Module 3.4: Learning Parameters : Gradient Descent

Now let us see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

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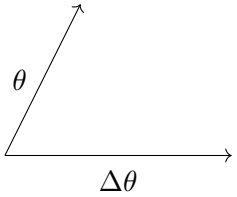

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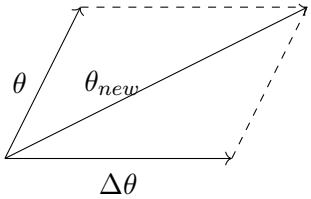

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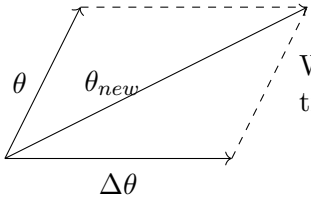

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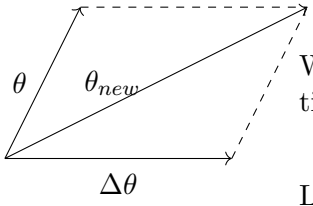

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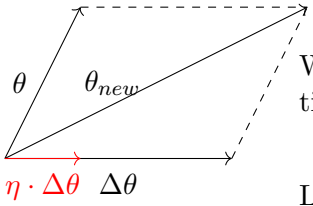

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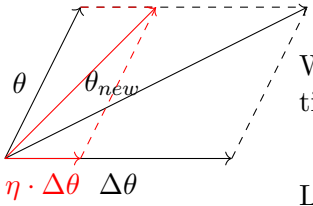

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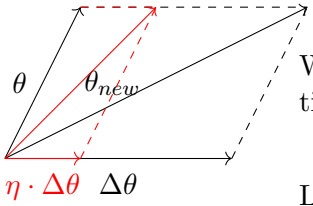
$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$



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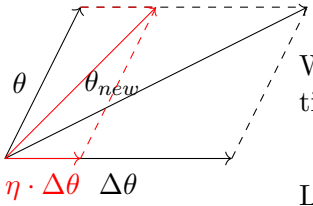

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Question: What is the right $\Delta\theta$ to use ?

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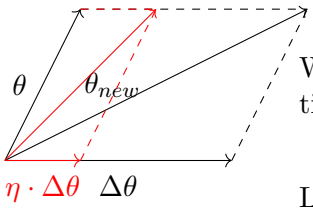

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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Note that the move (ηu) would be favorable only if,

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This implies,

$$u^T \nabla \mathscr{L}(\theta) < 0$$

Okay, so we have,

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Thus, $\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) = u^T \nabla \mathscr{L}(\theta) = k * cos(\beta)$ will be most negative when $cos(\beta) = -1$ *i.e.*, when β is 180°

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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$$w_{t+1} = w_t - \eta \nabla w_t$$

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So we now have a more principled way of moving in the w-b plane than our "guess work" algorithm

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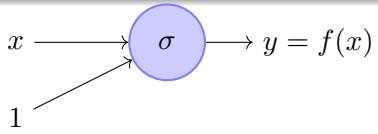
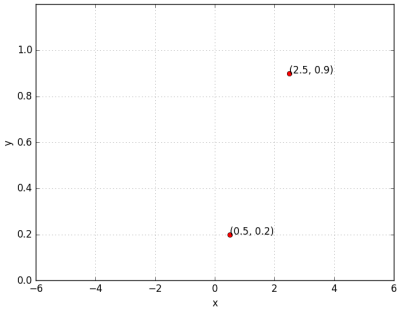
Algorithm: gradient_descent()

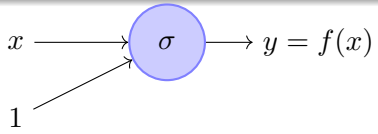
$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
while $t < max\_iterations$ **do**
    $w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
    $b_{t+1} \leftarrow b_t - \eta \nabla b_t$;
end

- To see this algorithm in practice lest first derive ∇w and ∇b for our toy neural network



$$f(x) = \frac{1}{1+e^{-(w \cdot x+b)}}$$





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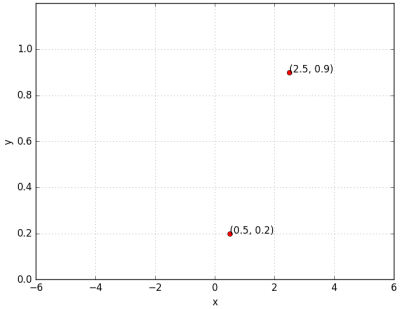

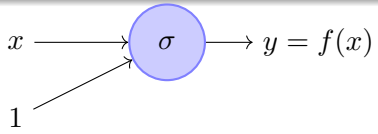

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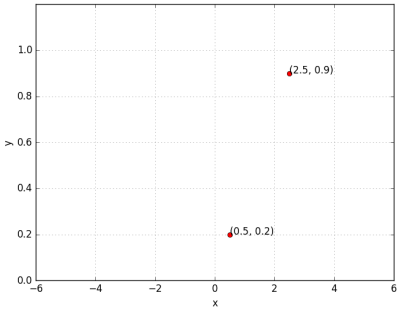

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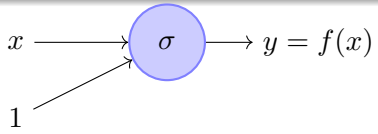

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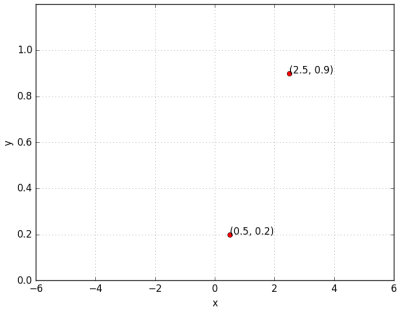

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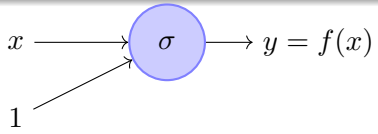
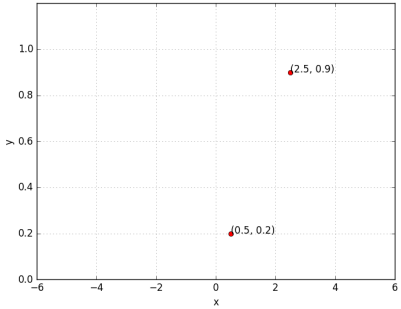
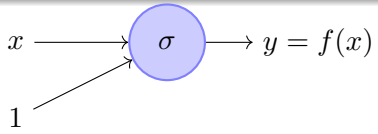


$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y), we have,





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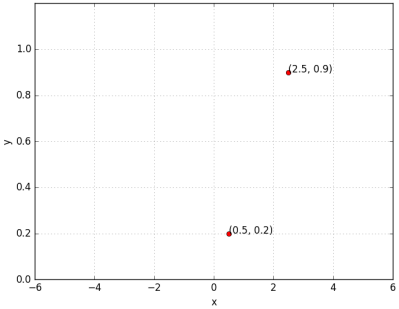

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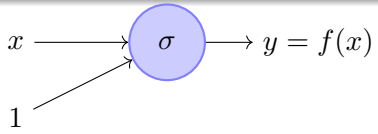
$$\nabla w = (f(x) - y) * f(x) * (1 - f(x)) * x$$



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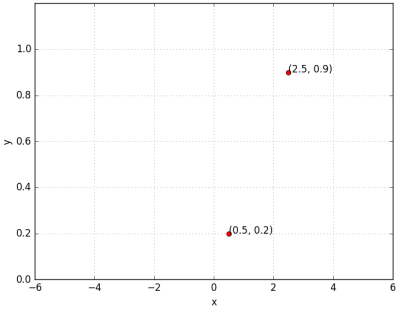

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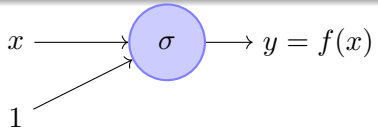

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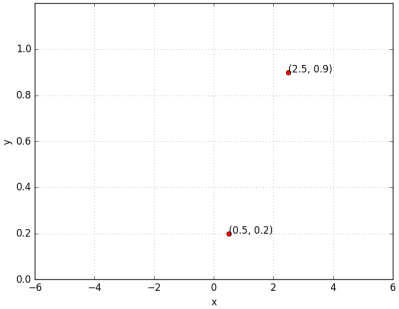

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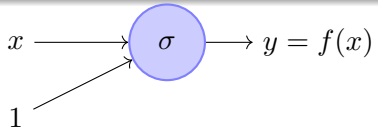

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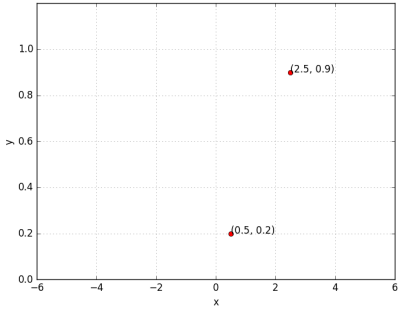

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X = [0.5, 2.5]
Y = [0.2, 0.9]
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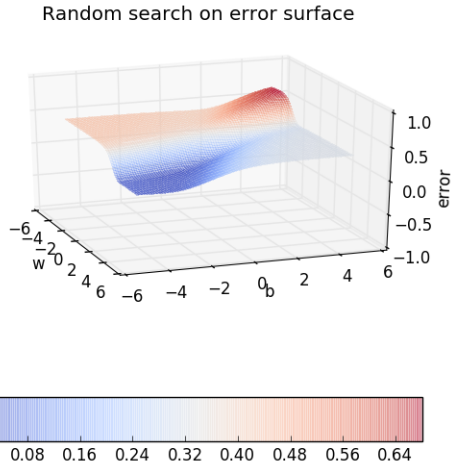

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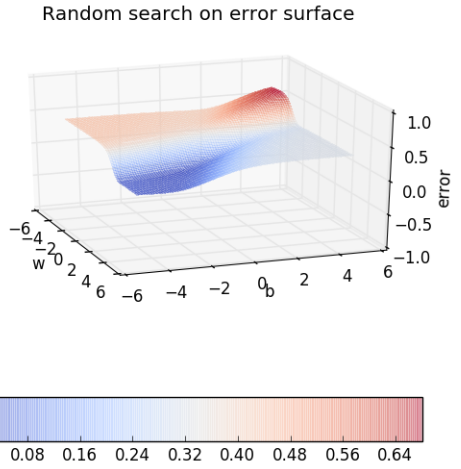

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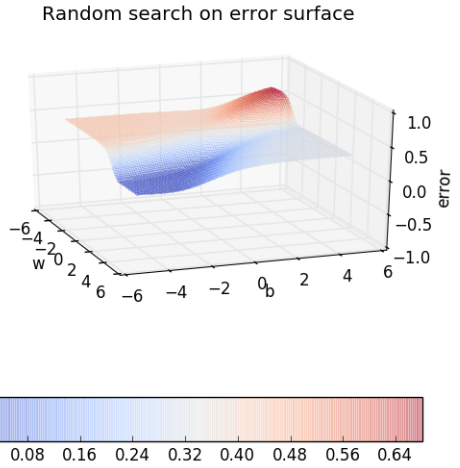

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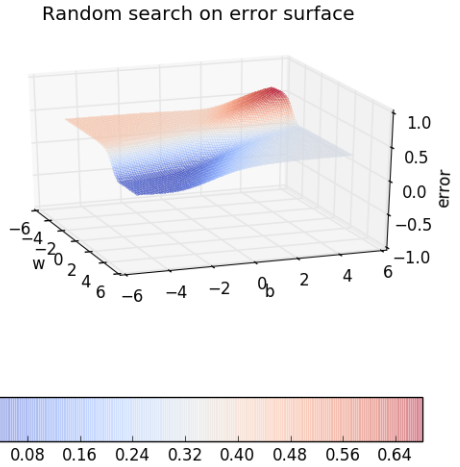

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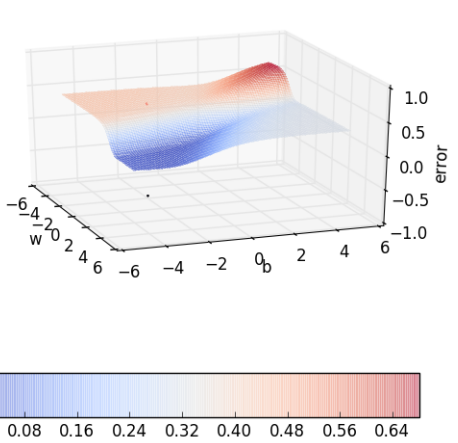

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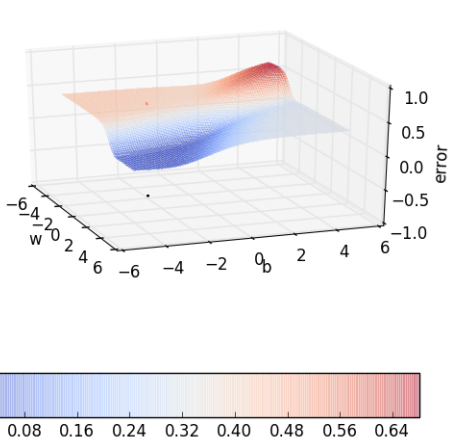

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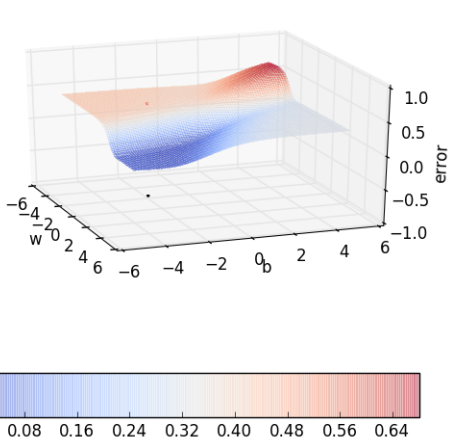

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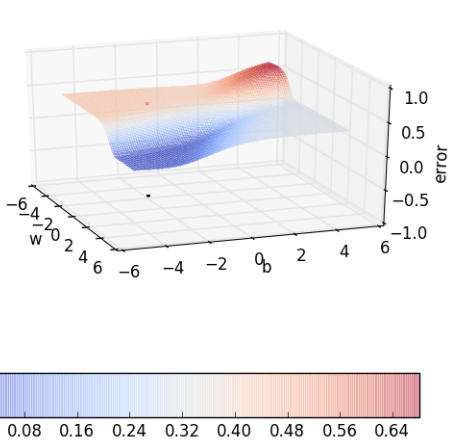

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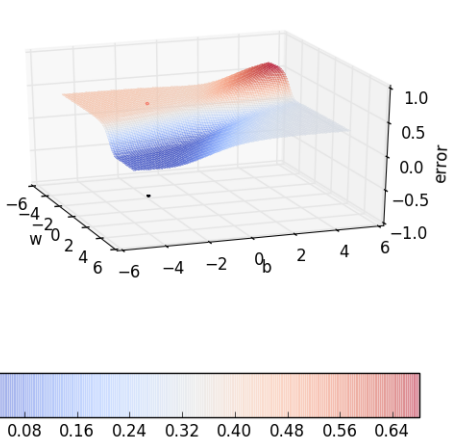

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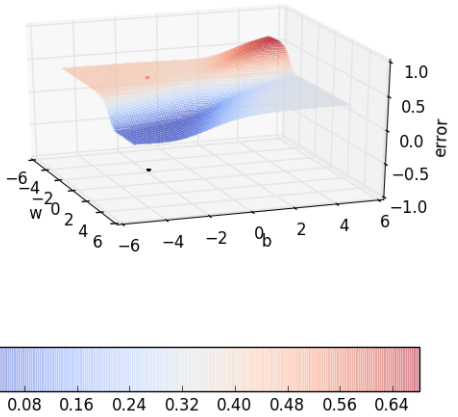
Gradient descent on the error surface



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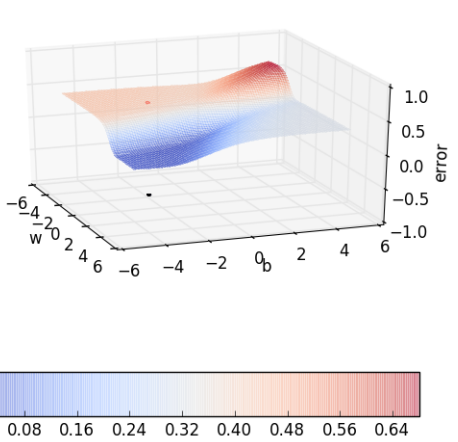

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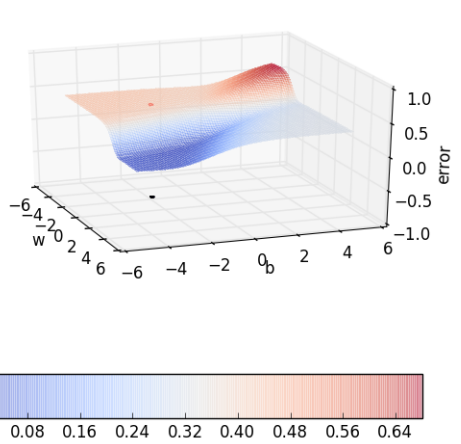

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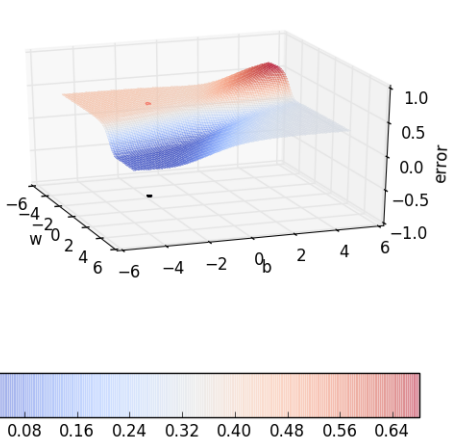

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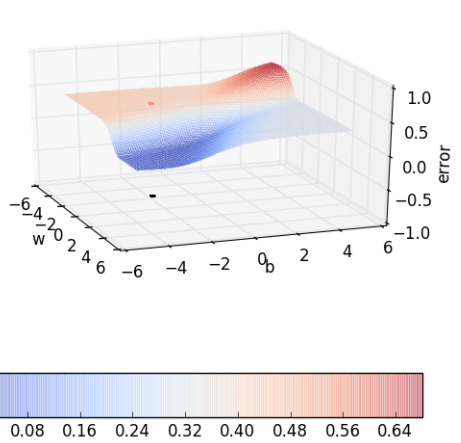

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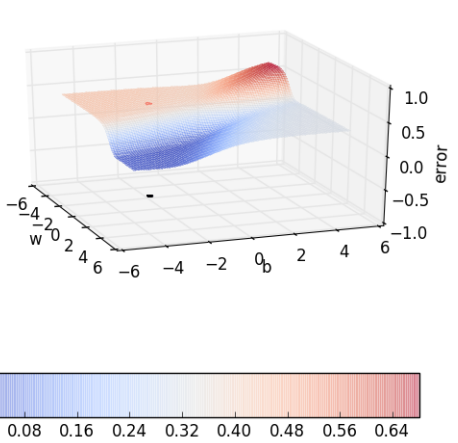

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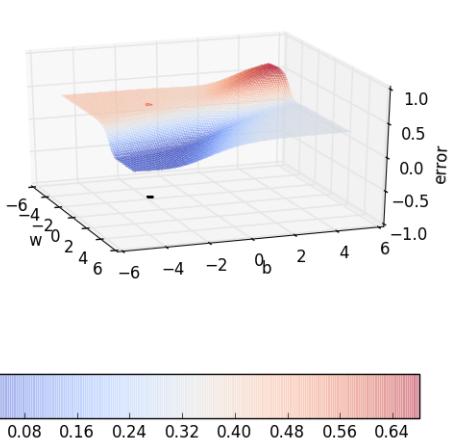

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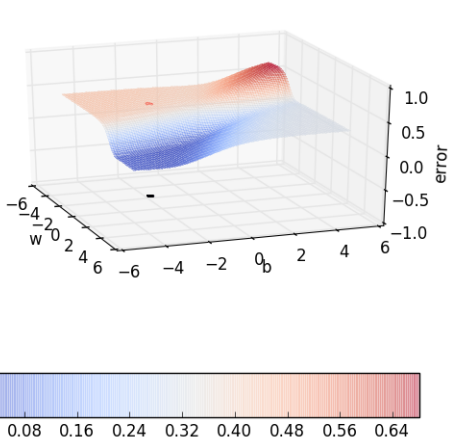

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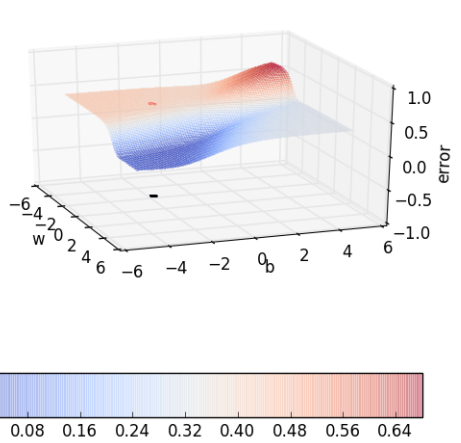

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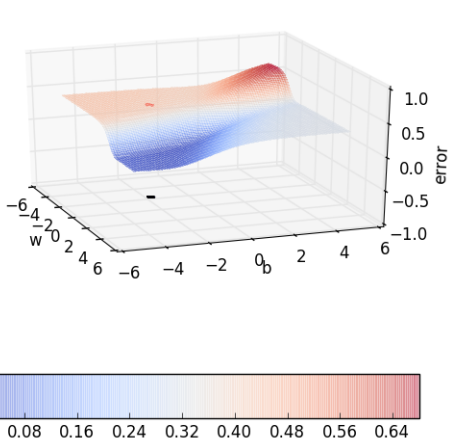

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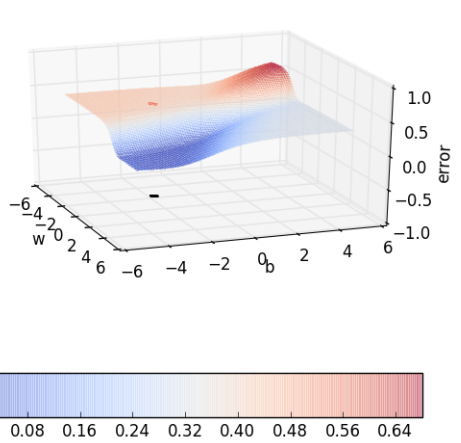

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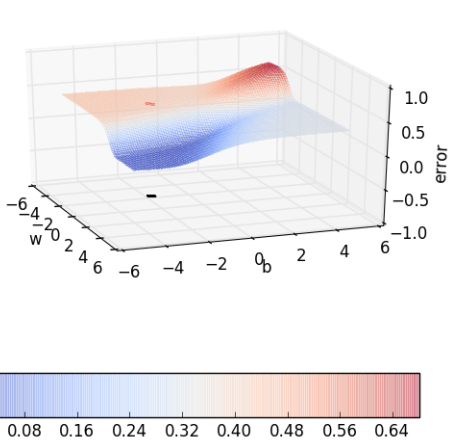

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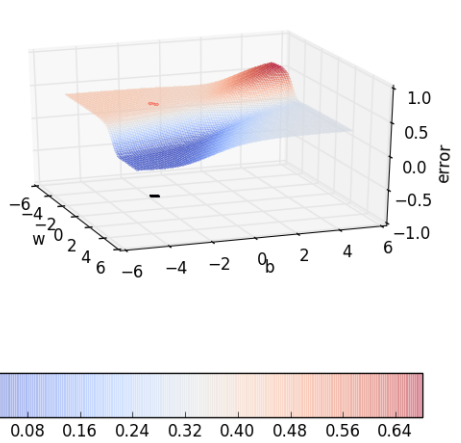

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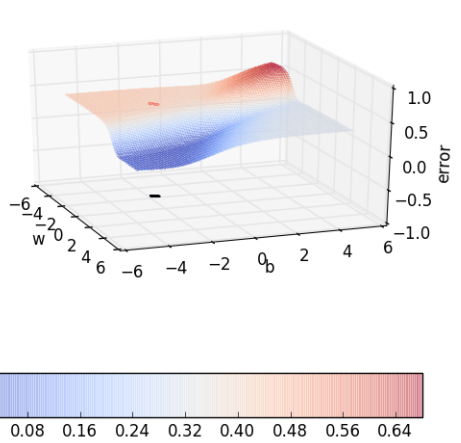

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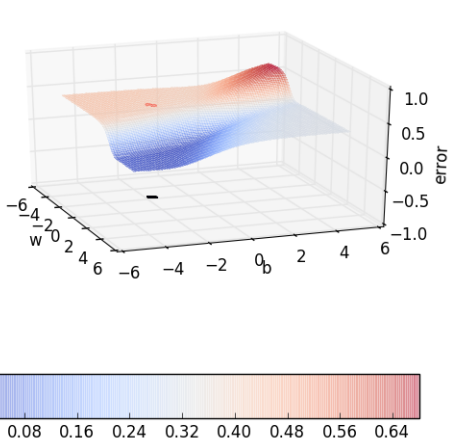

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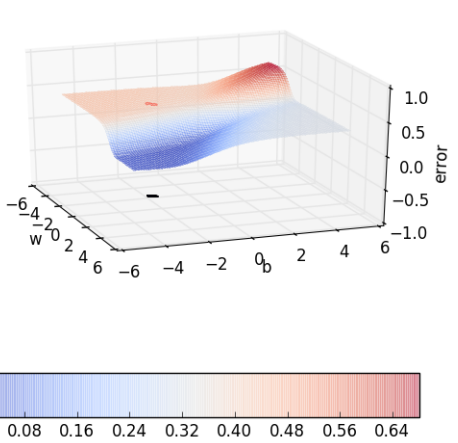

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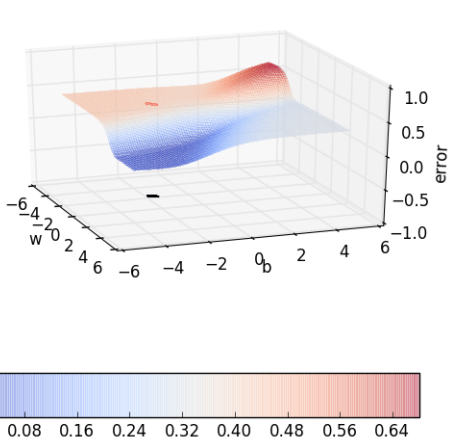

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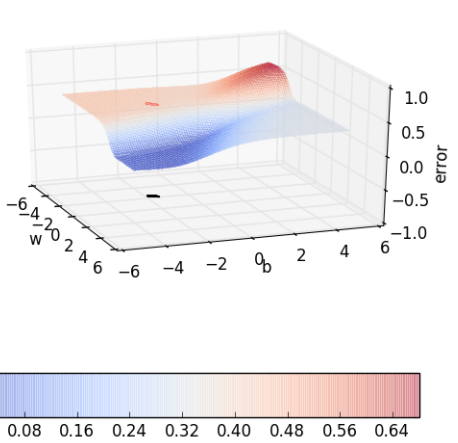

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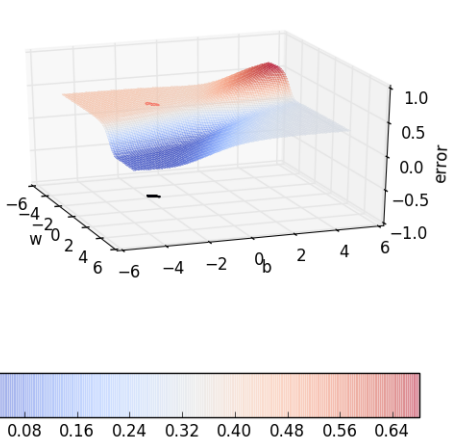

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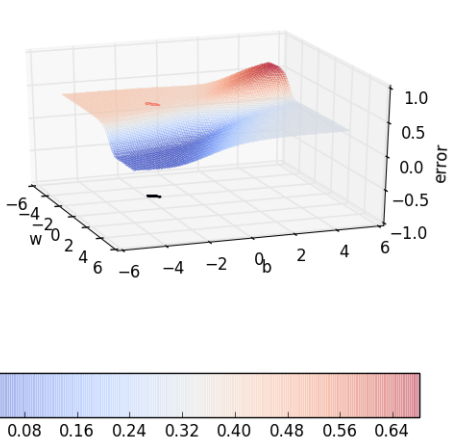

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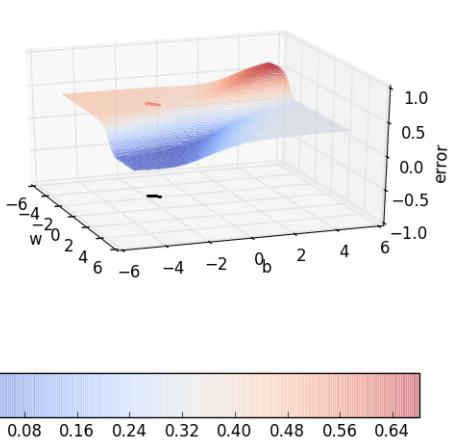

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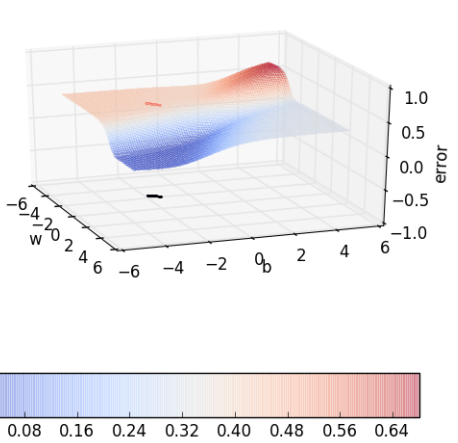

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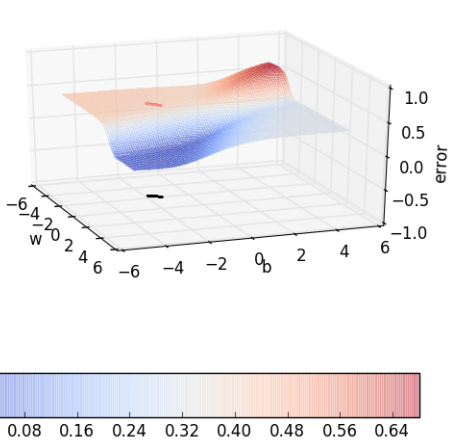

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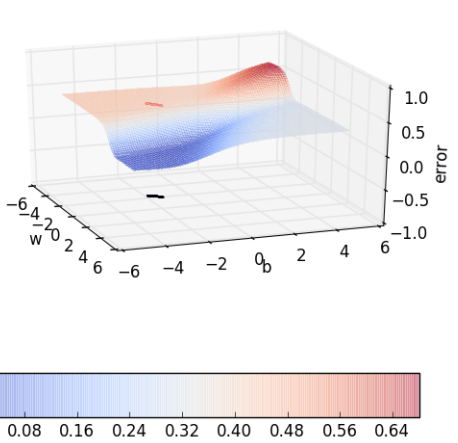

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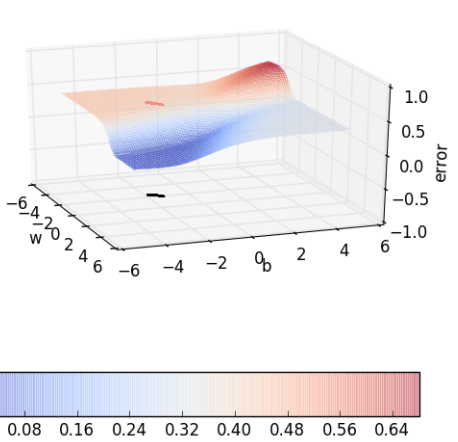

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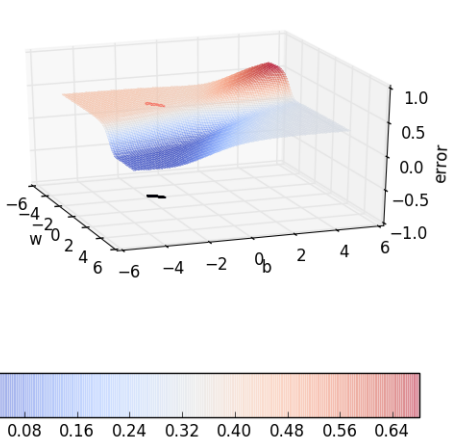

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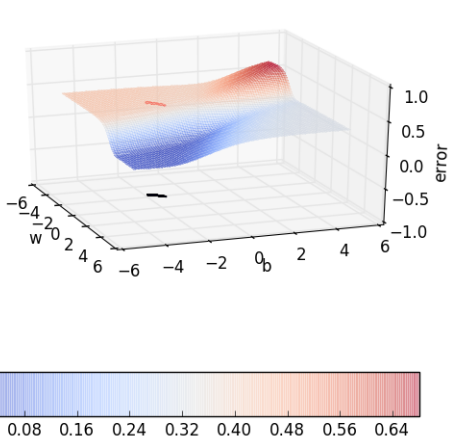

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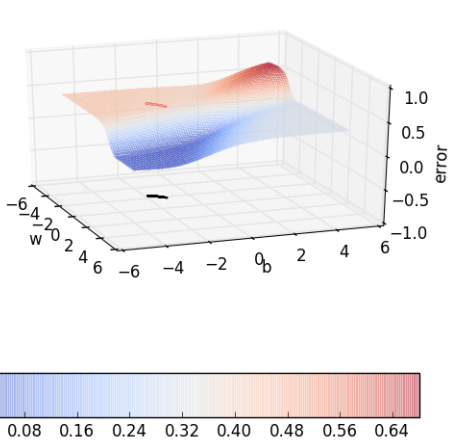

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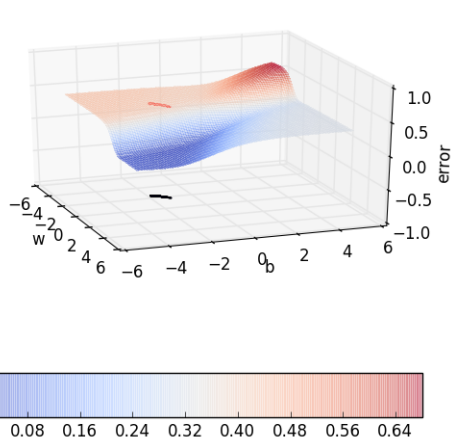

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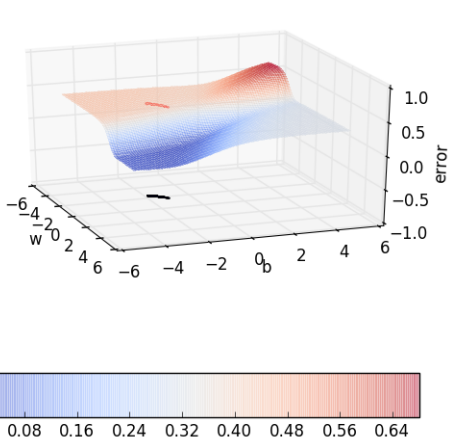

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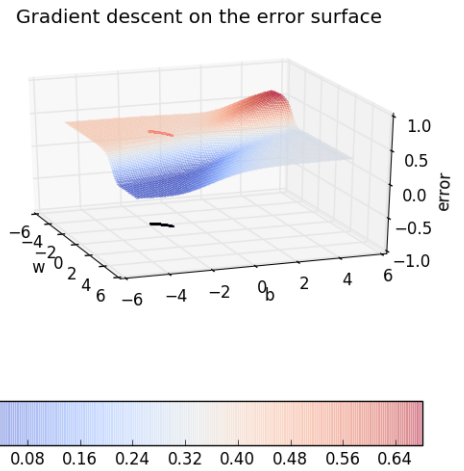

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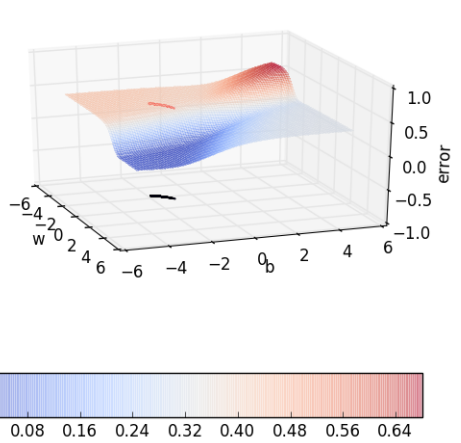

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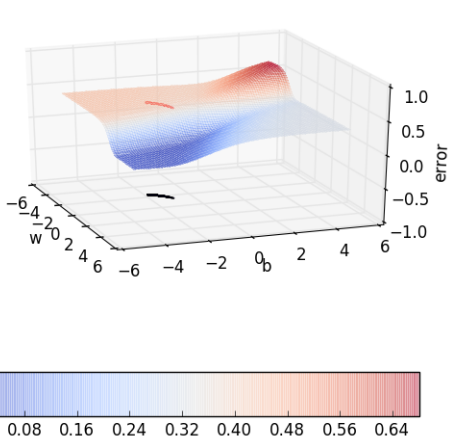

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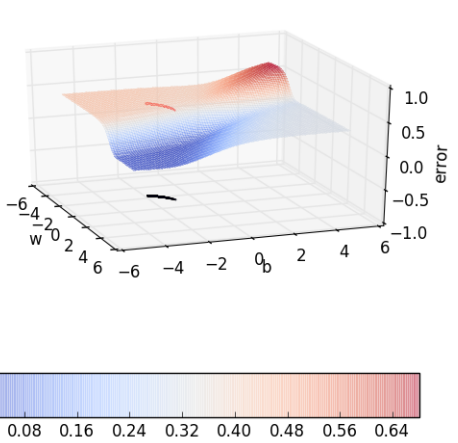

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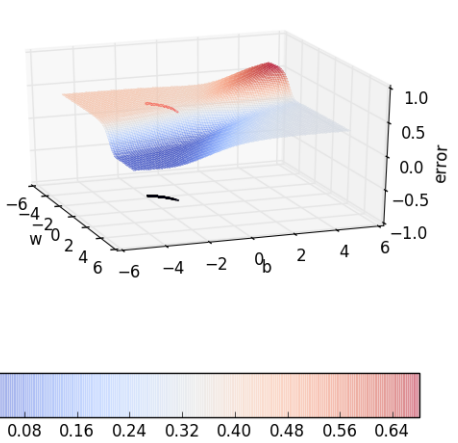

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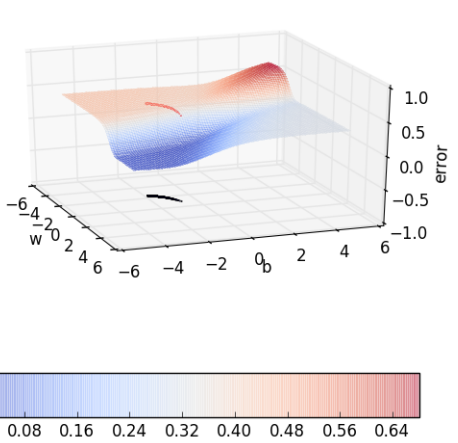

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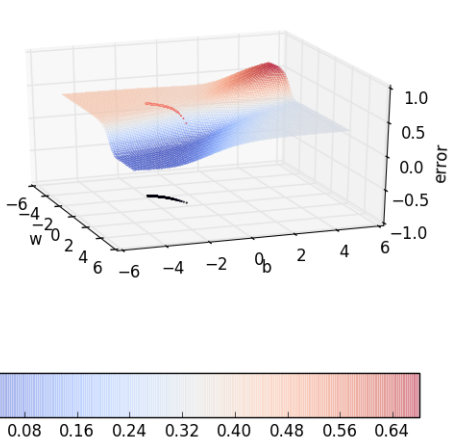

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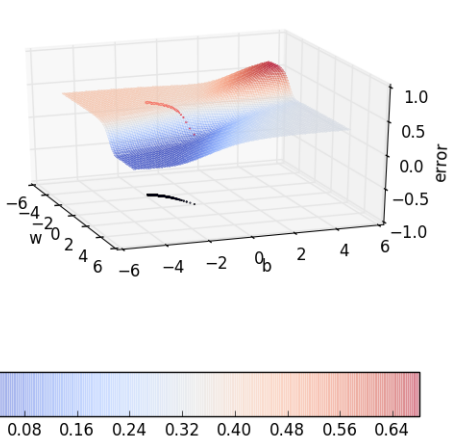

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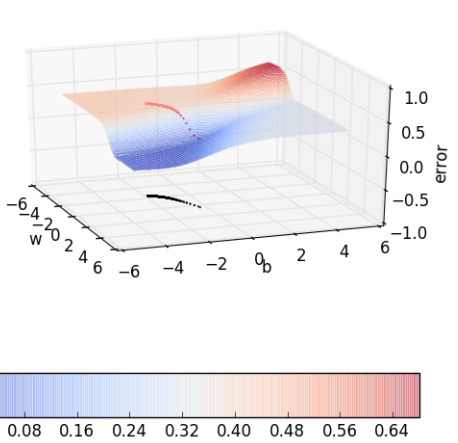

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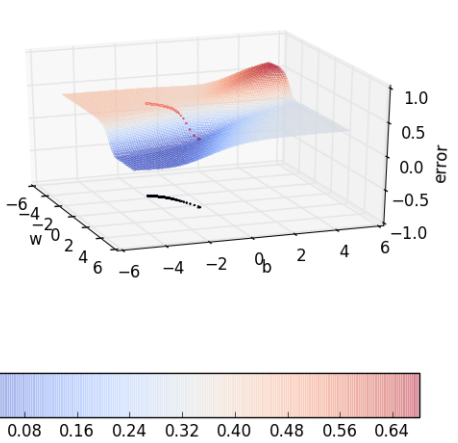

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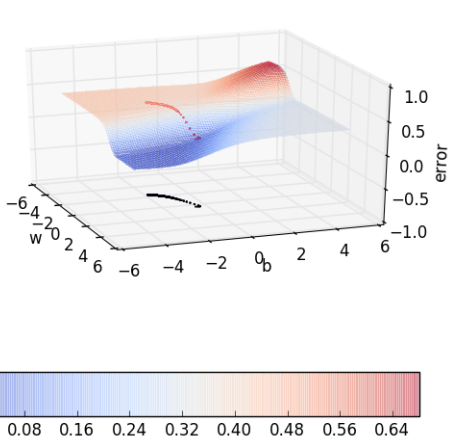

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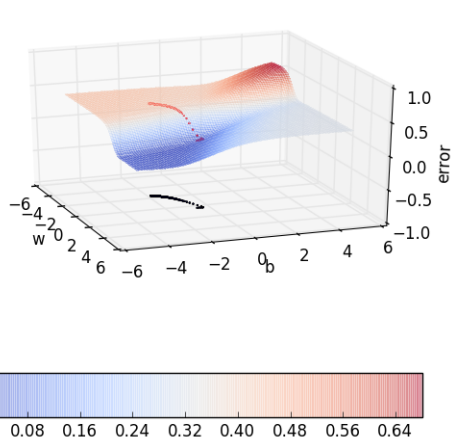

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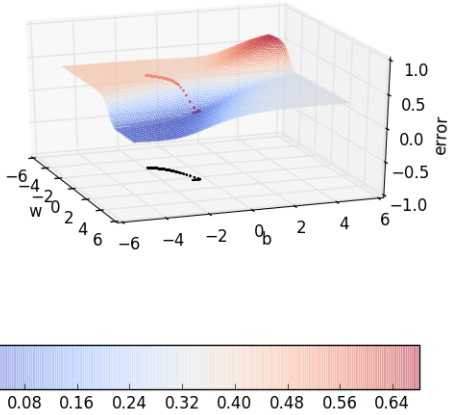
Gradient descent on the error surface



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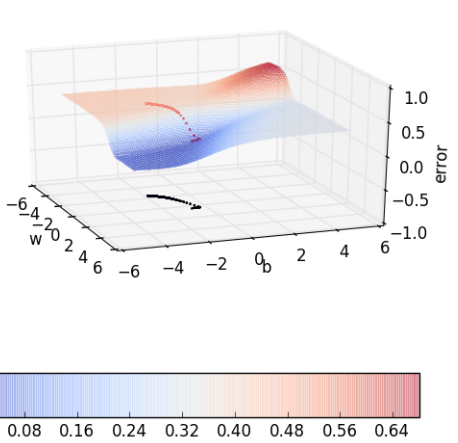

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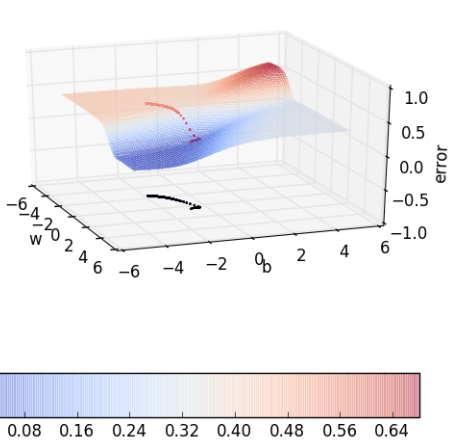

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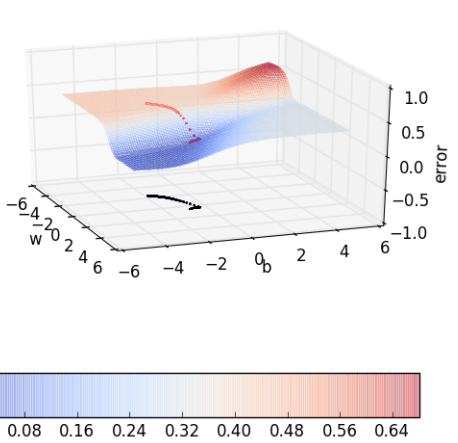

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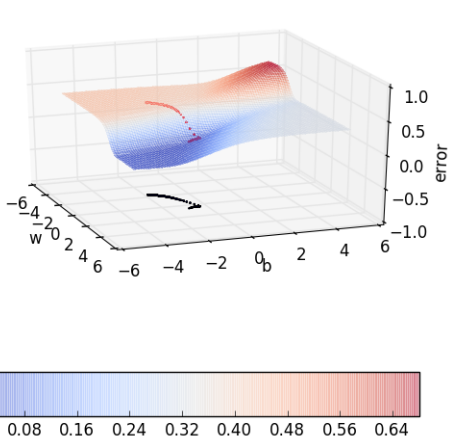

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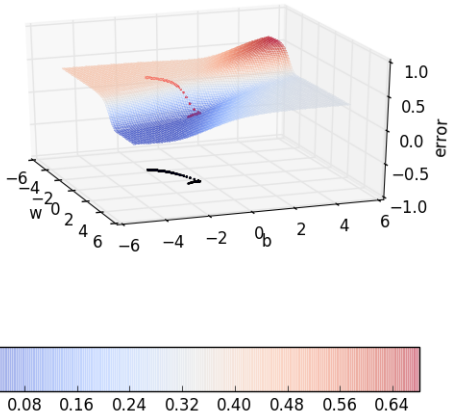
Gradient descent on the error surface



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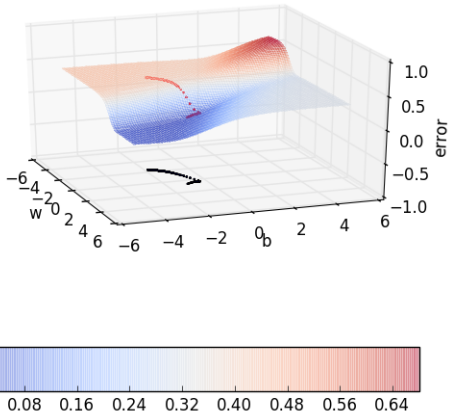

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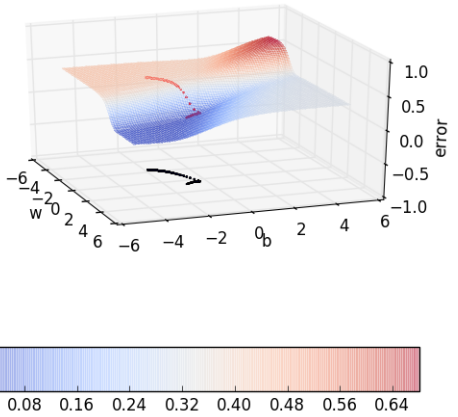

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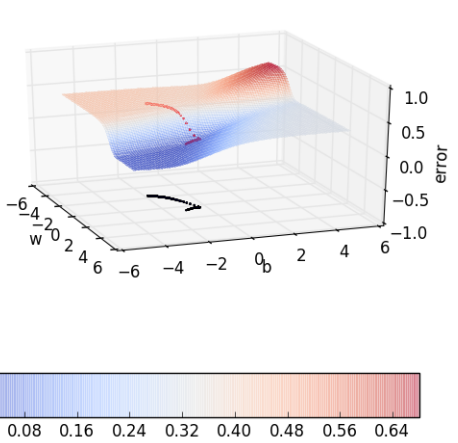

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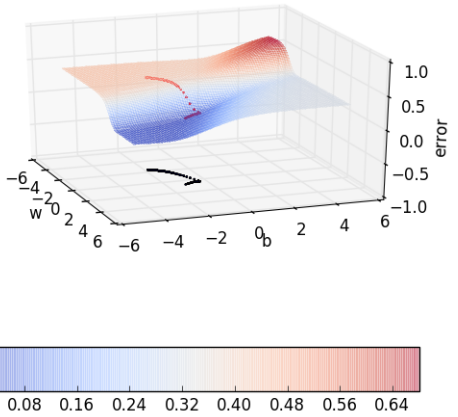

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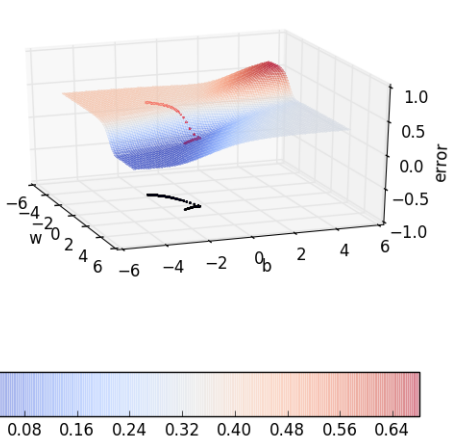

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def error (w, b) :
    err = 0.0
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        err += 0.5 * (fx - y) ** 2
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    w, b, eta, max_epochs = -2, -2, 1.0, 1000
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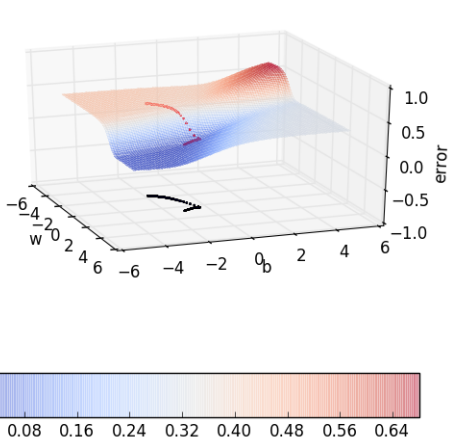
```



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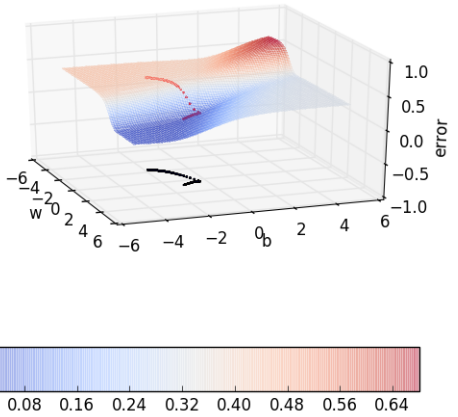
Gradient descent on the error surface



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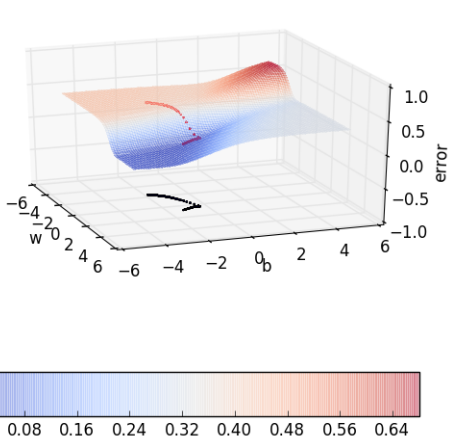

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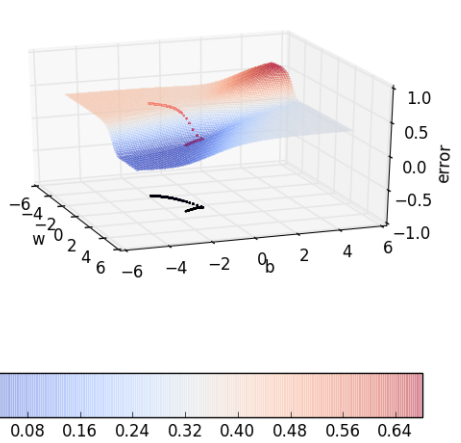

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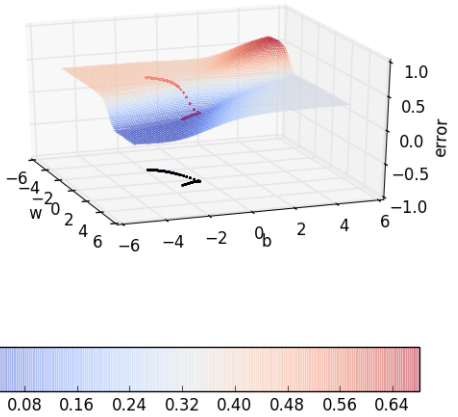
Gradient descent on the error surface



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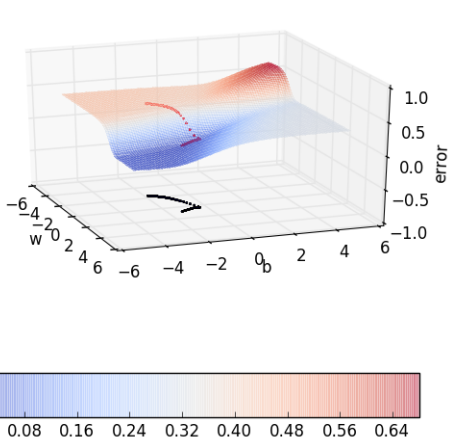

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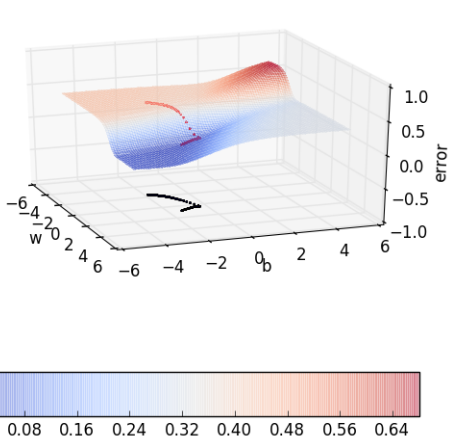

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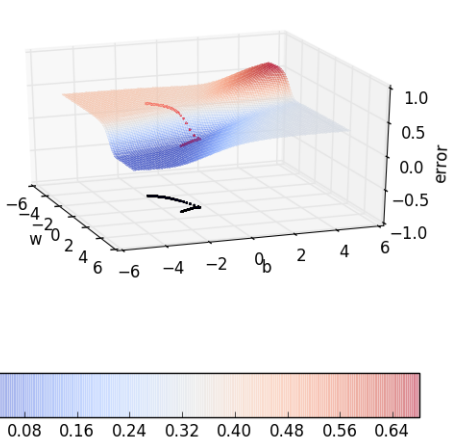

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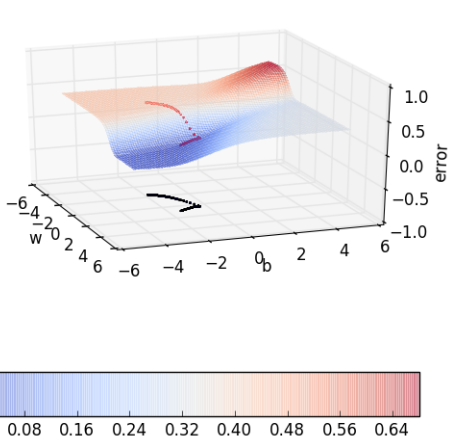

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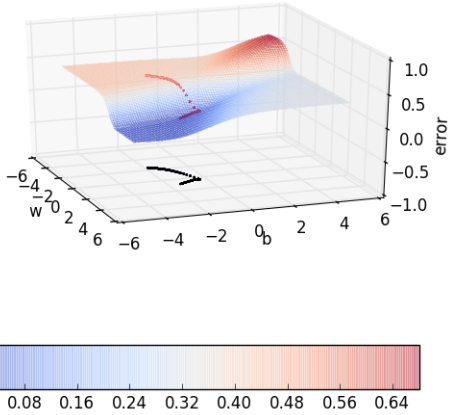
Gradient descent on the error surface



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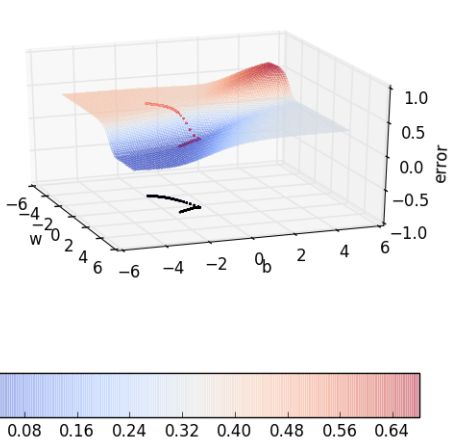

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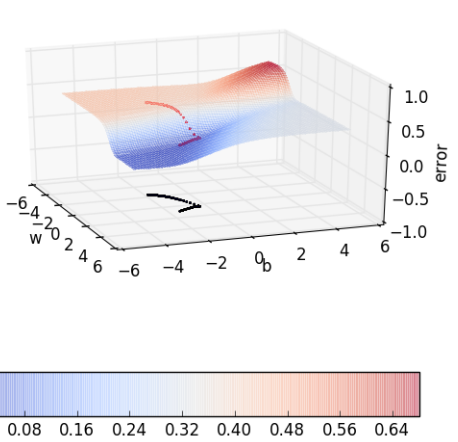

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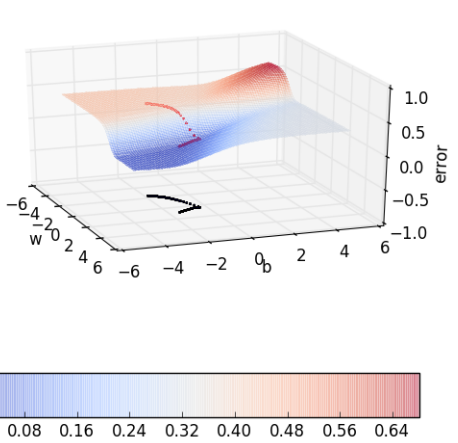

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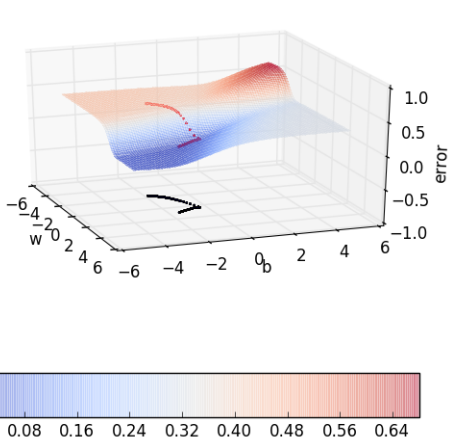

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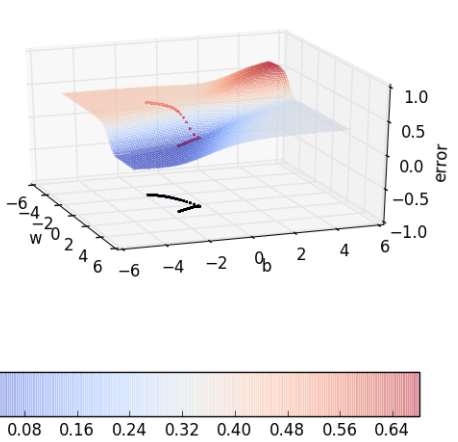

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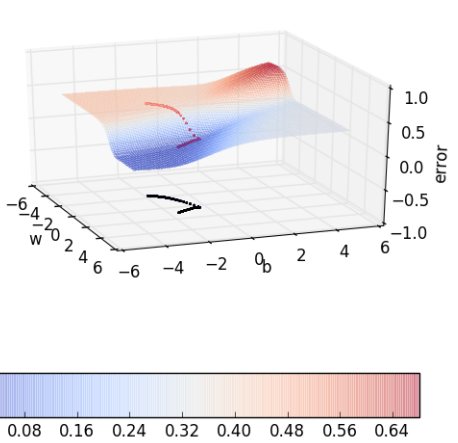

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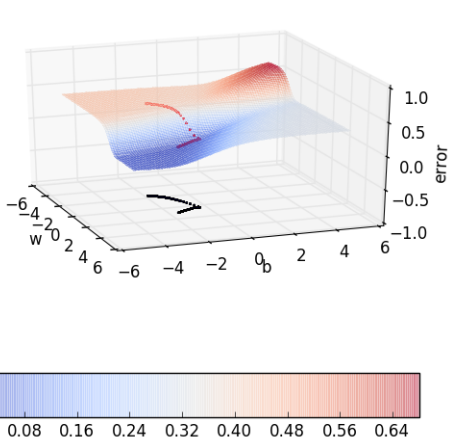

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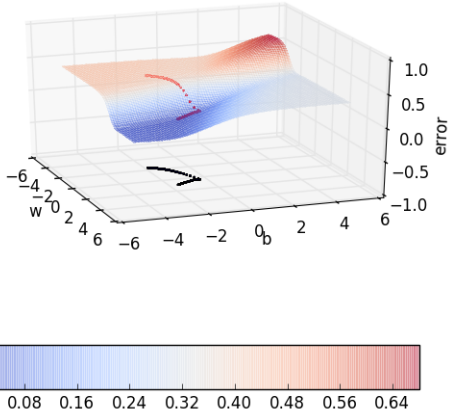
Gradient descent on the error surface



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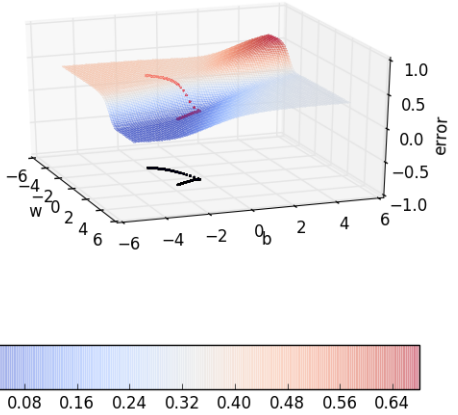

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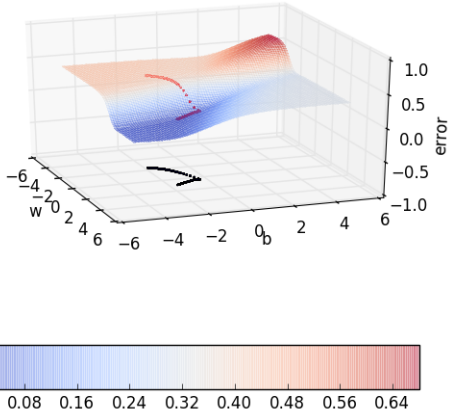

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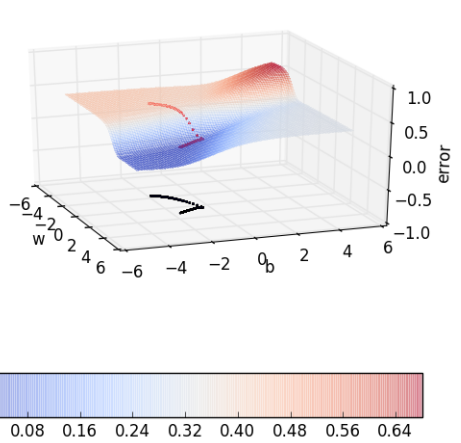

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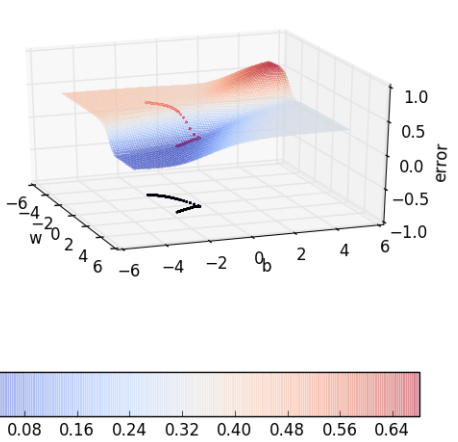

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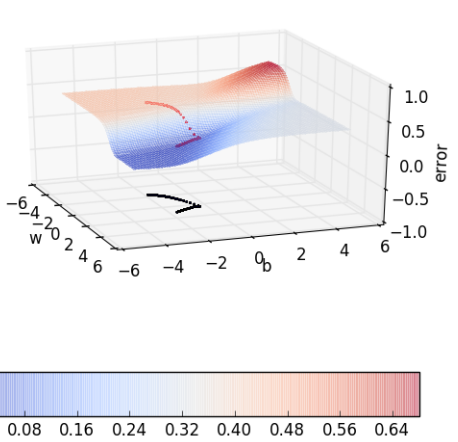

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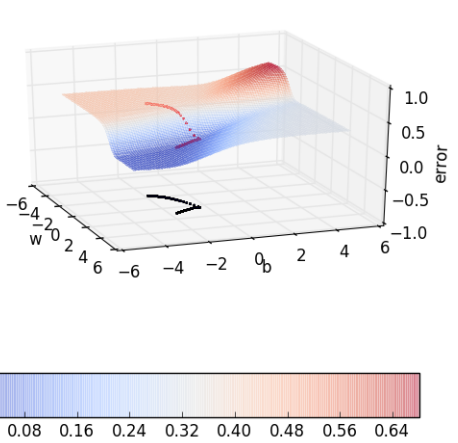

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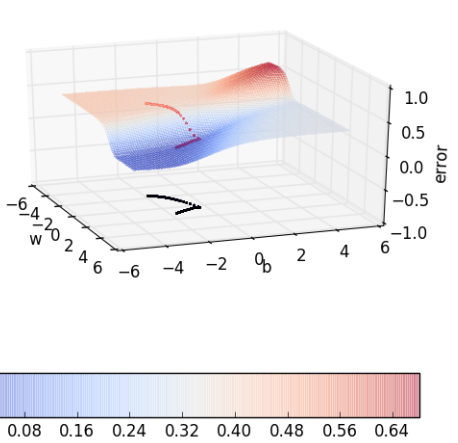

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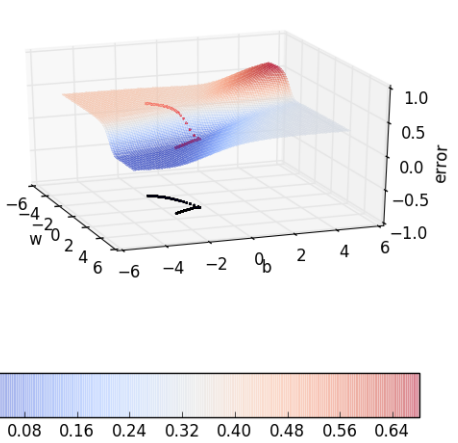

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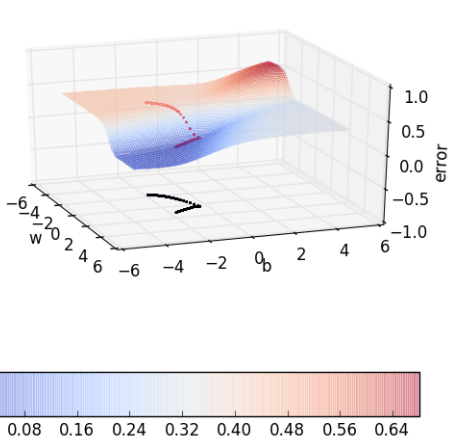

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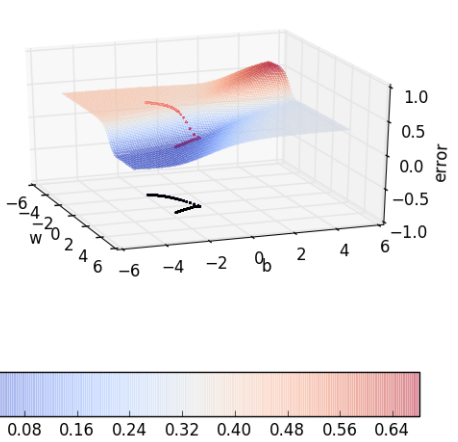

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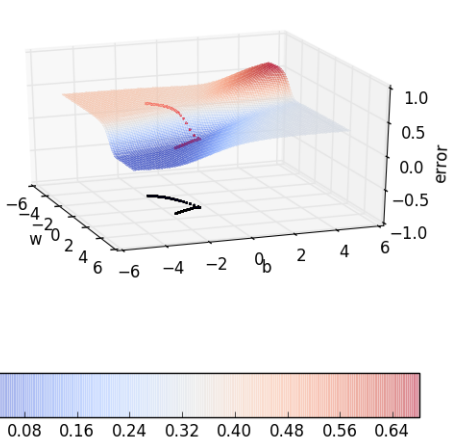

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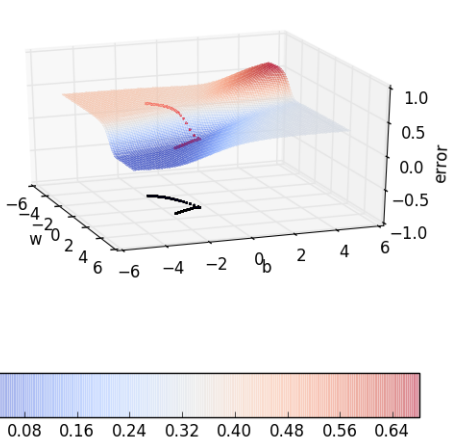

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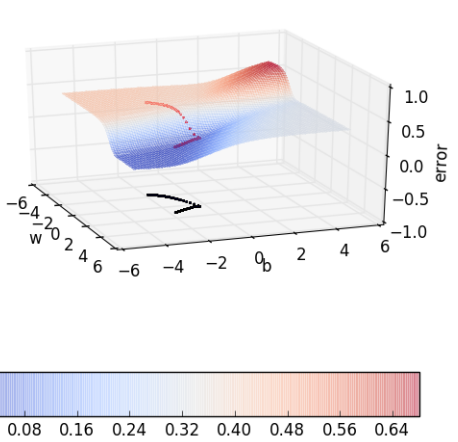

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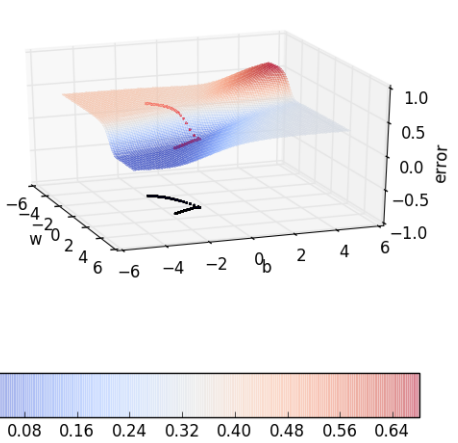

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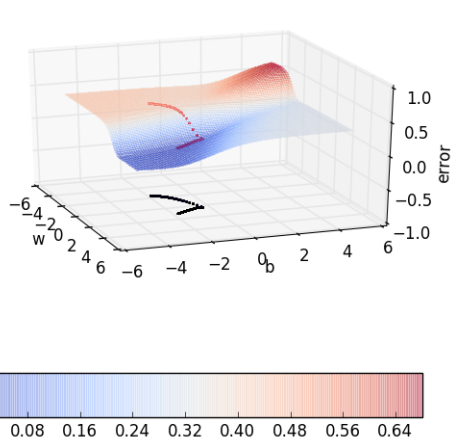

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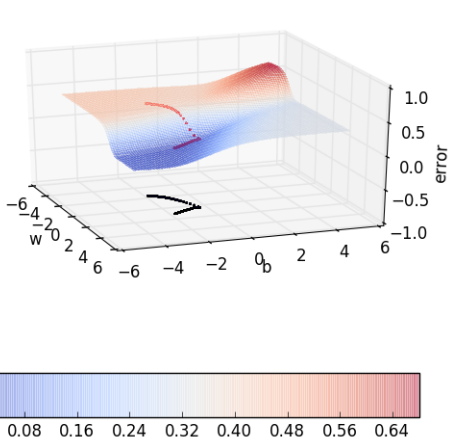

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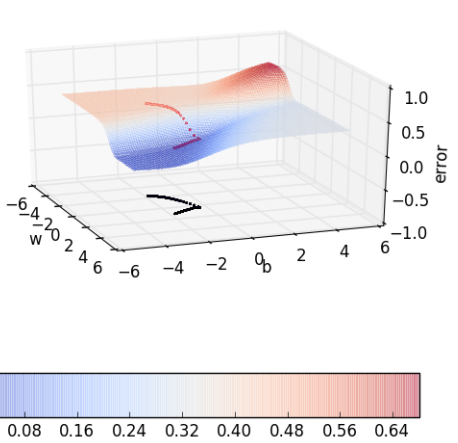

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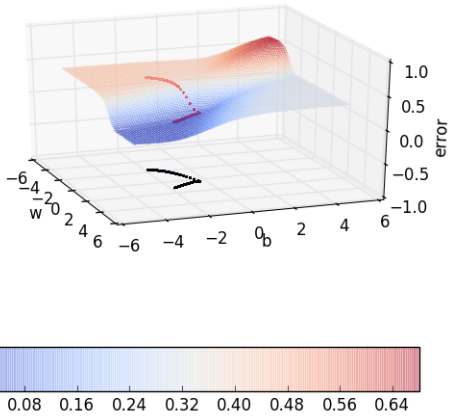
Gradient descent on the error surface



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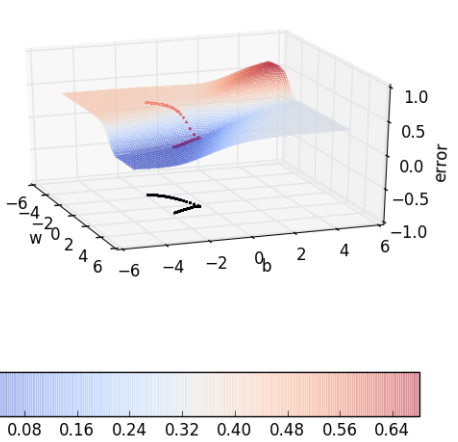

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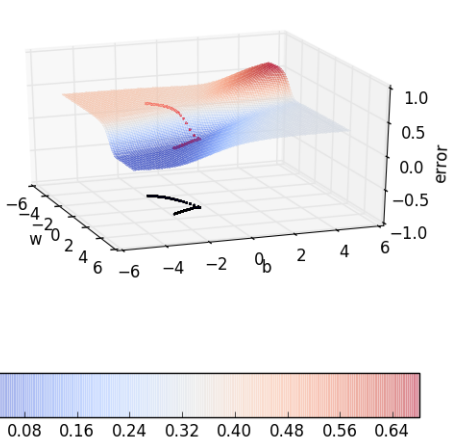

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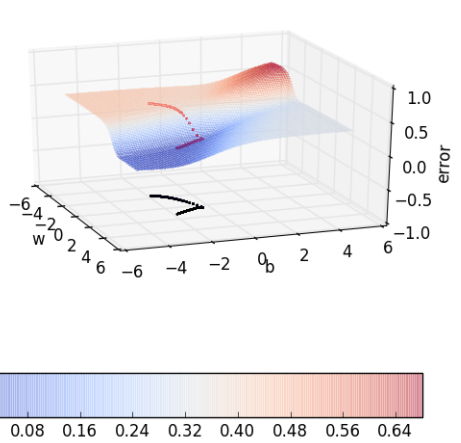

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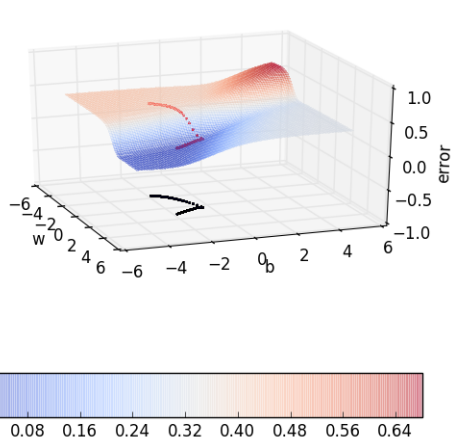

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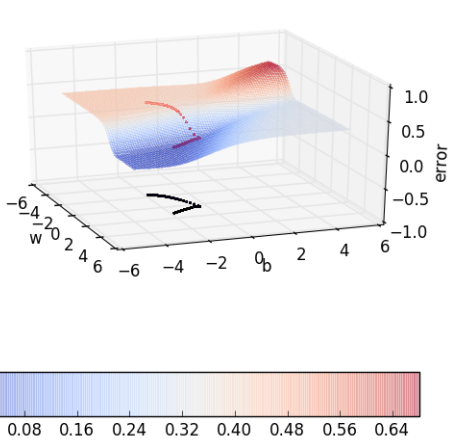

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