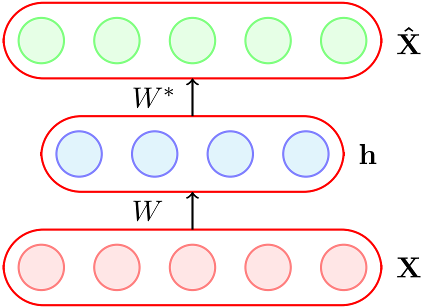

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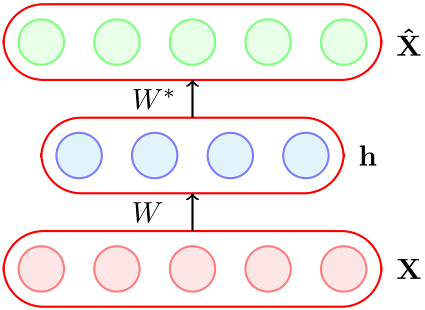

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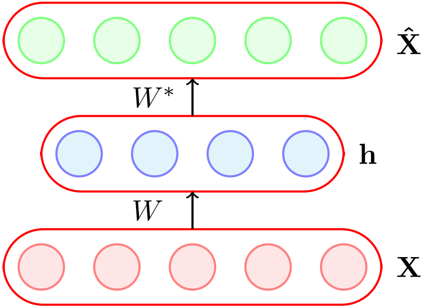

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- Of course, the fun lies in the fact that we are getting a good *abstraction* of the input
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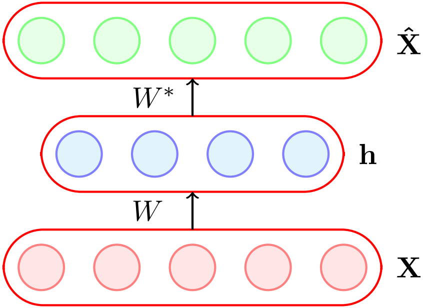
- Let us revisit *generation* in the context of autoencoders



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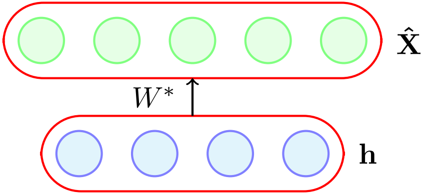

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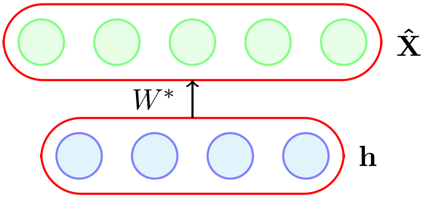
- In other words, once the autoencoder is trained can I remove the encoder, feed a hidden representation h to the decoder and decode a $\hat{X}$ from it ?



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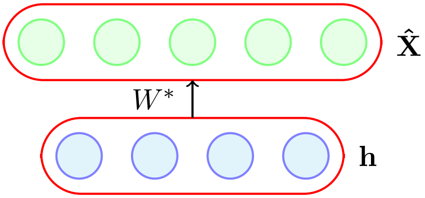

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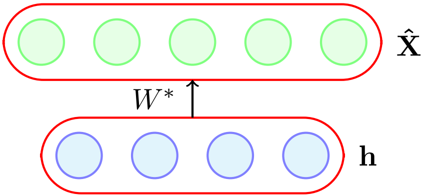
- h is a very high dimensional vector and only a few vectors in this space would actually correspond to meaningful latent representations of our input



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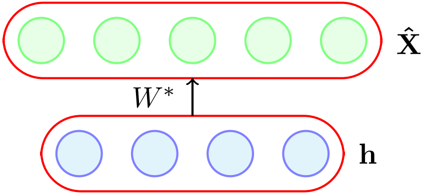
- In principle, yes! But in practice there is a problem with this approach
- h is a very high dimensional vector and only a few vectors in this space would actually correspond to meaningful latent representations of our input
- So of all the possible value of h which values should I feed to the decoder (we had asked a similar question before: slide 67, bullet 5 of lecture 19)



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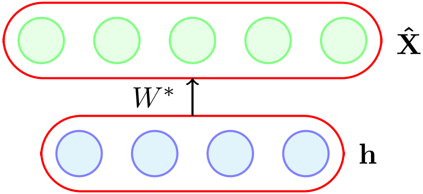

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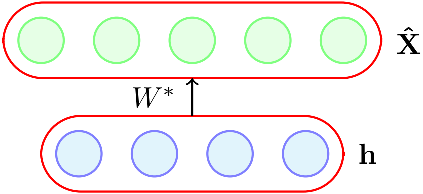

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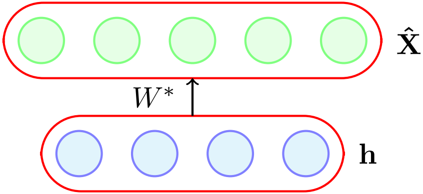

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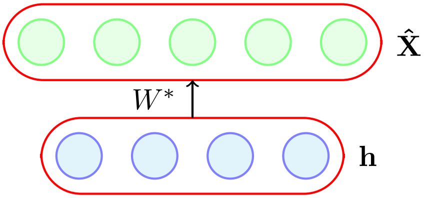
- They learn a hidden representation h but not a distribution $P(h|X)$



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- They learn a hidden representation h but not a distribution $P(h|X)$
- Similarly the decoder is also deterministic and does not learn a distribution over X (given a h we can get a X but not $P(X|h)$)

We will now look at variational autoencoders which have the same structure as autoencoders but they learn a distribution over the hidden variables