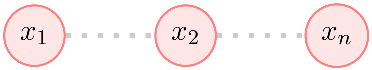
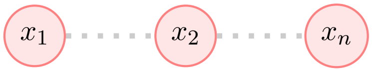
Module 4.1: Feedforward Neural Networks (a.k.a. multilayered network of neurons)

- The input to the network is an **n**-dimensional vector

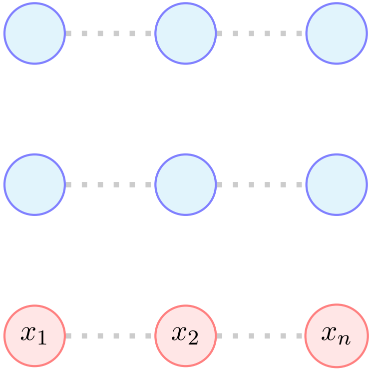
- The input to the network is an **n**-dimensional vector

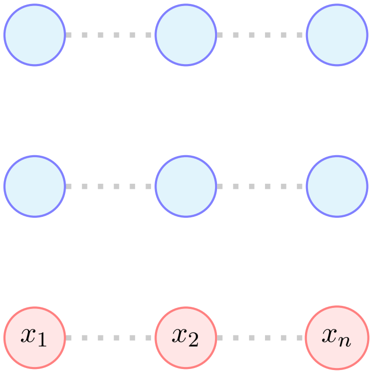


- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each

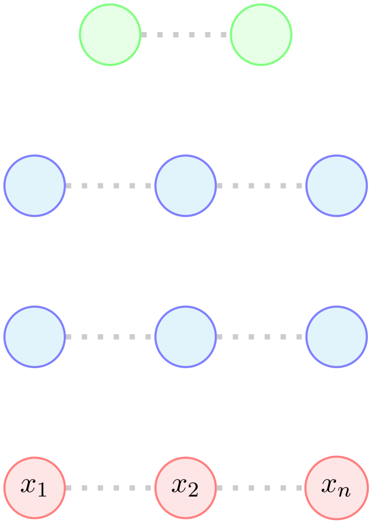


- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each

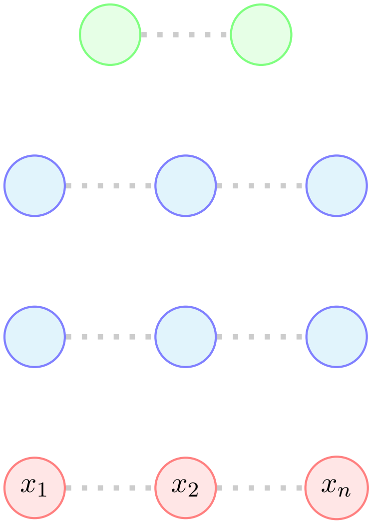




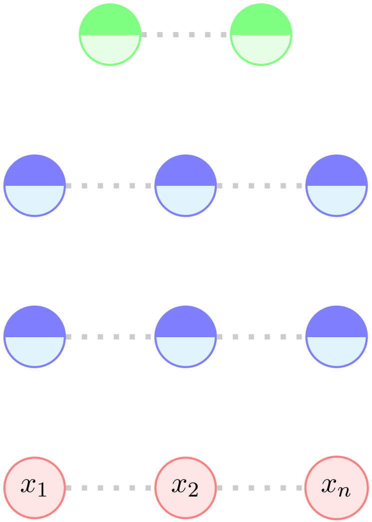
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)



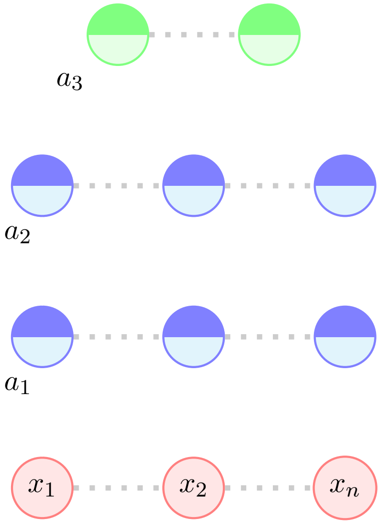
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)



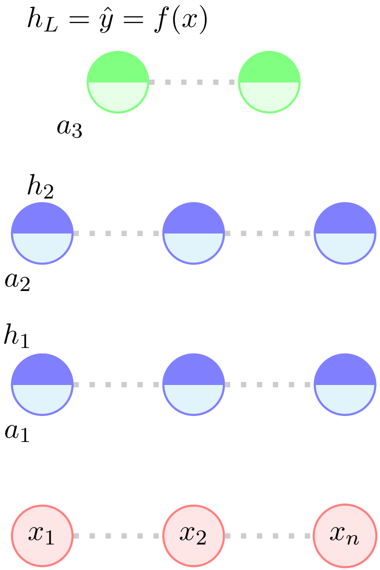
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts :



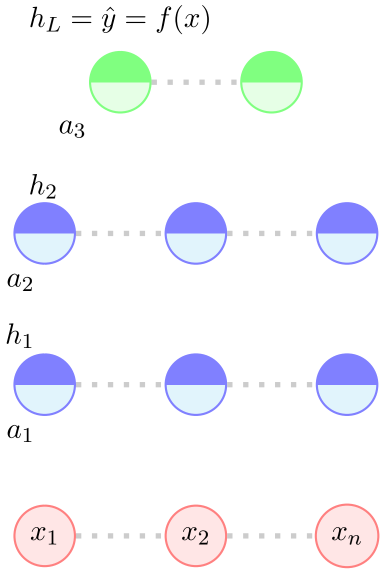
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts :



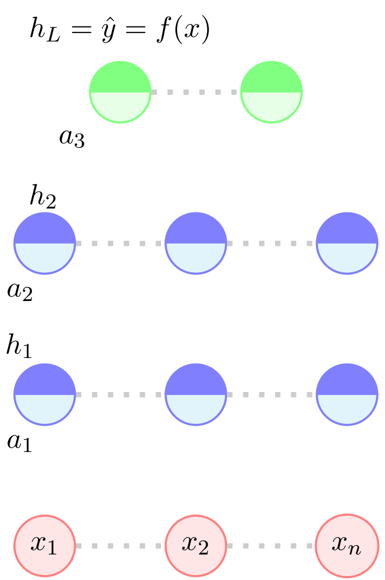
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation



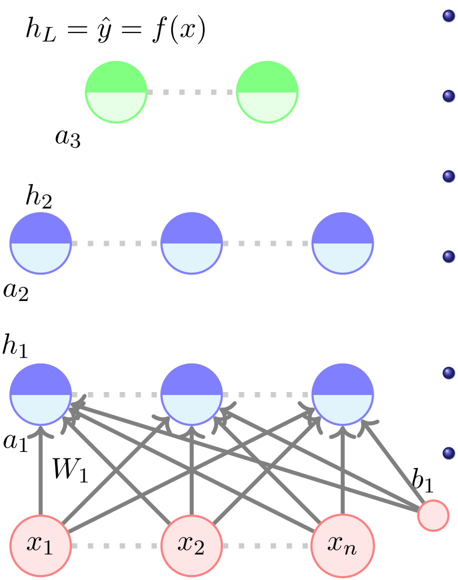
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation and activation



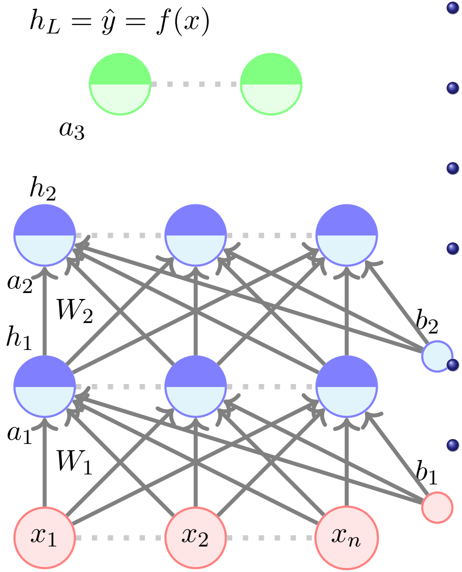
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation and activation (a_i and h_i are vectors)



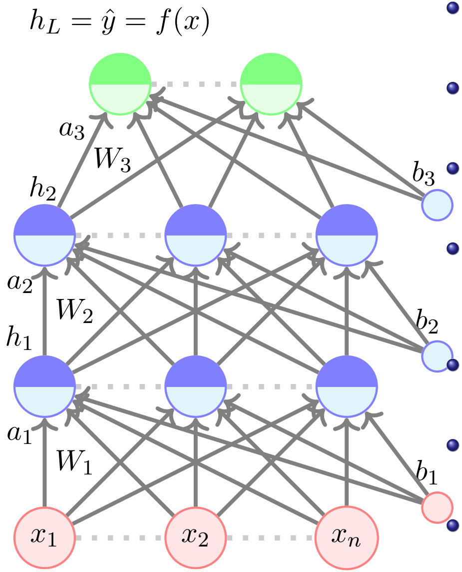
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation and activation (a_i and h_i are vectors)
- The input layer can be called the 0-th layer and the output layer can be called the (L)-th layer



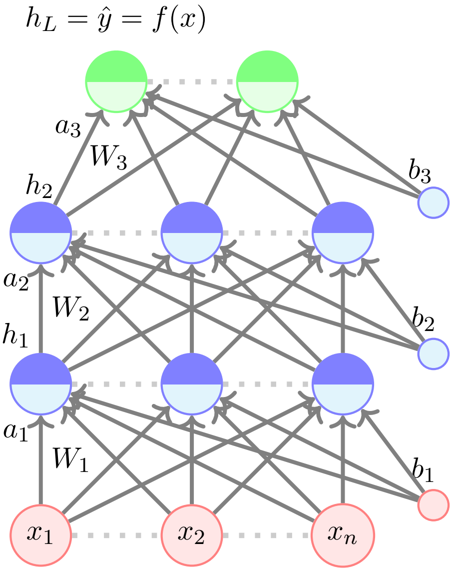
- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation and activation (a_i and h_i are vectors)
- The input layer can be called the 0-th layer and the output layer can be called the (L)-th layer
- $W_i \in \mathbb{R}^{n\times n}$ and $b_i \in \mathbb{R}^n$ are the weight and bias between layers $i-1$ and i $(0 < i < L)$



- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation and activation (a_i and h_i are vectors)
- The input layer can be called the 0-th layer and the output layer can be called the (L)-th layer
- $W_i \in \mathbb{R}^{n \times n}$ and $b_i \in \mathbb{R}^n$ are the weight and bias between layers $i-1$ and i ($0 < i < L$)

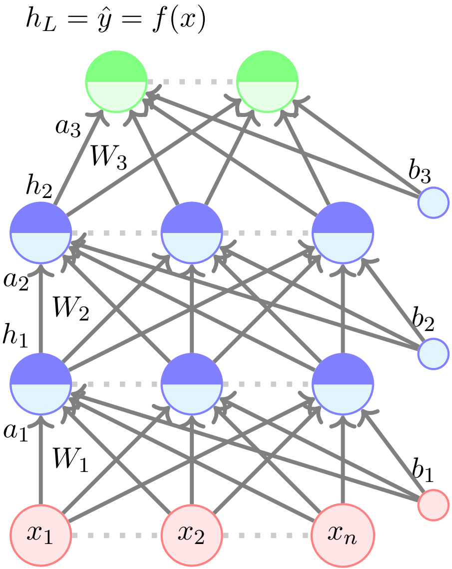


- The input to the network is an **n**-dimensional vector
- The network contains $\mathbf{L-1}$ hidden layers (2, in this case) having **n** neurons each
- Finally, there is one output layer containing **k** neurons (say, corresponding to **k** classes)
- Each neuron in the hidden layer and output layer can be split into two parts : pre-activation and activation (a_i and h_i are vectors)
- The input layer can be called the 0-th layer and the output layer can be called the (L)-th layer
- $W_i \in \mathbb{R}^{n \times n}$ and $b_i \in \mathbb{R}^n$ are the weight and bias between layers $i-1$ and i $(0 < i < L)$
- $W_L \in \mathbb{R}^{n \times k}$ and $b_L \in \mathbb{R}^k$ are the weight and bias between the last hidden layer and the output layer ($L = 3$ in this case)



- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

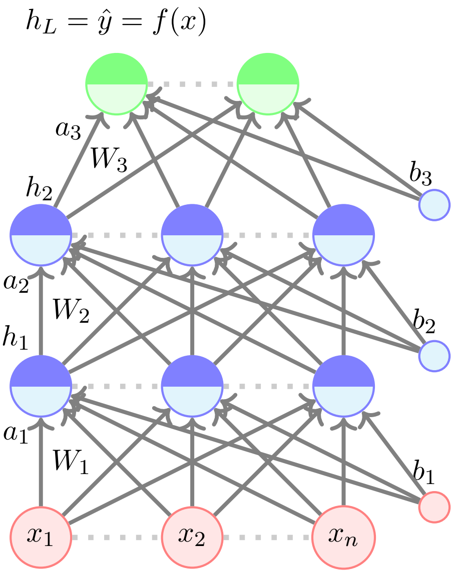


- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

- The activation at layer i is given by

$$h_i(x) = g(a_i(x))$$



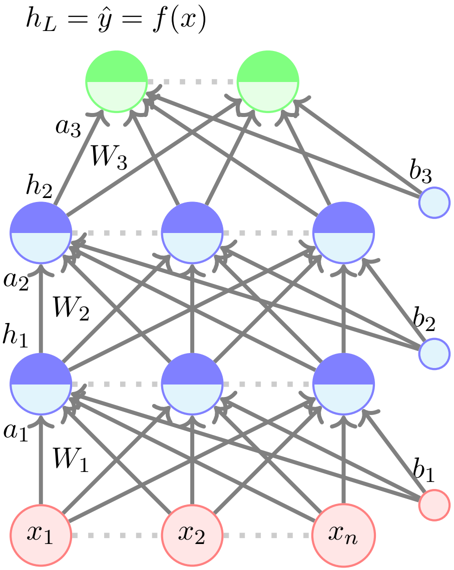
- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

- The activation at layer i is given by

$$h_i(x) = g(a_i(x))$$

where g is called the activation function (for example, logistic, tanh, linear, *etc.*)



- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

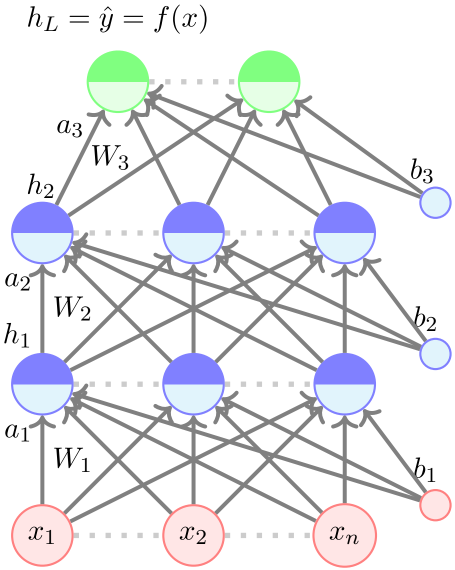
- The activation at layer i is given by

$$h_i(x) = g(a_i(x))$$

where g is called the activation function (for example, logistic, tanh, linear, *etc.*)

- The activation at the output layer is given by

$$f(x) = h_L(x) = O(a_L(x))$$



- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

- The activation at layer i is given by

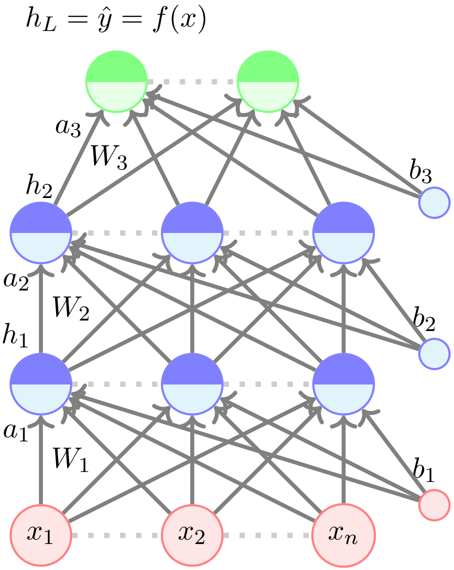
$$h_i(x) = g(a_i(x))$$

where g is called the activation function (for example, logistic, tanh, linear, *etc.*)

- The activation at the output layer is given by

$$f(x) = h_L(x) = O(a_L(x))$$

where O is the output activation function (for example, softmax, linear, *etc.*)



- The pre-activation at layer i is given by

$$a_i(x) = b_i + W_i h_{i-1}(x)$$

- The activation at layer i is given by

$$h_i(x) = g(a_i(x))$$

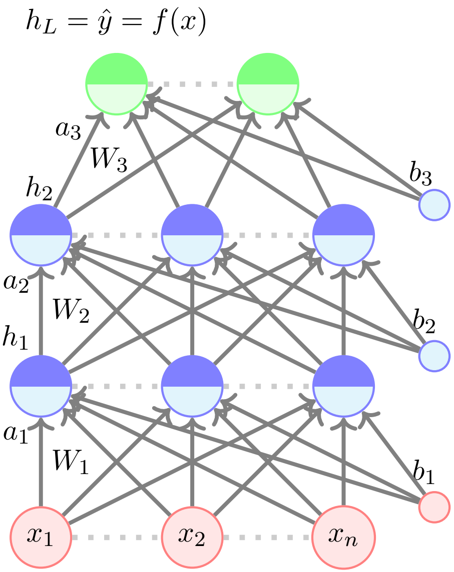
where g is called the activation function (for example, logistic, tanh, linear, *etc.*)

- The activation at the output layer is given by

$$f(x) = h_L(x) = O(a_L(x))$$

where O is the output activation function (for example, softmax, linear, *etc.*)

- To simplify notation we will refer to $a_i(x)$ as a_i and $h_i(x)$ as h_i



- The pre-activation at layer i is given by

$$a_i = b_i + W_i h_{i-1}$$

- The activation at layer i is given by

$$h_i = g(a_i)$$

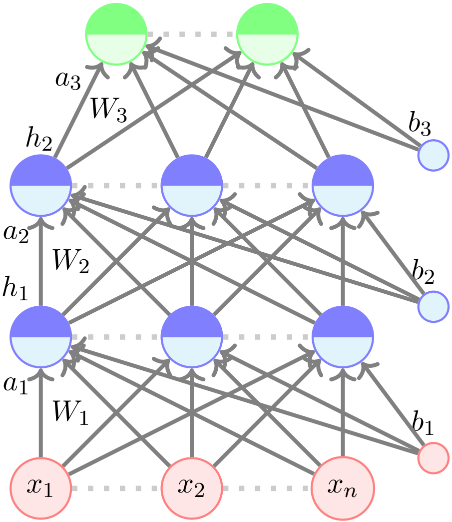
where g is called the activation function (for example, logistic, tanh, linear, *etc.*)

- The activation at the output layer is given by

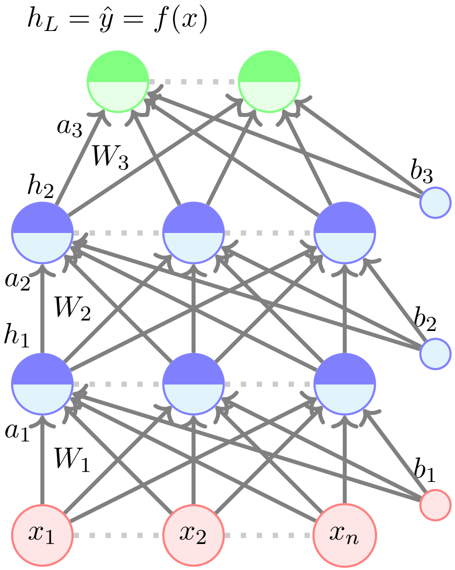
$$f(x) = h_L = O(a_L)$$

where O is the output activation function (for example, softmax, linear, *etc.*)

$$h_L = \hat{y} = f(x)$$

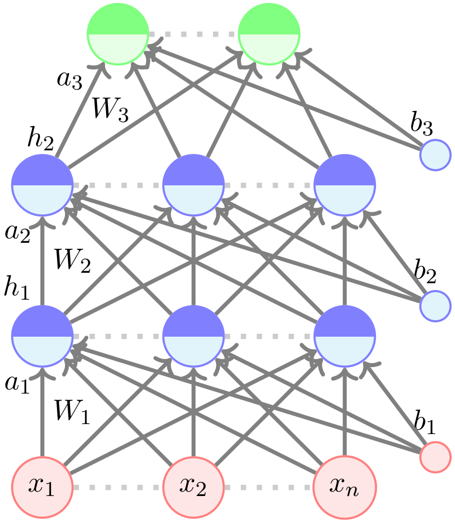


- **Data:** $\{x_i, y_i\}_{i=1}^{N}$



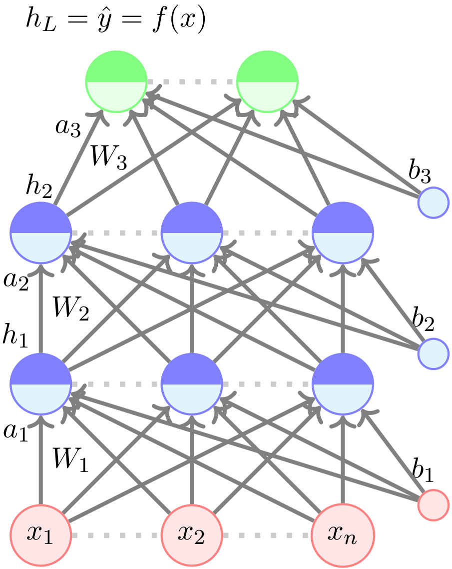
- **Data:** $\{x_i, y_i\}_{i=1}^N$
- **Model:**

$h_L = \hat{y} = f(x)$



- **Data:** $\{x_i, y_i\}_{i=1}^N$
- **Model:**

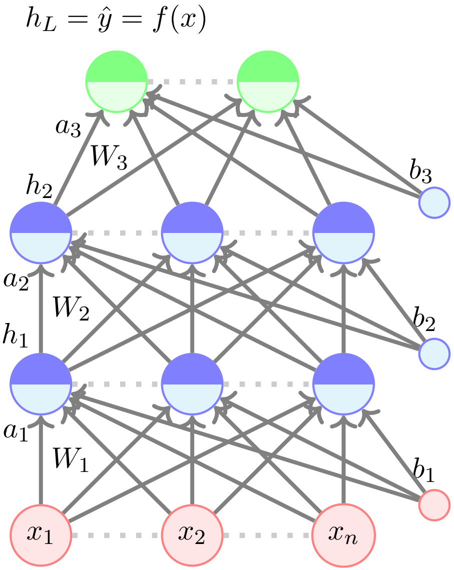
$$\hat{y}_i = f(x_i) = O(W_3 g(W_2 g(W_1 x + b_1) + b_2) + b_3)$$



- **Data:** $\{x_i, y_i\}_{i=1}^{N}$
- **Model:**

$$\hat{y}_i = f(x_i) = O(W_3 g(W_2 g(W_1 x + b_1) + b_2) + b_3)$$

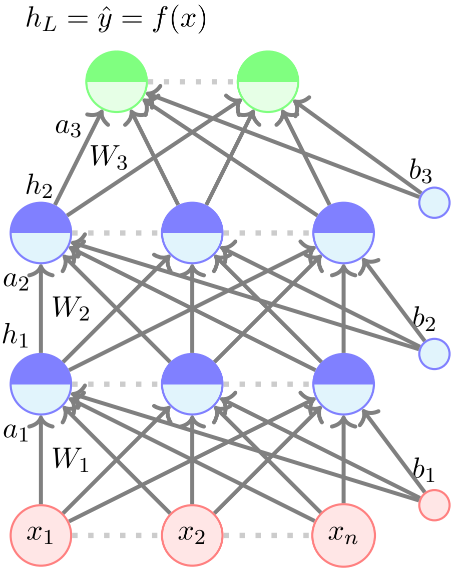
- **Parameters:**
  $\theta = W_1, .., W_L, b_1, b_2, ..., b_L (L = 3)$



- **Data:** $\{x_i, y_i\}_{i=1}^{N}$
- **Model:**

$$\hat{y}_i = f(x_i) = O(W_3 g(W_2 g(W_1 x + b_1) + b_2) + b_3)$$

- **Parameters:**
  $\theta = W_1, .., W_L, b_1, b_2, ..., b_L (L = 3)$
- **Algorithm:** Gradient Descent with Back-propagation (we will see soon)



- **Data:** $\{x_i, y_i\}_{i=1}^{N}$
- **Model:**

$$\hat{y}_i = f(x_i) = O(W_3 g(W_2 g(W_1 x + b_1) + b_2) + b_3)$$

- **Parameters:**
  $\theta = W_1, .., W_L, b_1, b_2, ..., b_L (L = 3)$
- **Algorithm:** Gradient Descent with Back-propagation (we will see soon)
- **Objective/Loss/Error function:** Say,

$$min \frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{k} (\hat{y}_{ij} - y_{ij})^2$$

In general, $min \mathscr{L}(\theta)$

where $\mathscr{L}(\theta)$ is some function of the parameters