


Module 5.2: Learning Parameters : Gradient Descent

Now let's see if there is a more efficient and principled way of doing this


Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!


vector of parameters,
say, randomly initialized


$$\theta = [w, b]$$

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change in the
values of w, b

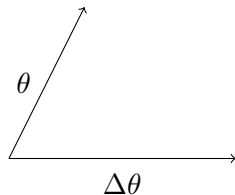

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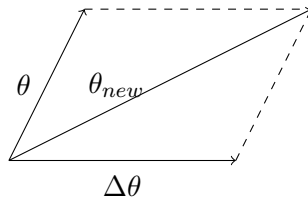


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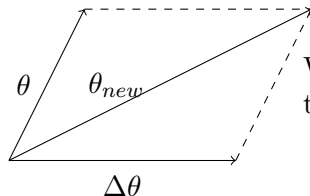


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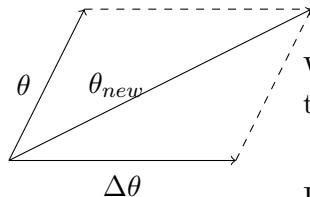
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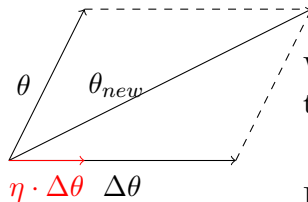
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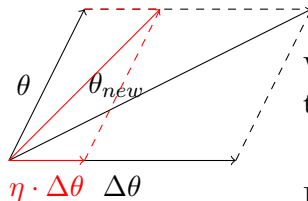
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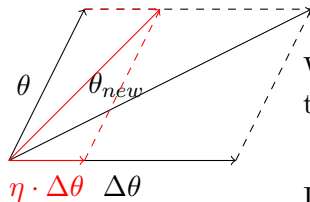
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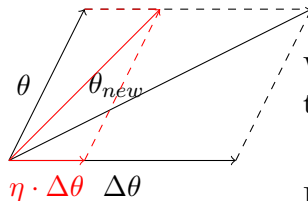
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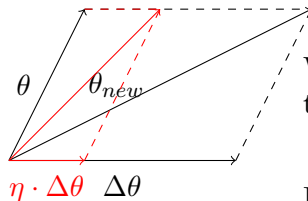
Question: What is the right $\Delta\theta$ to use?

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The answer comes from Taylor series

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Note that the move (ηu) would be favorable only if,

$$\mathcal{L}(\theta + \eta u) - \mathcal{L}(\theta) < 0 \text{ [i.e., if the new loss is less than the previous loss]}$$

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This implies,

$$u^T \nabla \mathcal{L}(\theta) < 0$$

Okay, so we have,

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$$-1 \leq \cos(\beta) = \frac{u^T \nabla \mathcal{L}(\theta)}{\|u\| * \|\nabla \mathcal{L}(\theta)\|} \leq 1$$

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Multiply throughout by $k = \|u\| * \|\nabla \mathcal{L}(\theta)\|$

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Thus, $\mathcal{L}(\theta + \eta u) - \mathcal{L}(\theta) = u^T \nabla \mathcal{L}(\theta) = k * \cos(\beta)$ will be most negative when $\cos(\beta) = -1$ i.e., when β is 180°

Gradient Descent Rule

- The direction u that we intend to move in should be at 180° w.r.t. the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$

$$b_{t+1} = b_t - \eta \nabla b_t$$

$$\text{where, } \nabla w_t = \frac{\partial \mathcal{L}(w, b)}{\partial w} \text{ at } w = w_t, b = b_t, \nabla b_t = \frac{\partial \mathcal{L}(w, b)}{\partial b} \text{ at } w = w_t, b = b_t$$

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So we now have a more principled way of moving in the w - b plane than our “guess work” algorithm

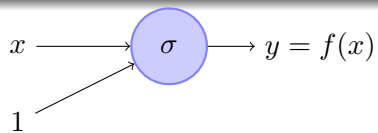
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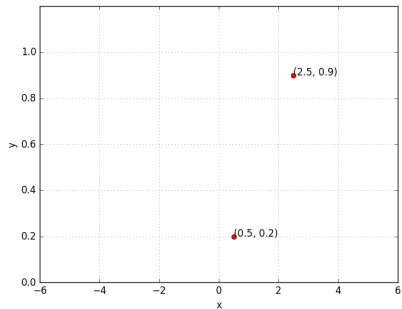
Algorithm 1: `gradient_descent()`

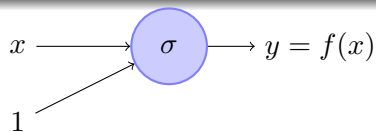
```
 $t \leftarrow 0;$   
 $max\_iterations \leftarrow 1000;$   
while  $t < max\_iterations$  do  
     $w_{t+1} \leftarrow w_t - \eta \nabla w_t;$   
     $b_{t+1} \leftarrow b_t - \eta \nabla b_t;$   
end
```

- To see this algorithm in practice let us first derive ∇w and ∇b for our toy neural network

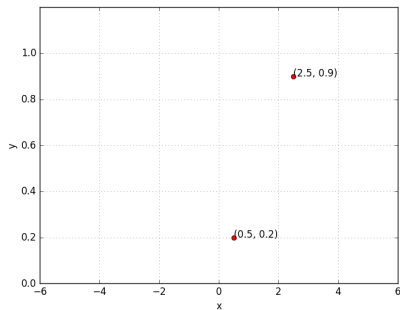


$$f(x) = \frac{1}{1 + e^{-(w \cdot x + b)}}$$

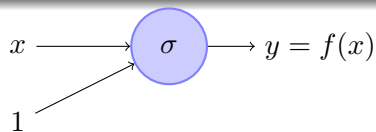




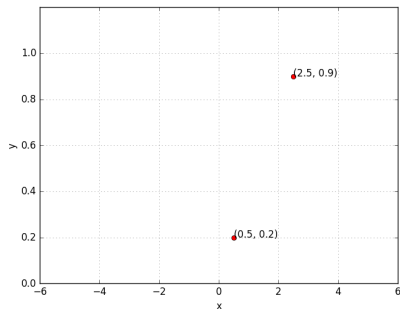
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Let's assume there is only 1 point to fit (x, y)

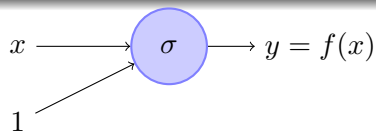


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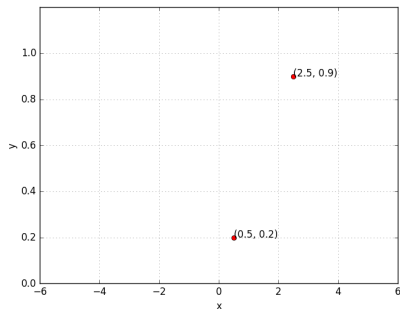


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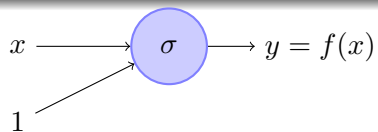
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&= \frac{1}{(1 + e^{-(wx+b)})} * \frac{e^{-(wx+b)}}{(1 + e^{-(wx+b)})} * (x) \\
&= f(x) * (1 - f(x)) * x
\end{aligned}$$

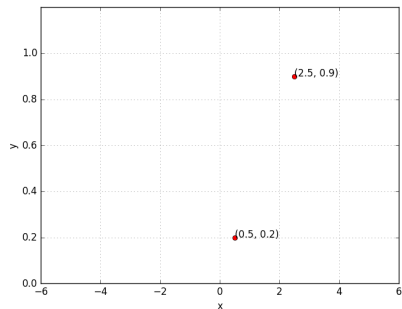
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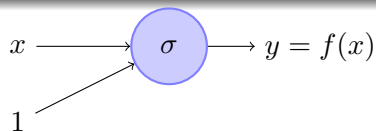
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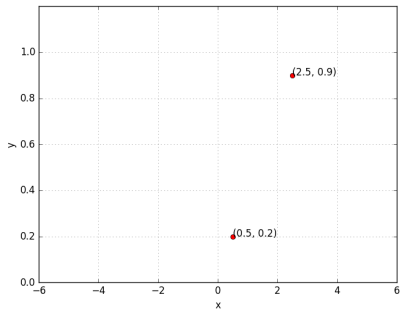
$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

So if there is only 1 point (x, y) , we have,



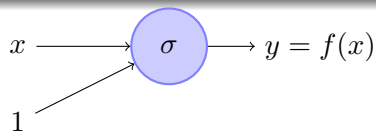


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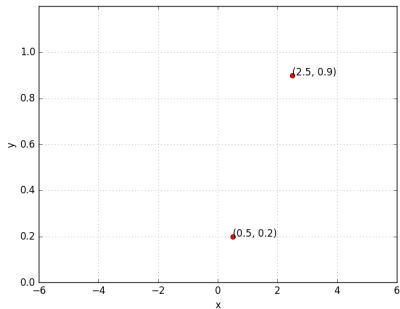


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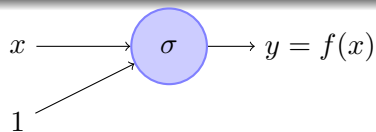
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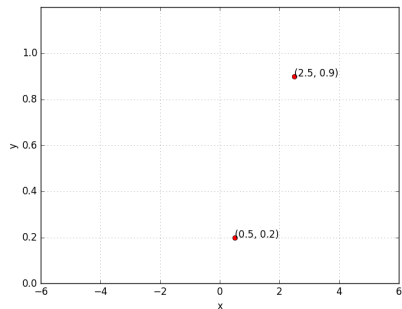
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For two points,



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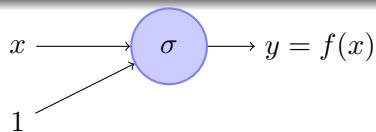


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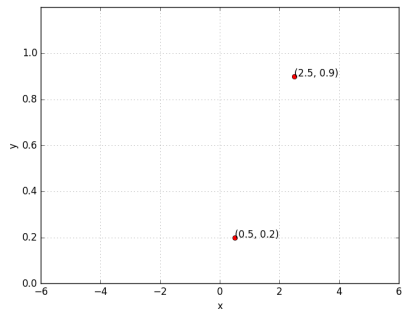
$$\nabla w = (f(x) - y) * f(x) * (1 - f(x)) * x$$

For two points,

$$\nabla w = \sum_{i=1}^2 (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$



$$f(x) = \frac{1}{1 + e^{-(w \cdot x + b)}}$$



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X = [0.5, 2.5]
Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
    err = 0.0
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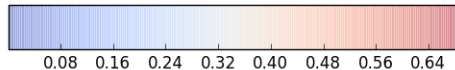
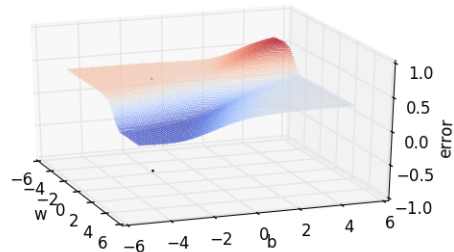
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Gradient descent on the error surface



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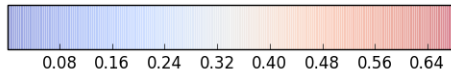
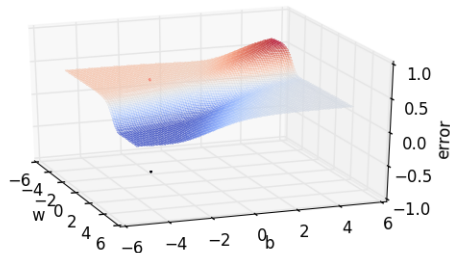
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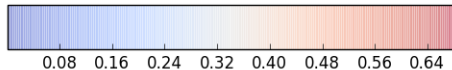
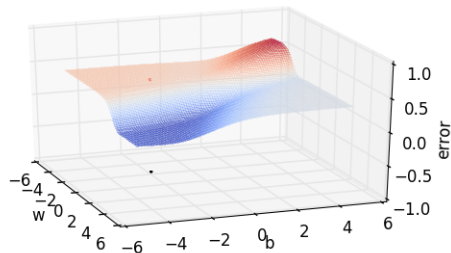
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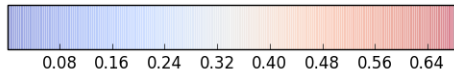
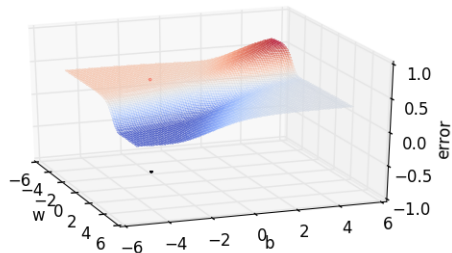
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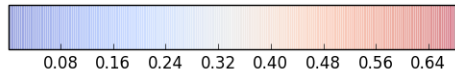
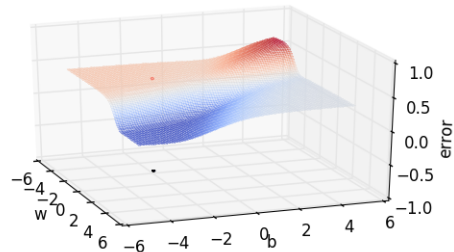
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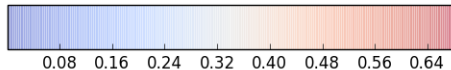
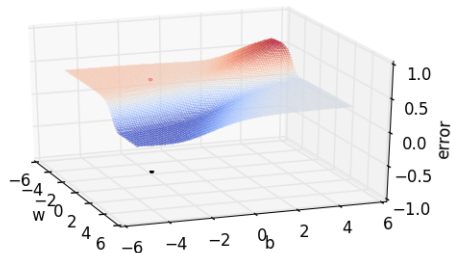
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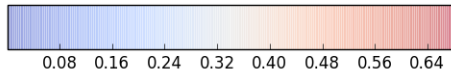
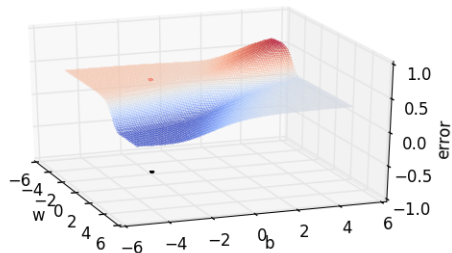
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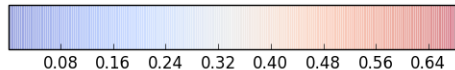
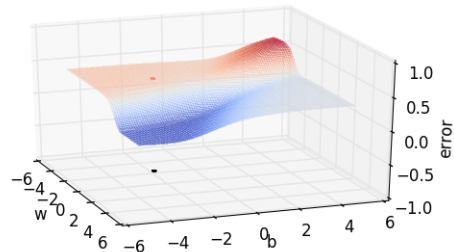
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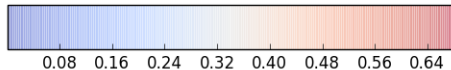
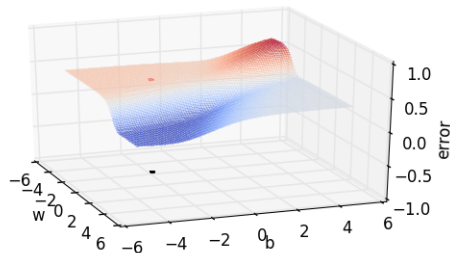
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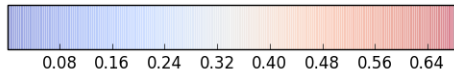
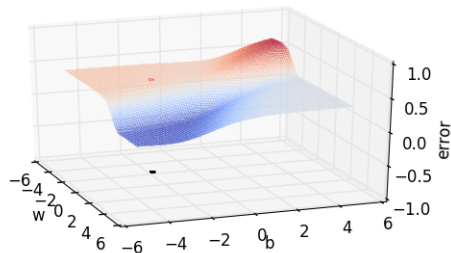
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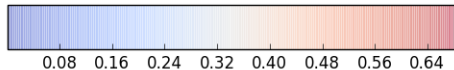
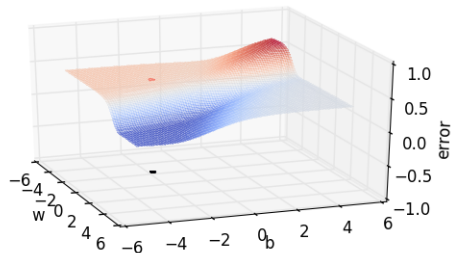
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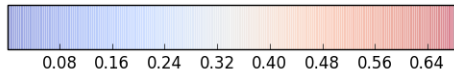
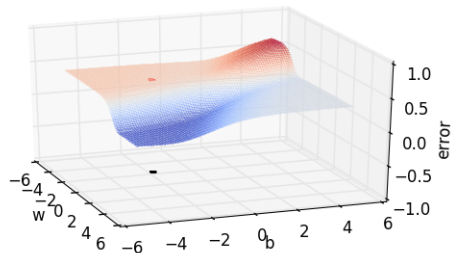
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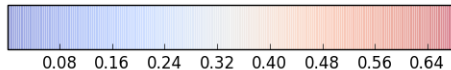
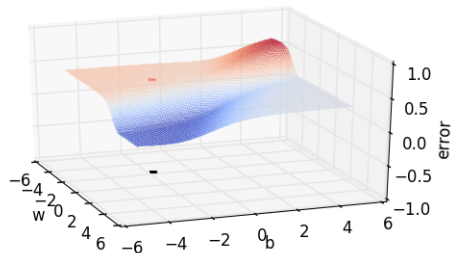
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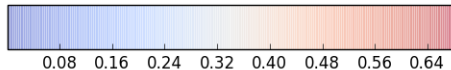
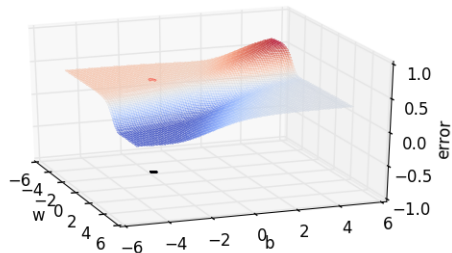
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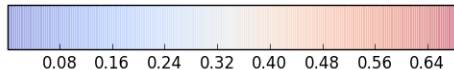
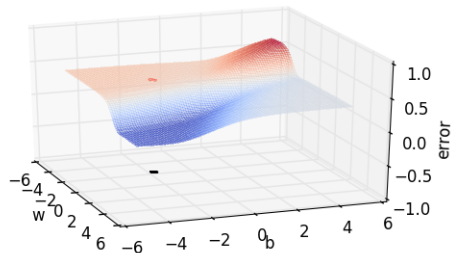
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```

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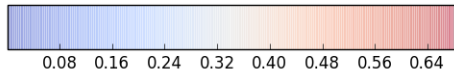
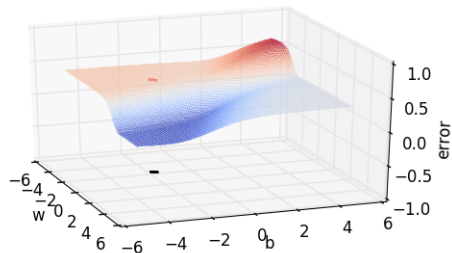
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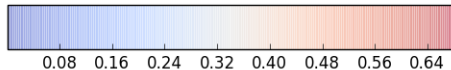
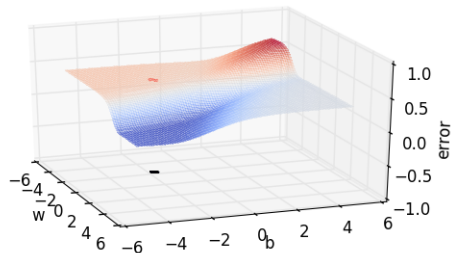
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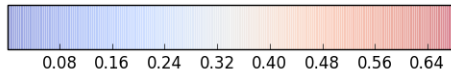
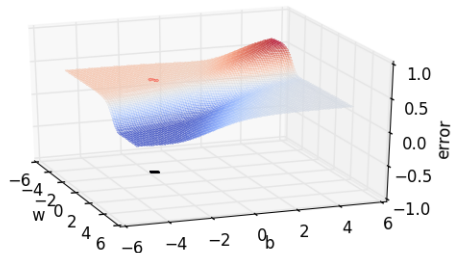
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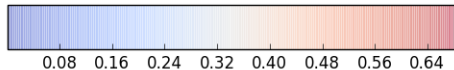
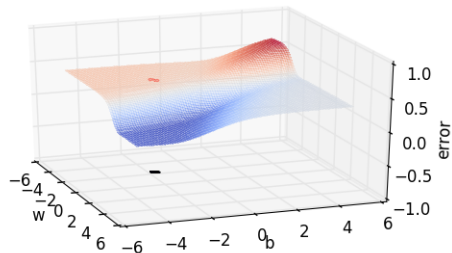
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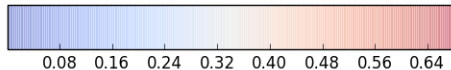
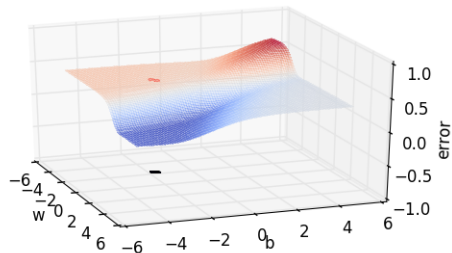
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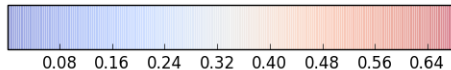
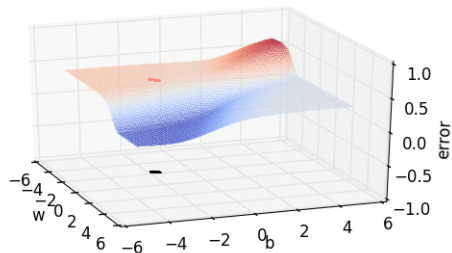
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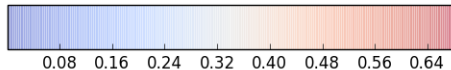
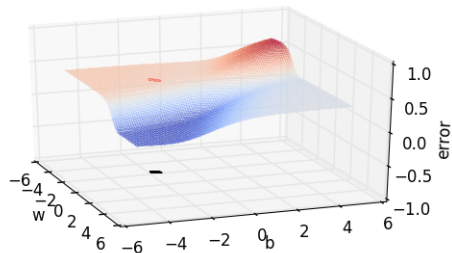
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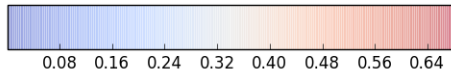
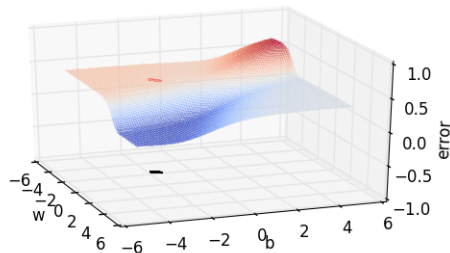
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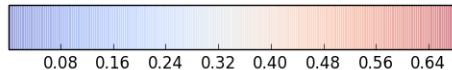
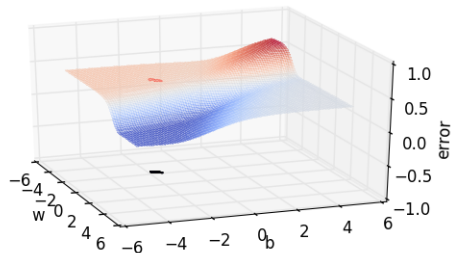
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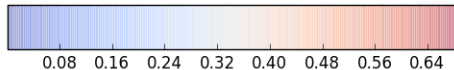
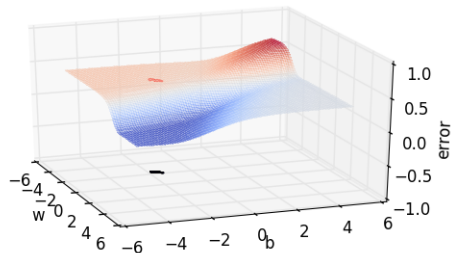
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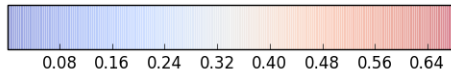
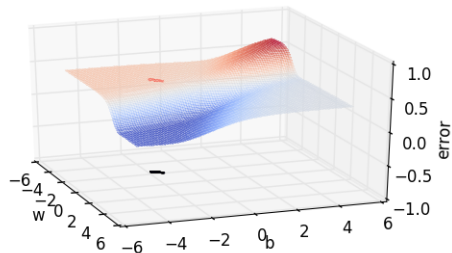
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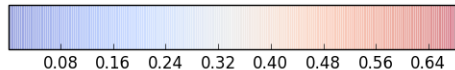
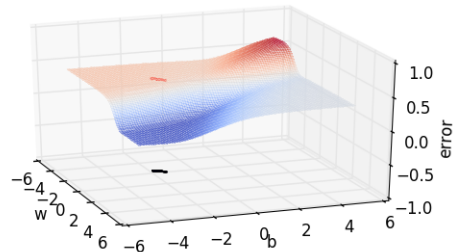
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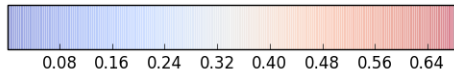
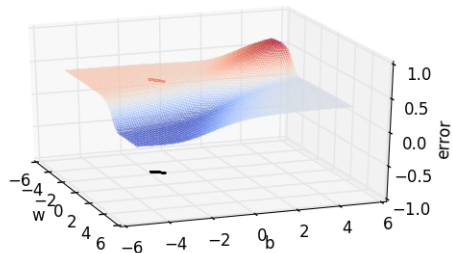
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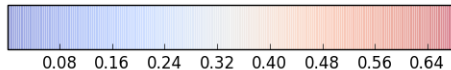
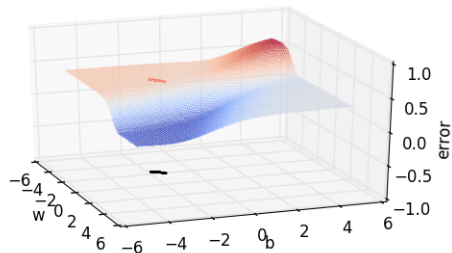
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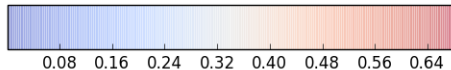
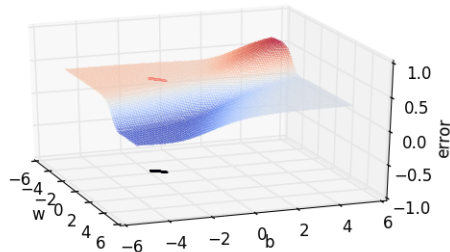
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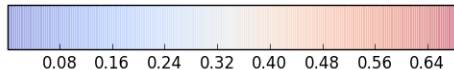
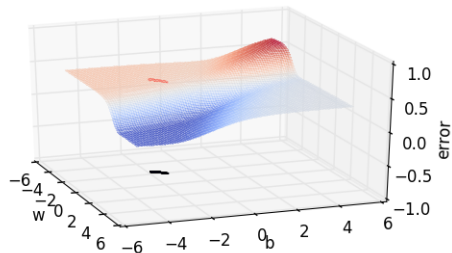
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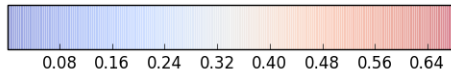
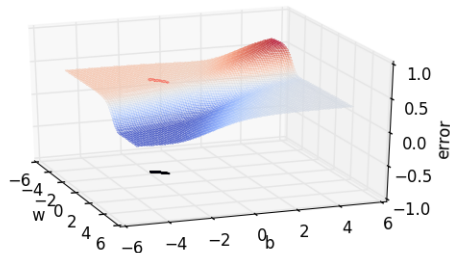
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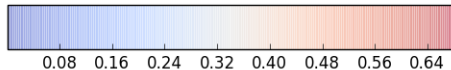
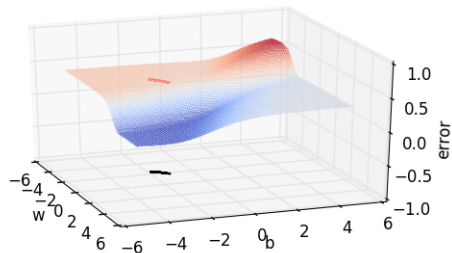
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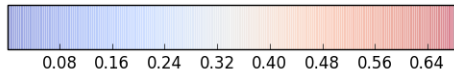
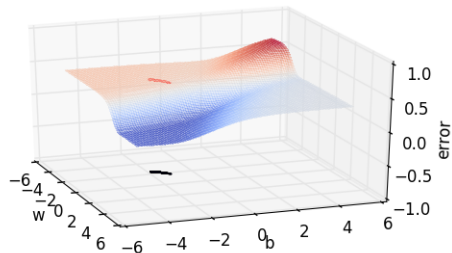
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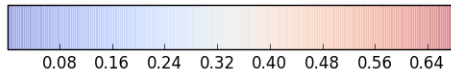
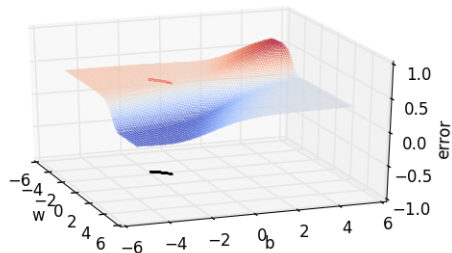
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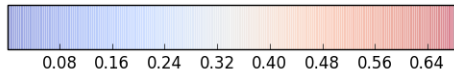
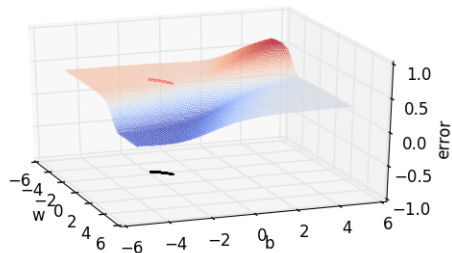
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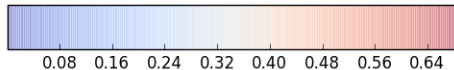
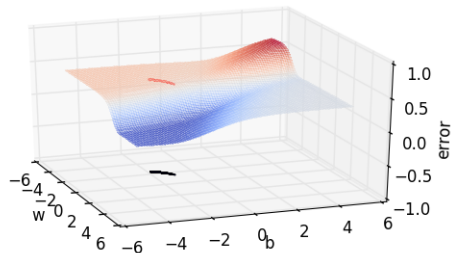
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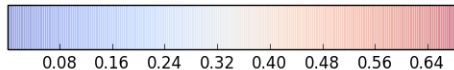
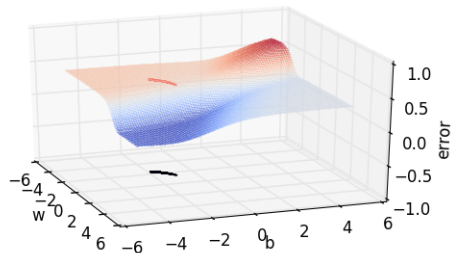
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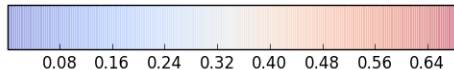
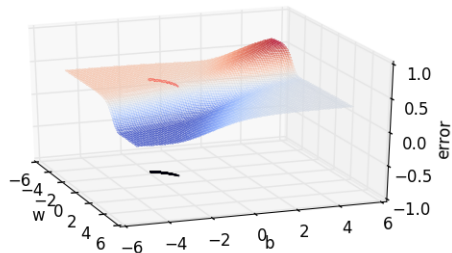
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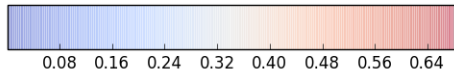
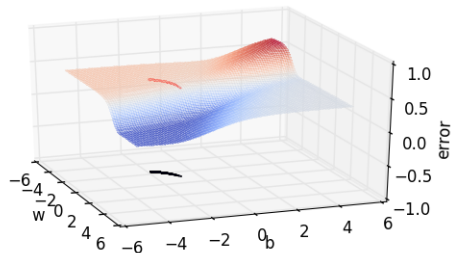
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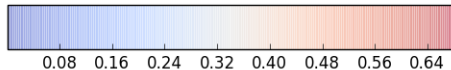
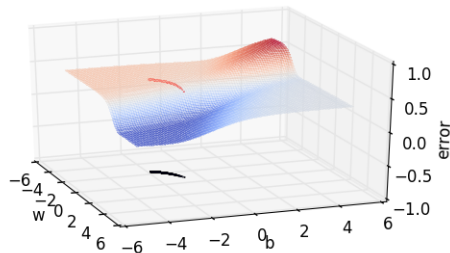
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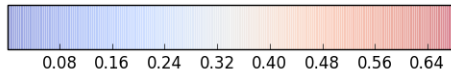
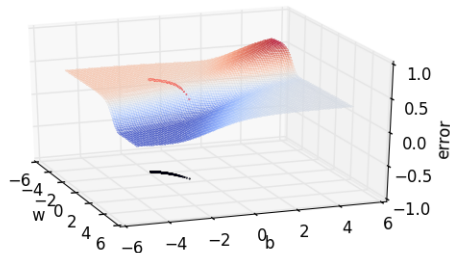
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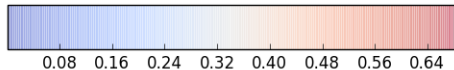
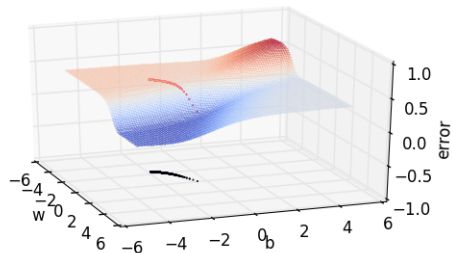
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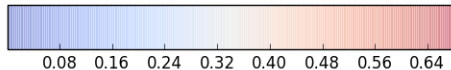
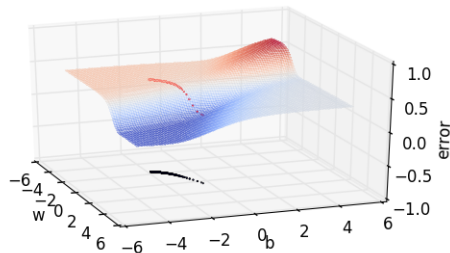
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Gradient descent on the error surface



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X = [0.5, 2.5]
Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
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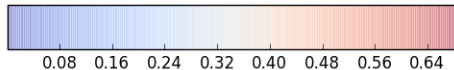
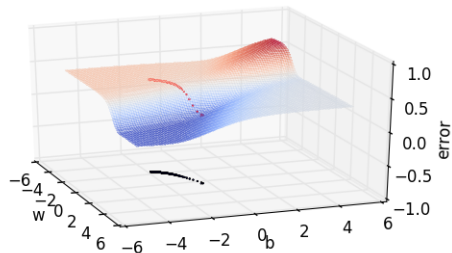
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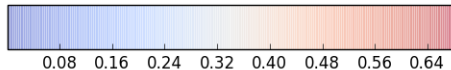
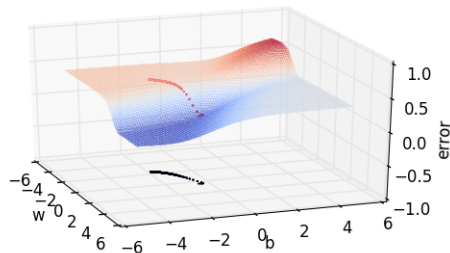
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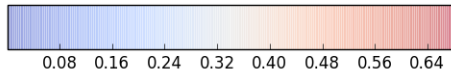
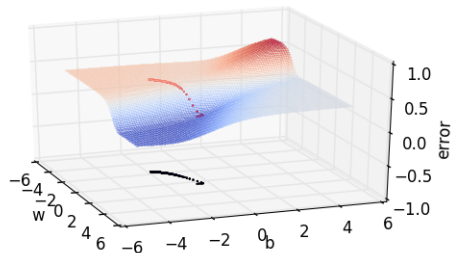
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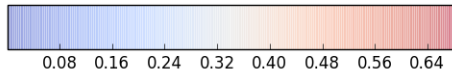
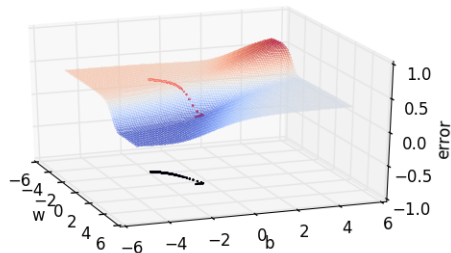
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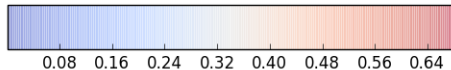
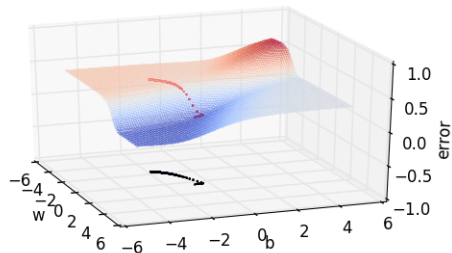
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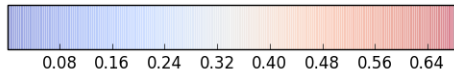
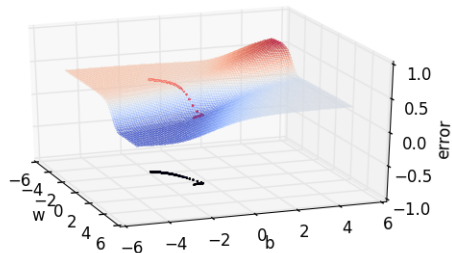
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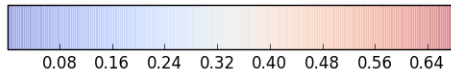
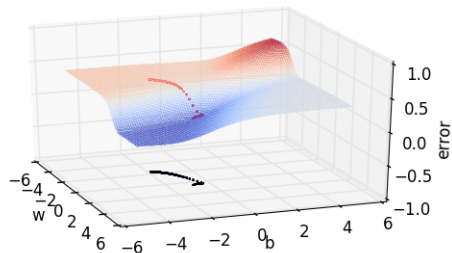
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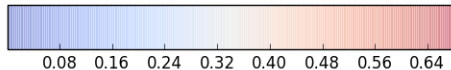
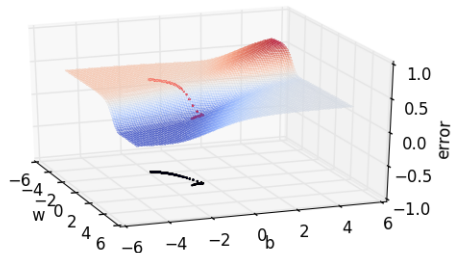
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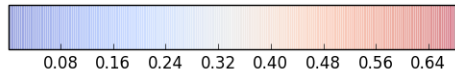
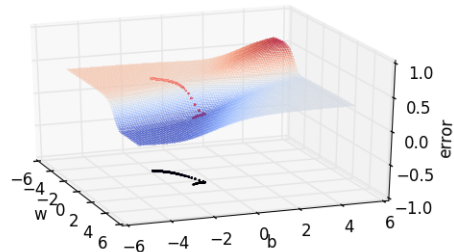
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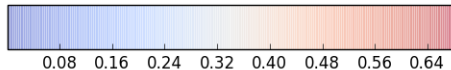
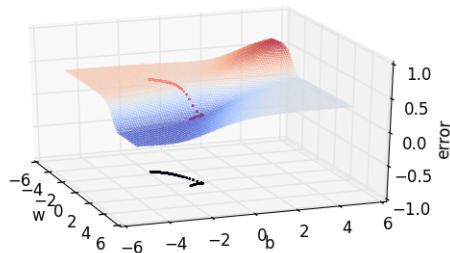
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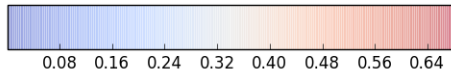
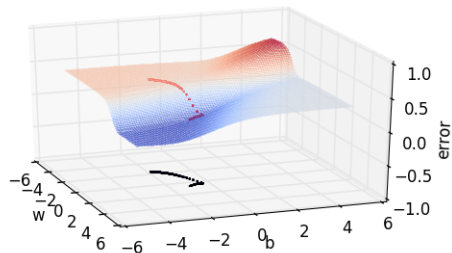
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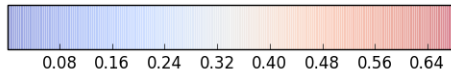
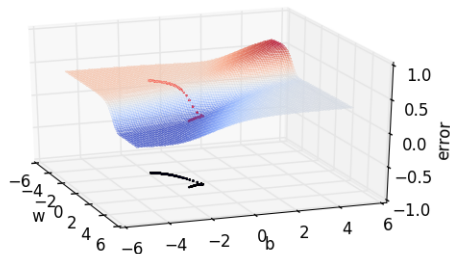
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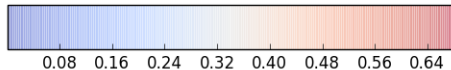
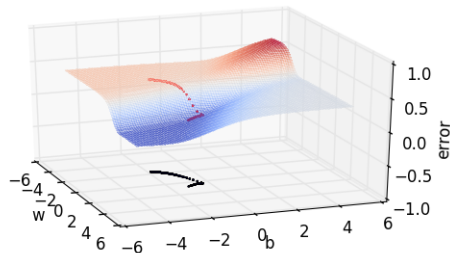
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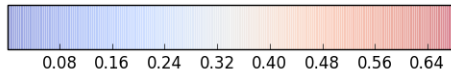
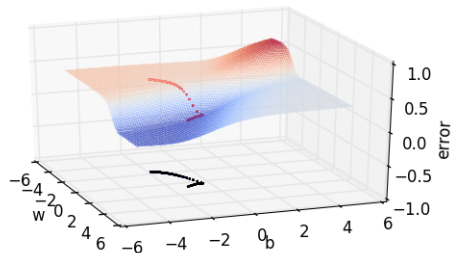
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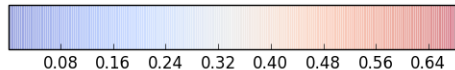
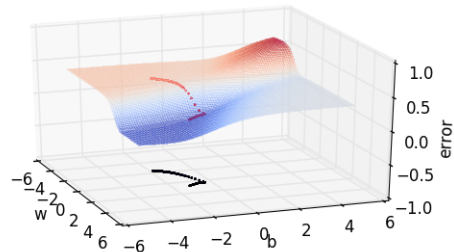
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Gradient descent on the error surface



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Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

def error (w, b) :
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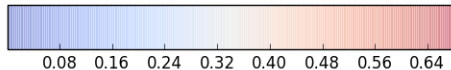
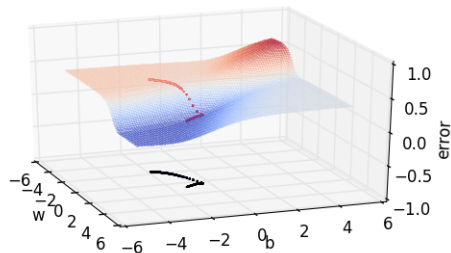
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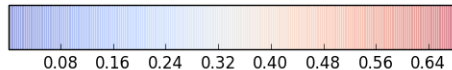
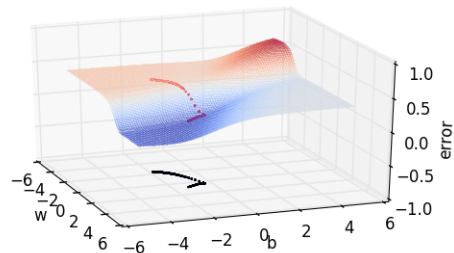
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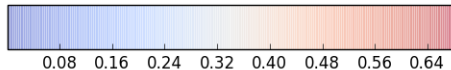
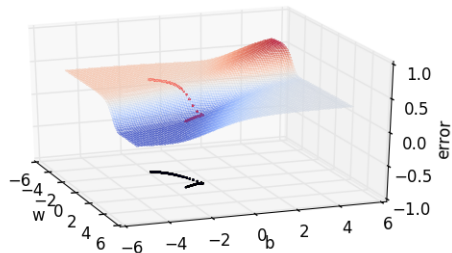
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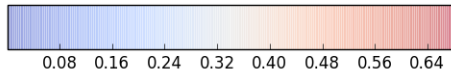
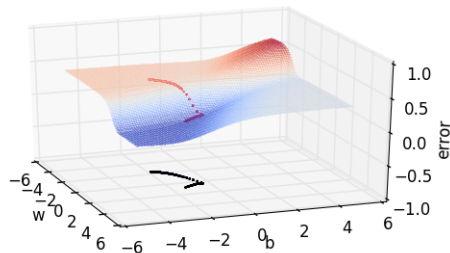
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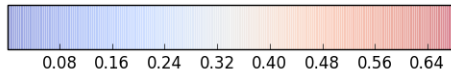
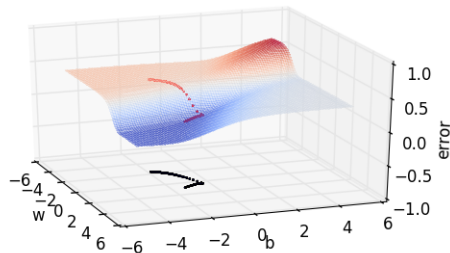
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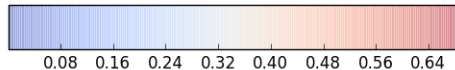
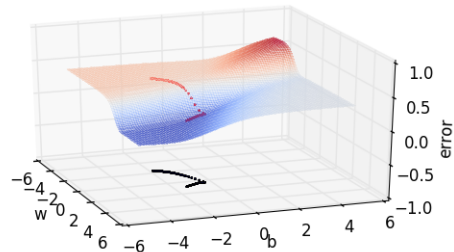
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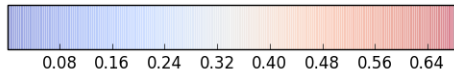
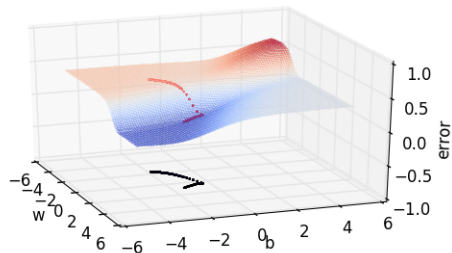
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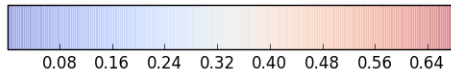
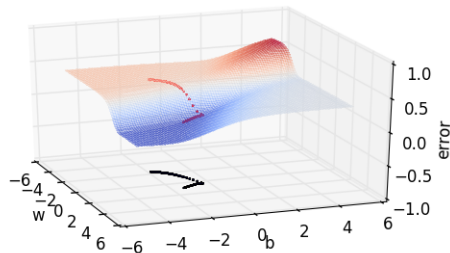
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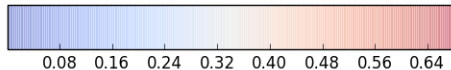
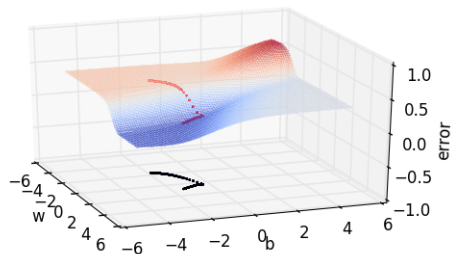
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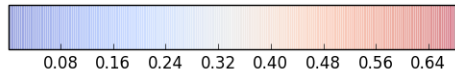
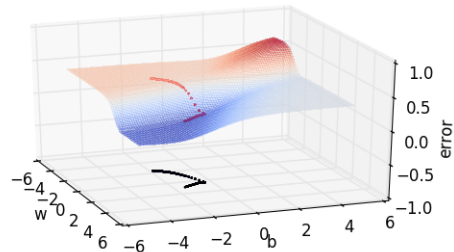
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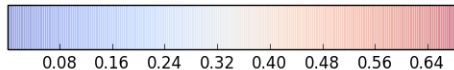
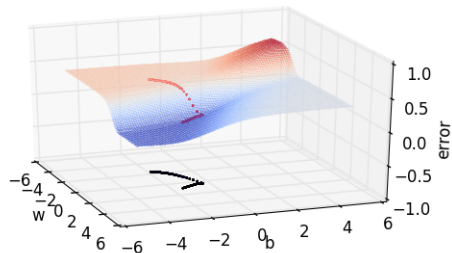
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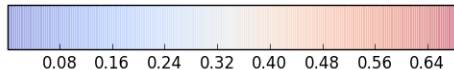
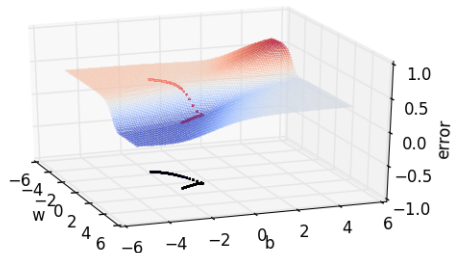
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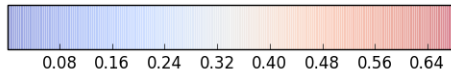
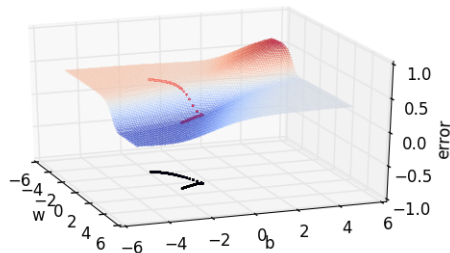
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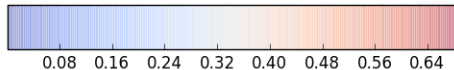
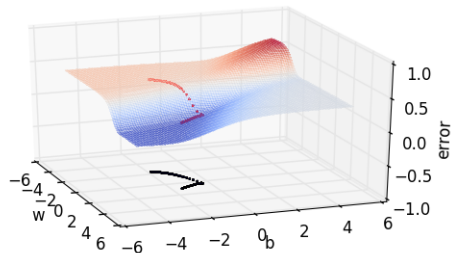
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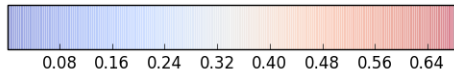
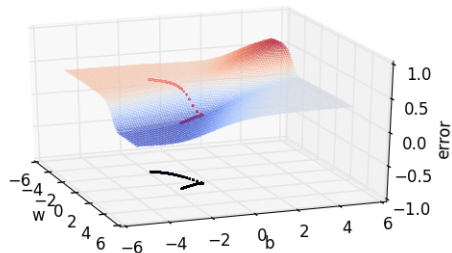
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Y = [0.2, 0.9]

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    return 1.0 / (1.0 + np.exp(-(w*x + b)))

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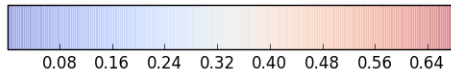
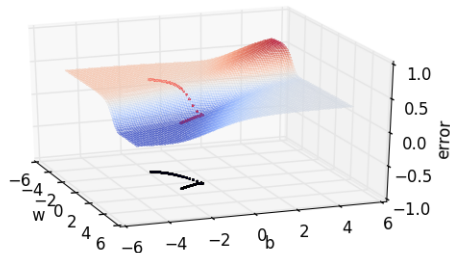
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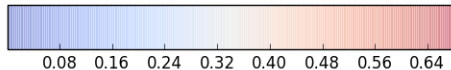
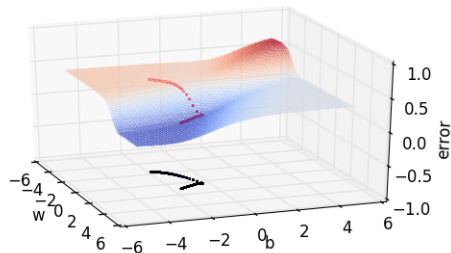
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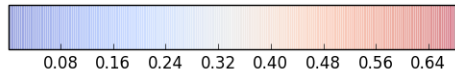
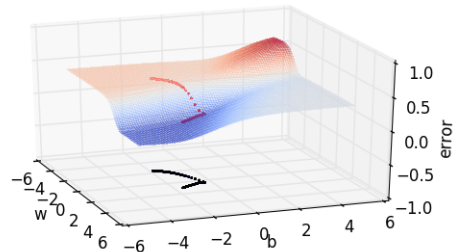
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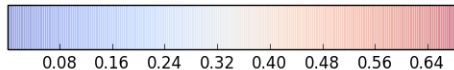
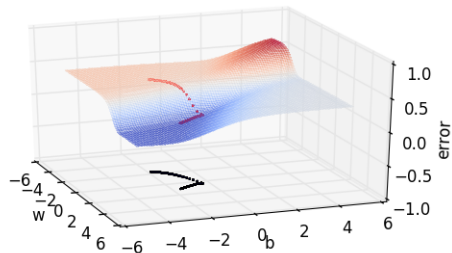
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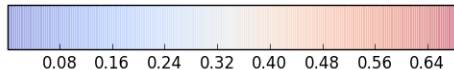
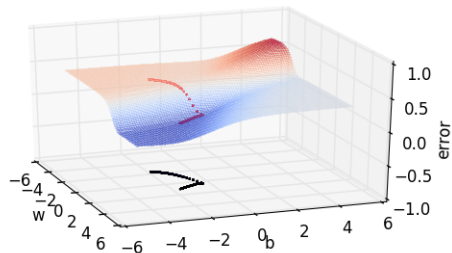
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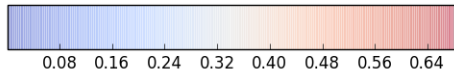
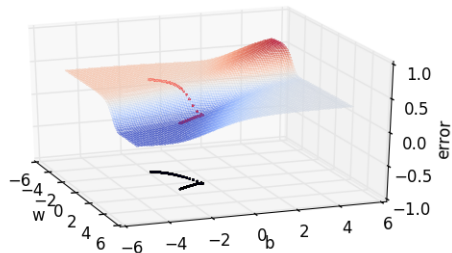
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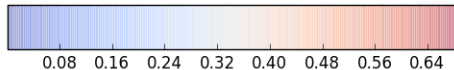
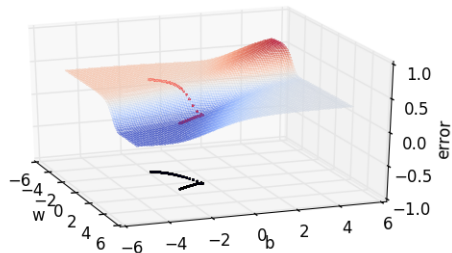
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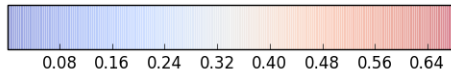
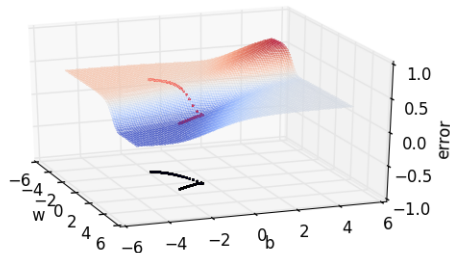
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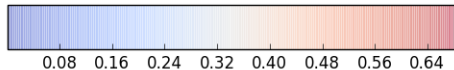
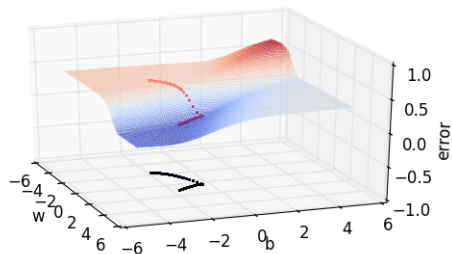
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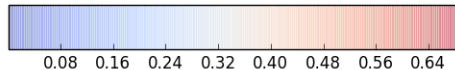
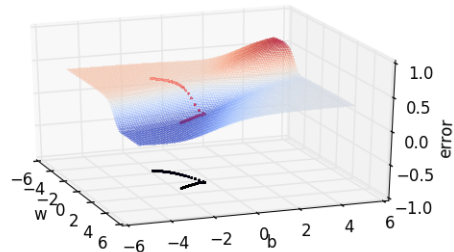
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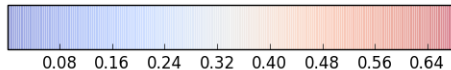
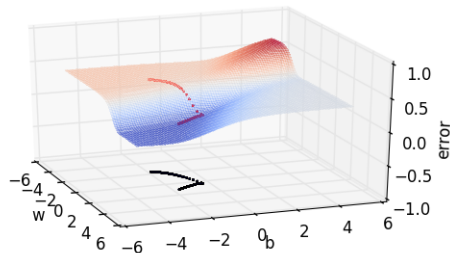
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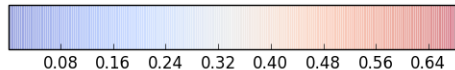
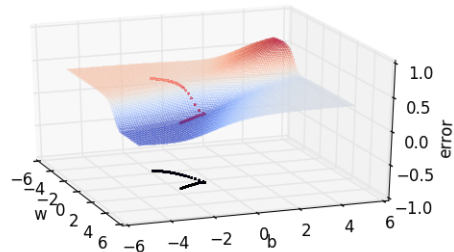
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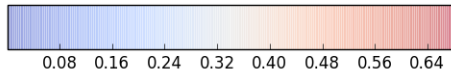
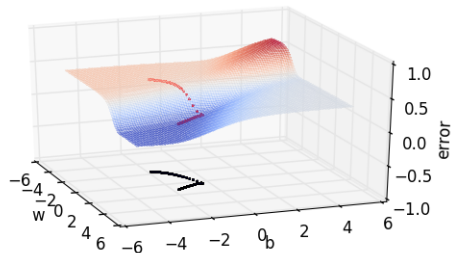
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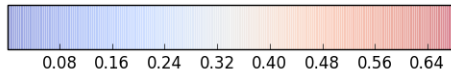
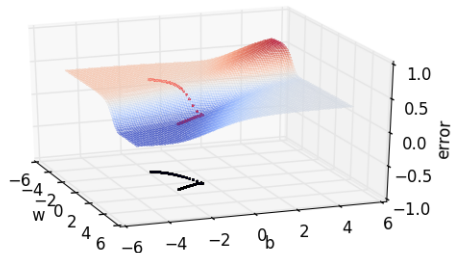
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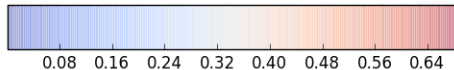
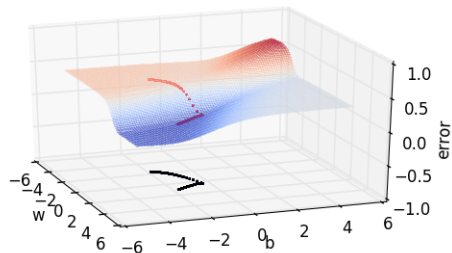
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def do_gradient_descent() :
    w, b, eta, max_epochs = -2, -2, 1.0, 1000
    for i in range(max_epochs) :
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        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
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        w = w - eta * dw
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Gradient descent on the error surface



```

X = [0.5, 2.5]
Y = [0.2, 0.9]

def f(w,b,x) : #sigmoid with parameters w,b
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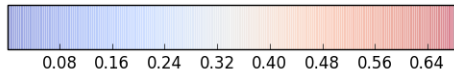
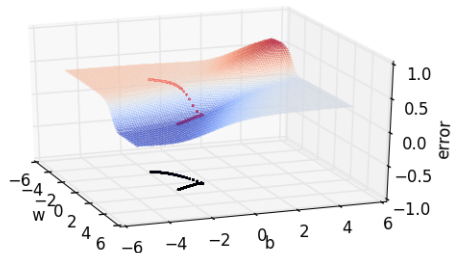
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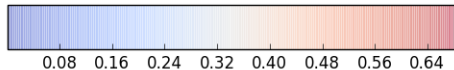
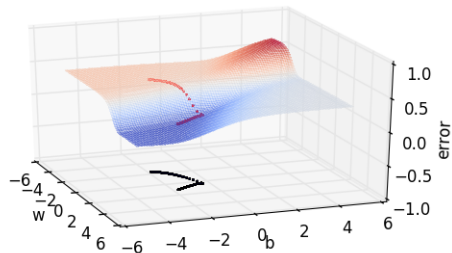
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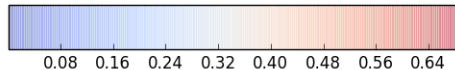
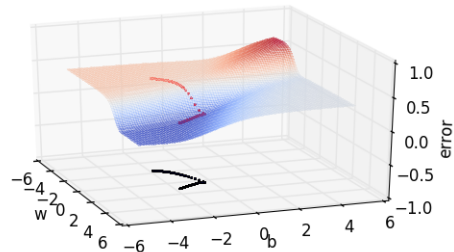
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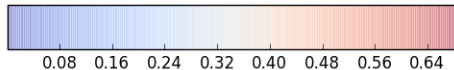
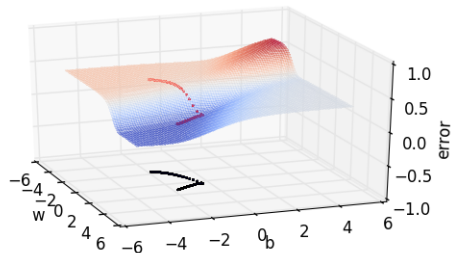
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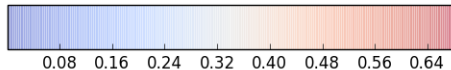
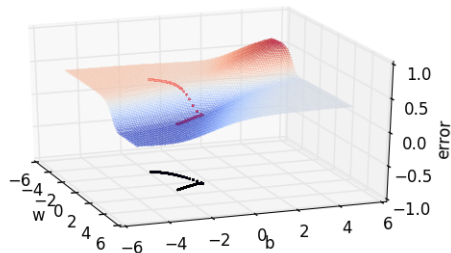
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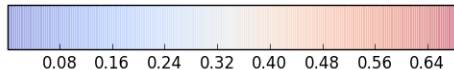
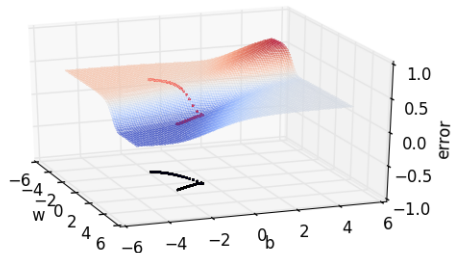
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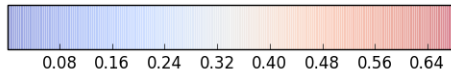
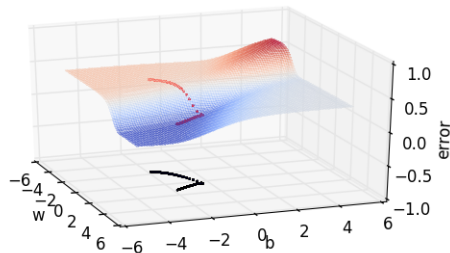
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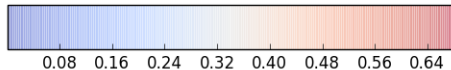
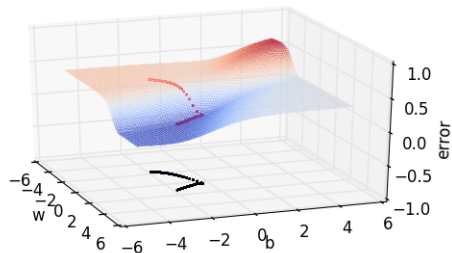
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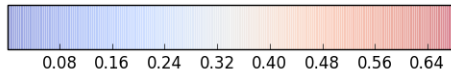
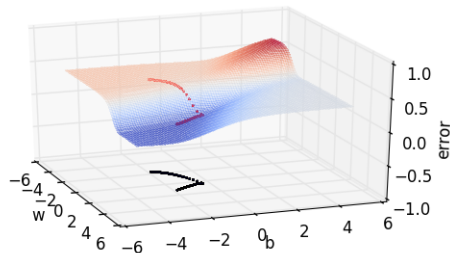
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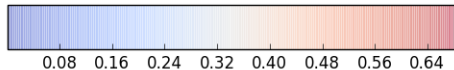
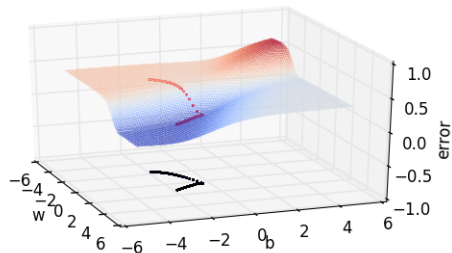
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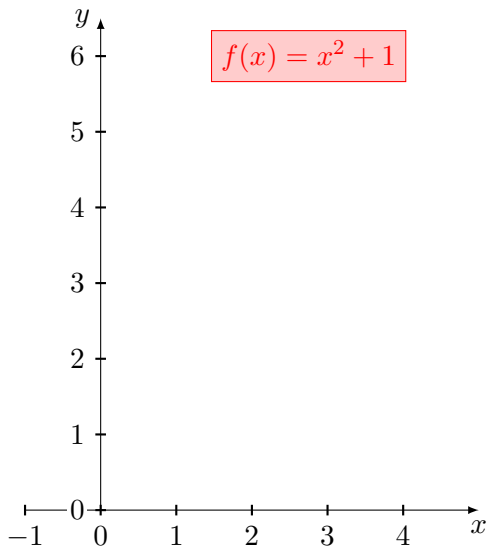
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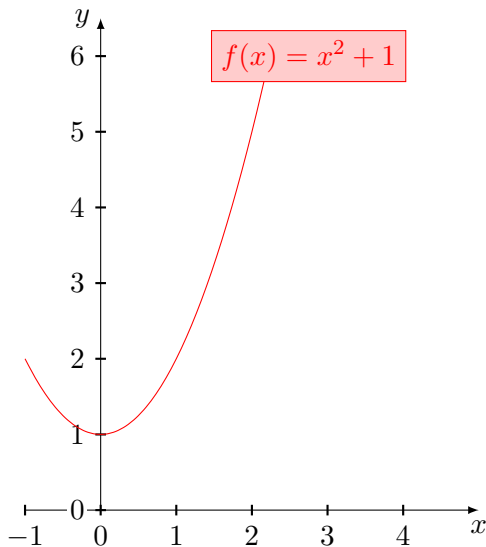
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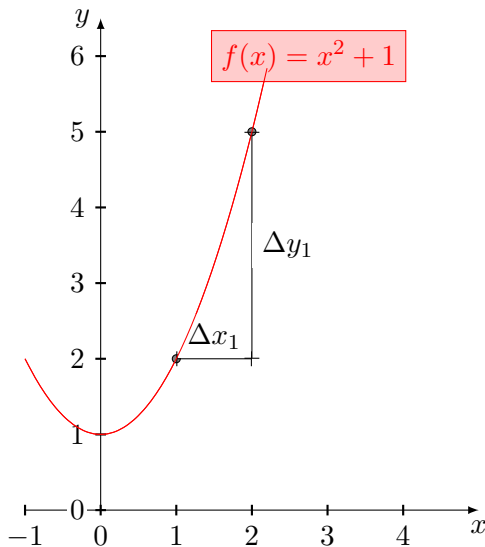
Gradient descent on the error surface



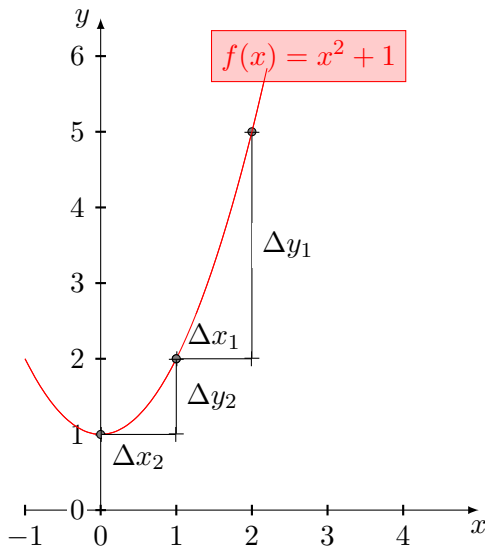




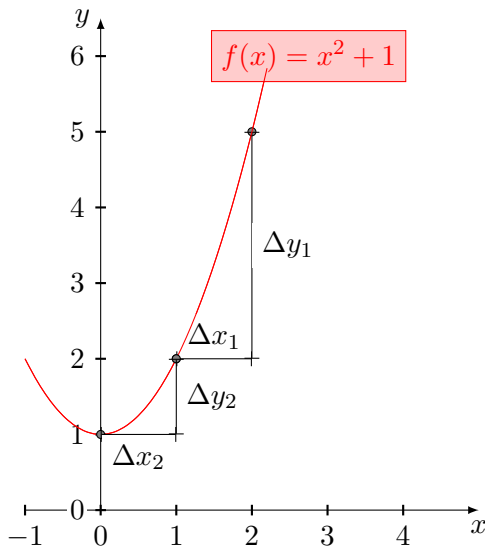
- When the curve is steep the gradient $(\frac{\Delta y_1}{\Delta x_1})$ is large



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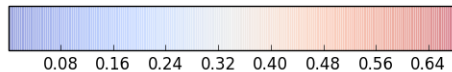
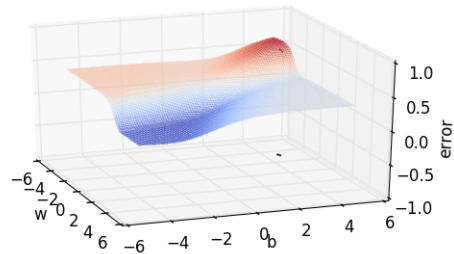


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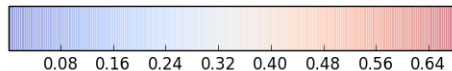
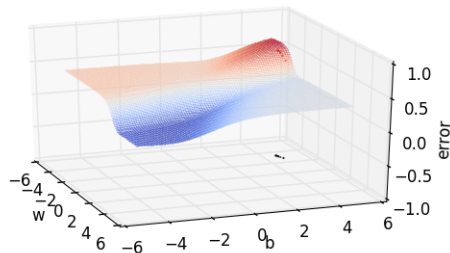


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- When the curve is gentle the gradient $(\frac{\Delta y_2}{\Delta x_2})$ is small
- Recall that our weight updates are proportional to the gradient $w = w - \eta \nabla w$
- Hence in the areas where the curve is gentle the updates are small whereas in the areas where the curve is steep the updates are large

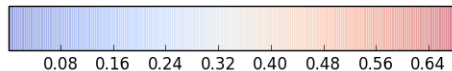
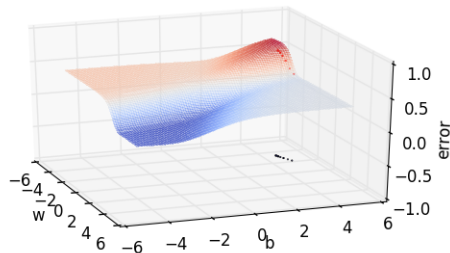
- *Let's see what happens when we start from a different point*



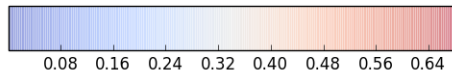
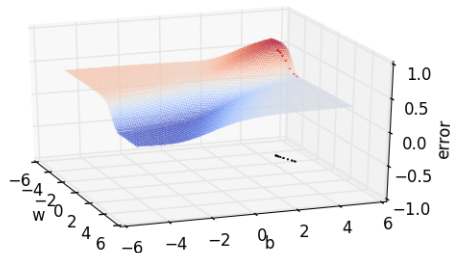
- Irrespective of where we start from once we hit a surface which has a gentle slope, the progress slows down



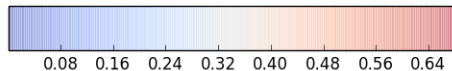
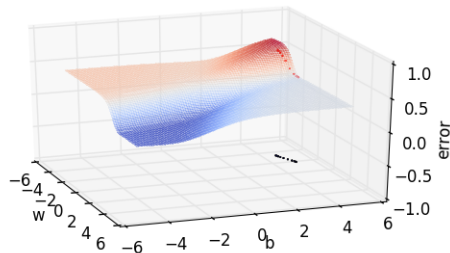
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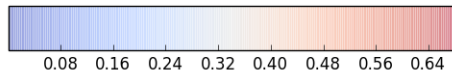
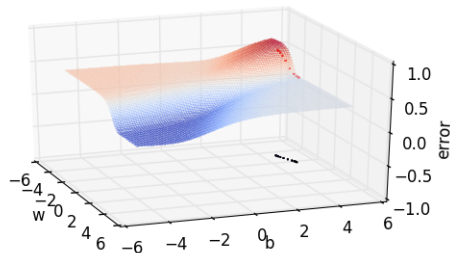
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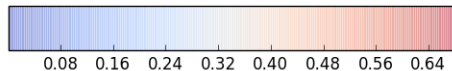
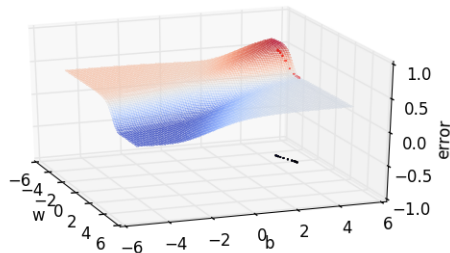
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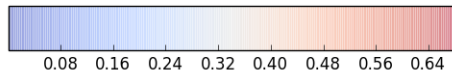
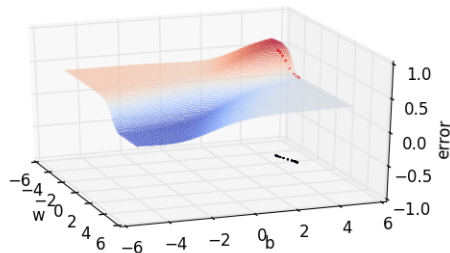
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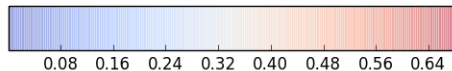
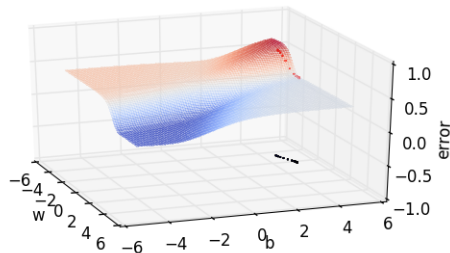
- Irrespective of where we start from once we hit a surface which has a gentle slope, the progress slows down



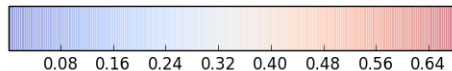
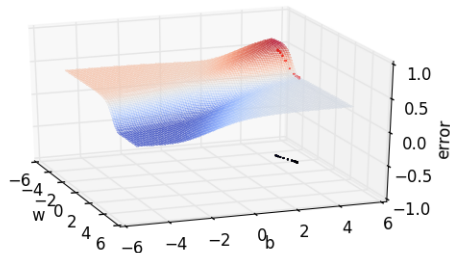
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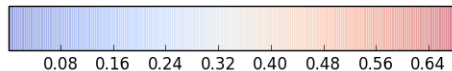
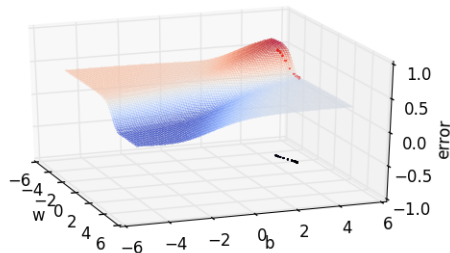
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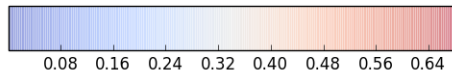
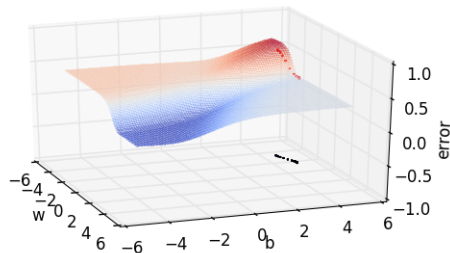
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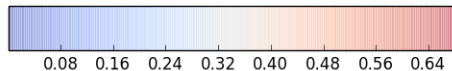
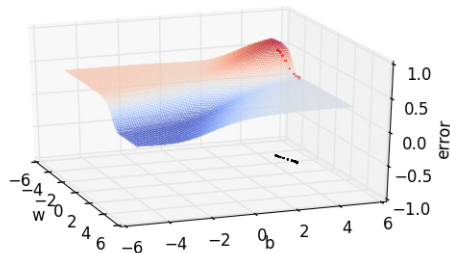
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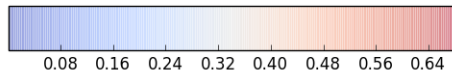
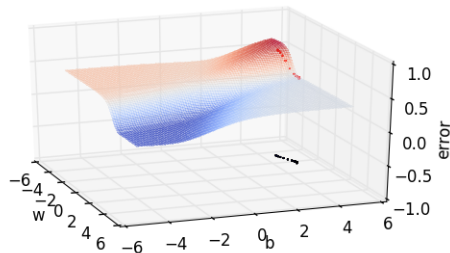
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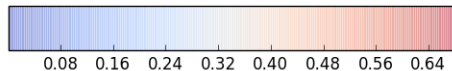
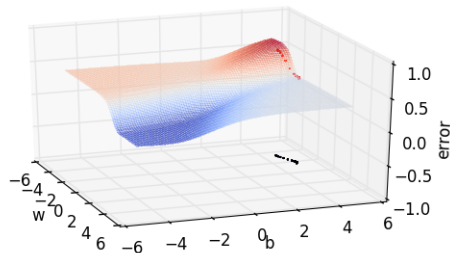
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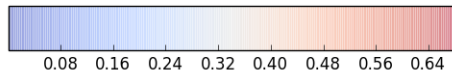
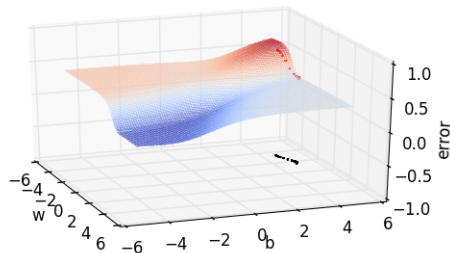
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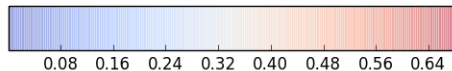
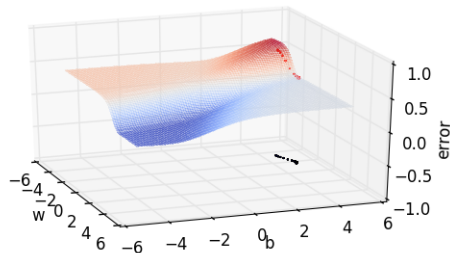
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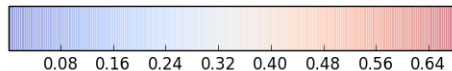
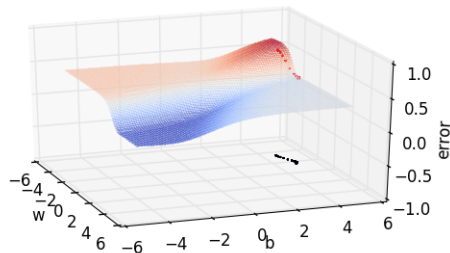
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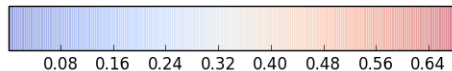
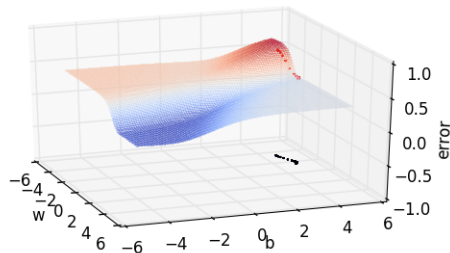
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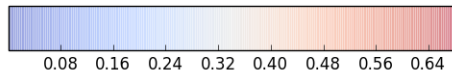
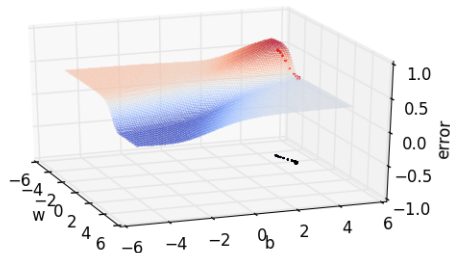
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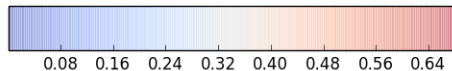
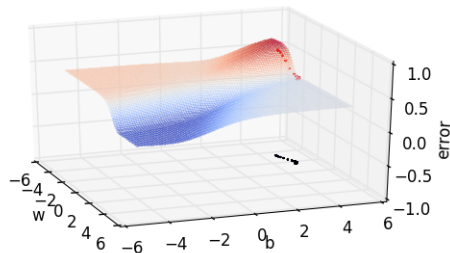
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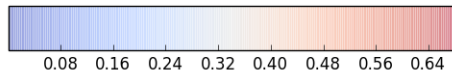
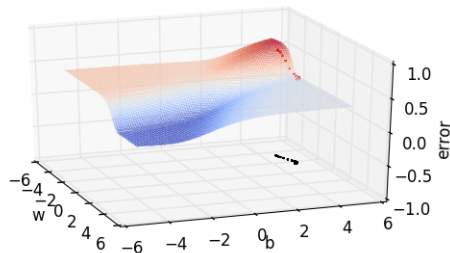
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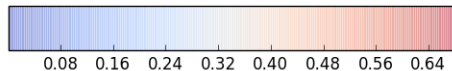
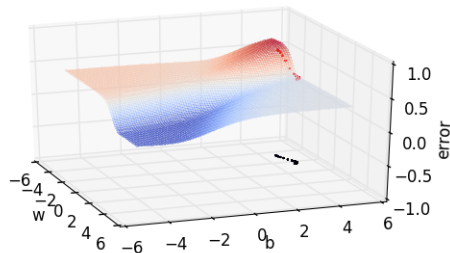
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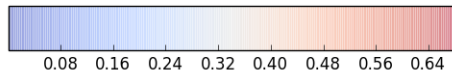
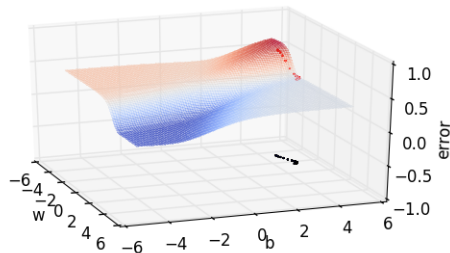
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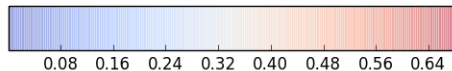
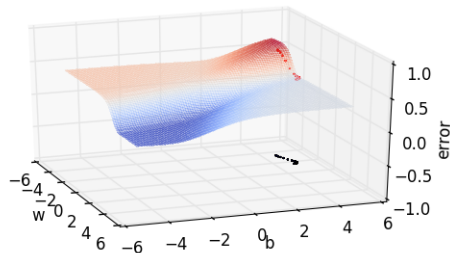
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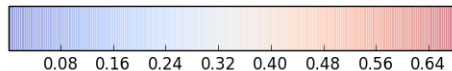
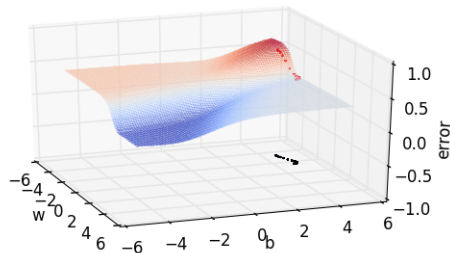
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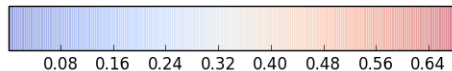
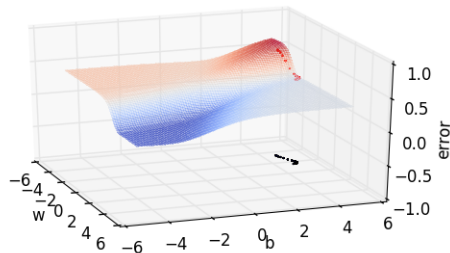
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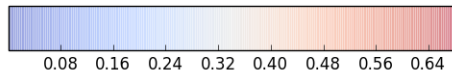
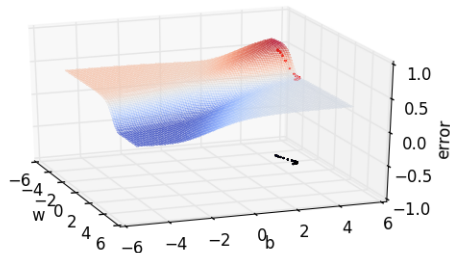
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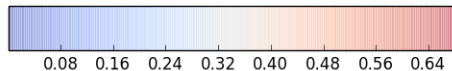
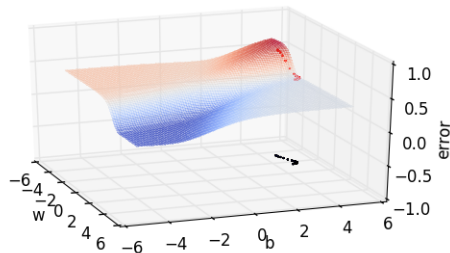
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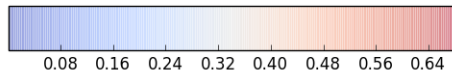
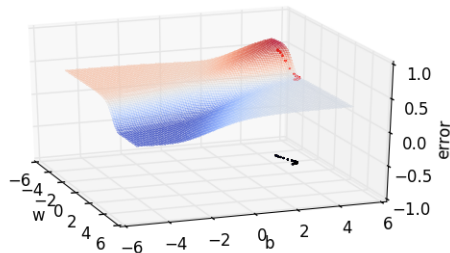
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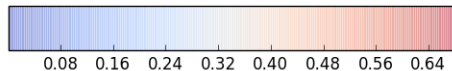
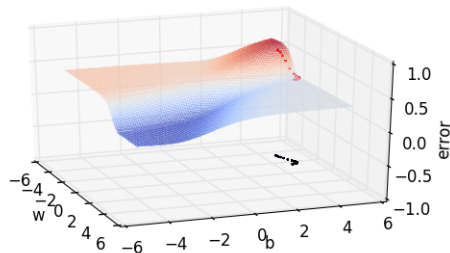
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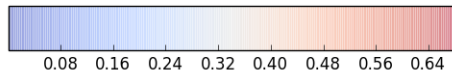
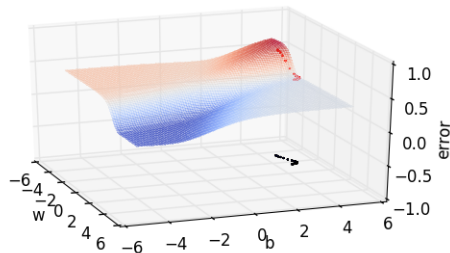
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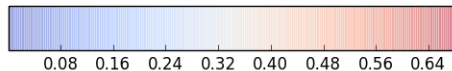
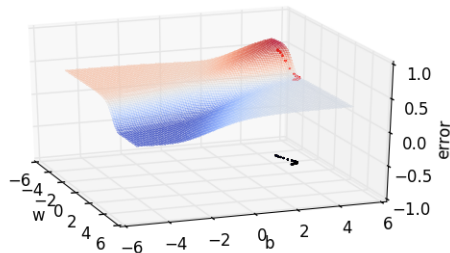
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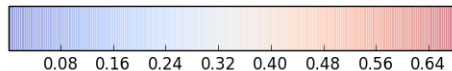
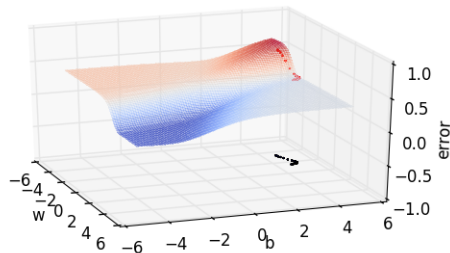
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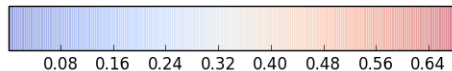
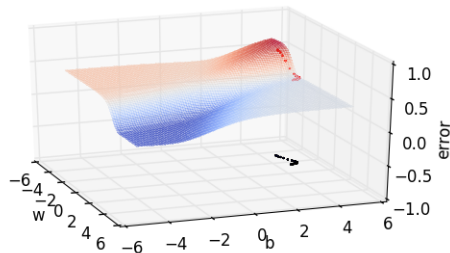
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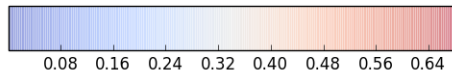
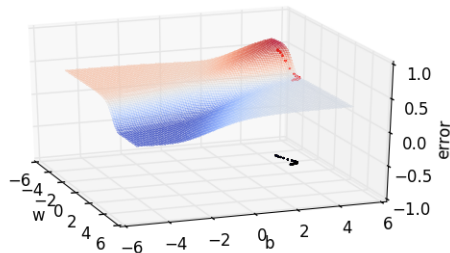
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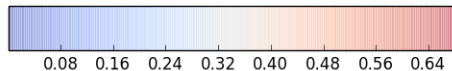
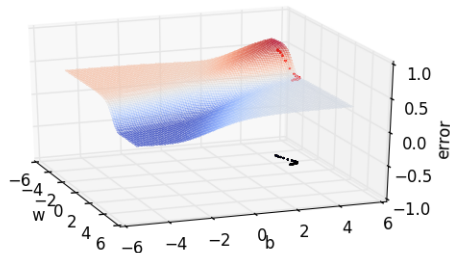
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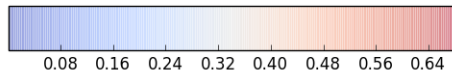
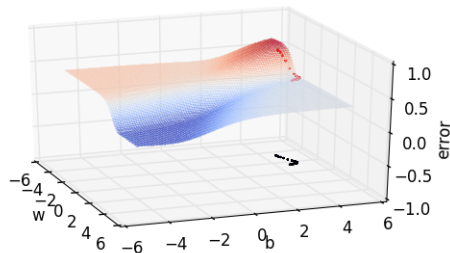
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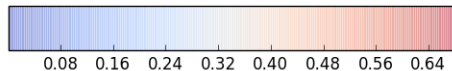
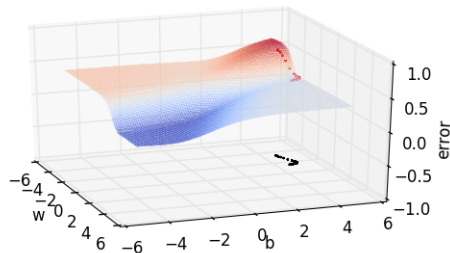
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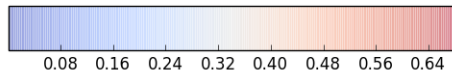
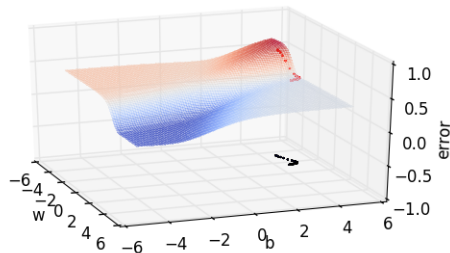
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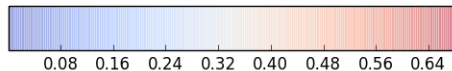
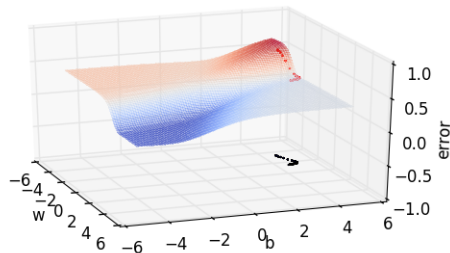
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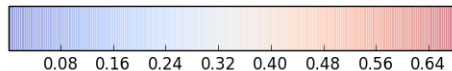
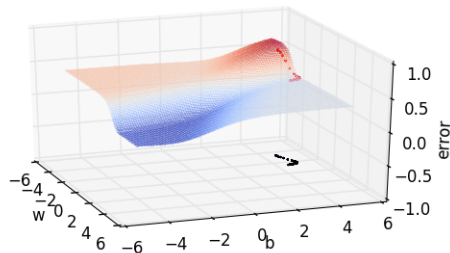
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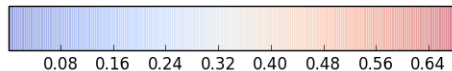
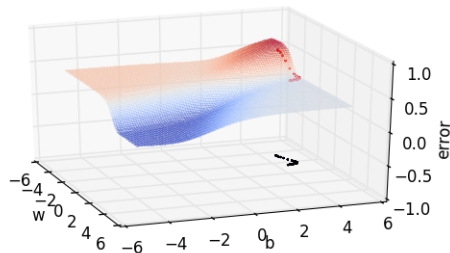
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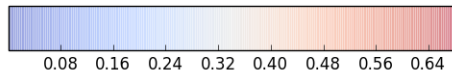
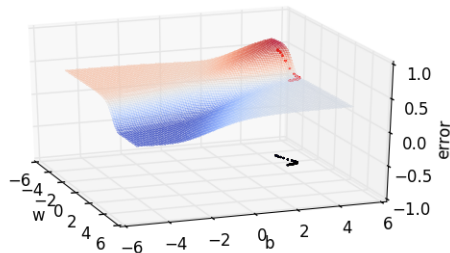
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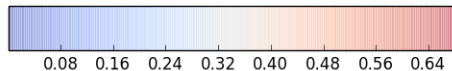
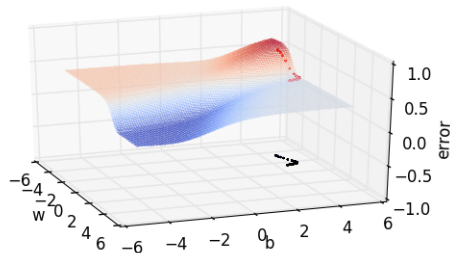
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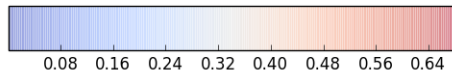
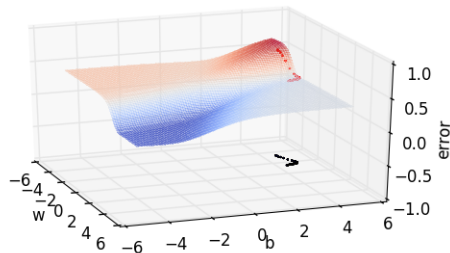
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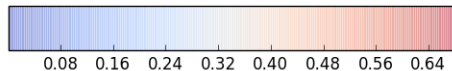
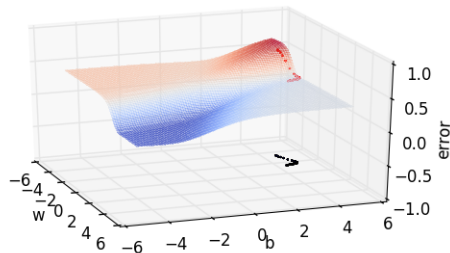
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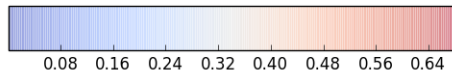
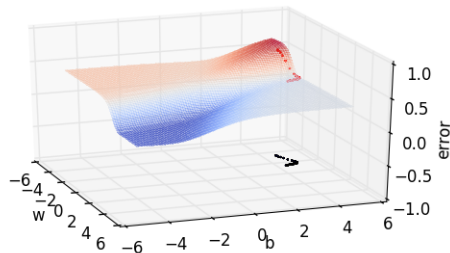
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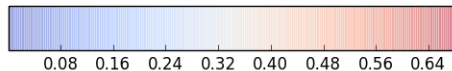
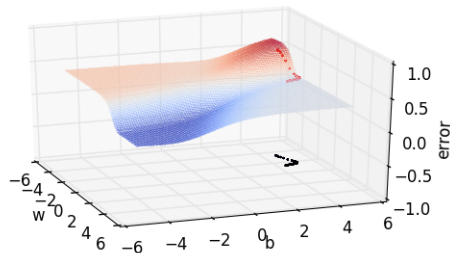
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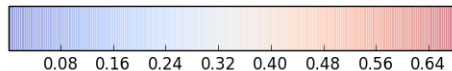
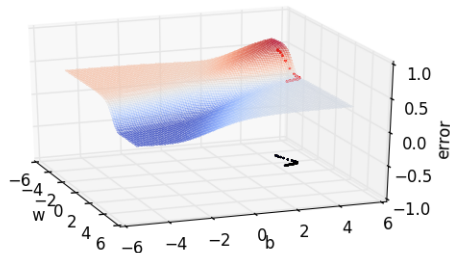
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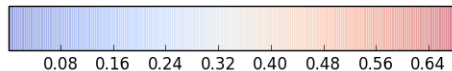
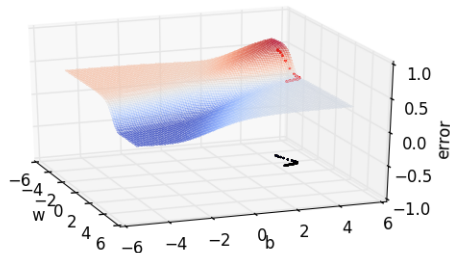
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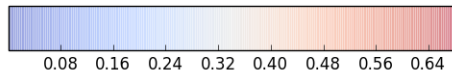
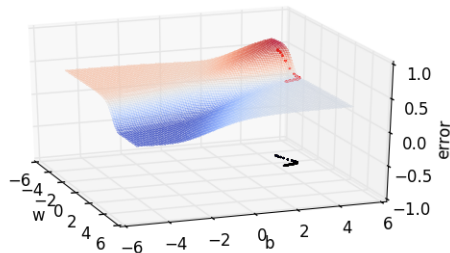
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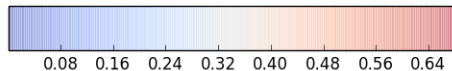
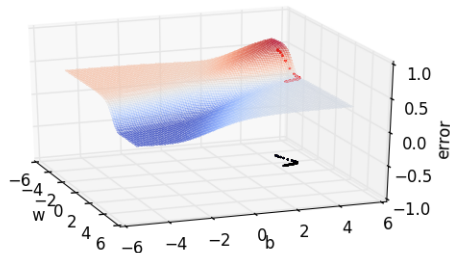
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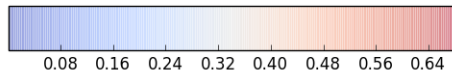
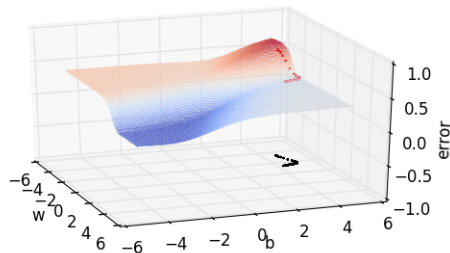
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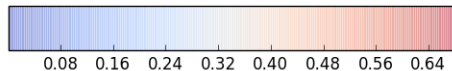
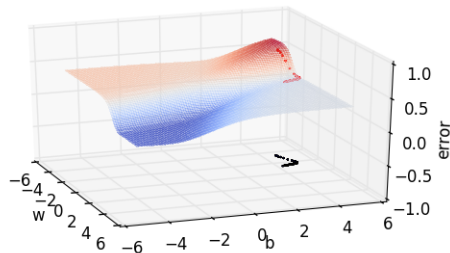
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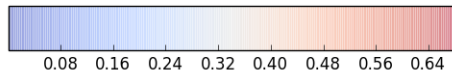
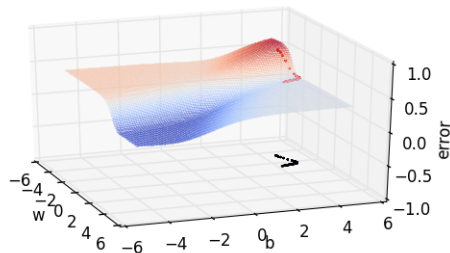
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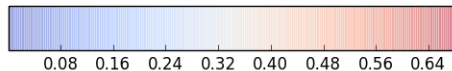
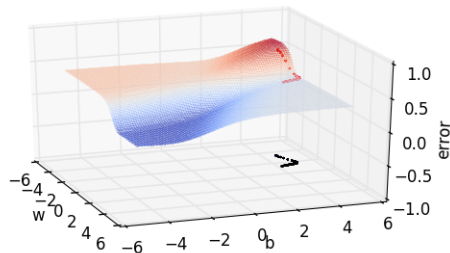
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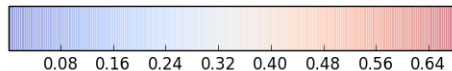
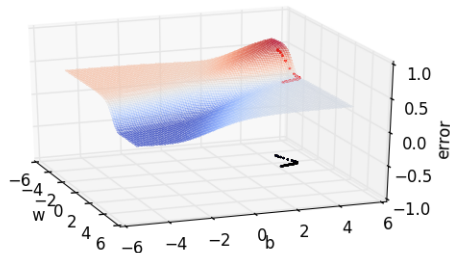
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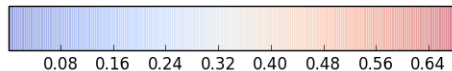
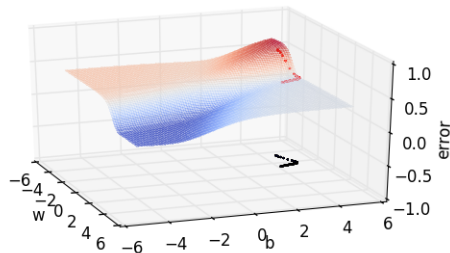
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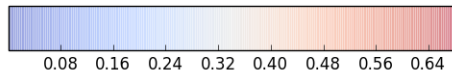
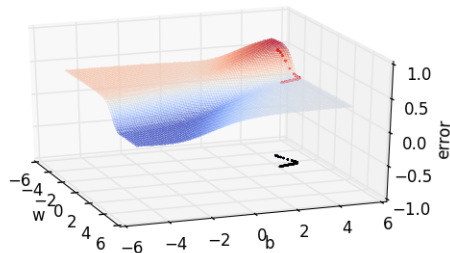
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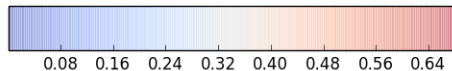
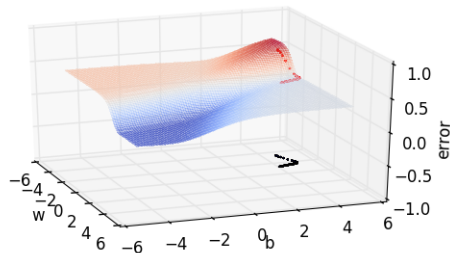
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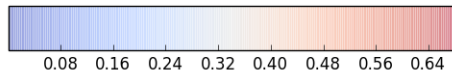
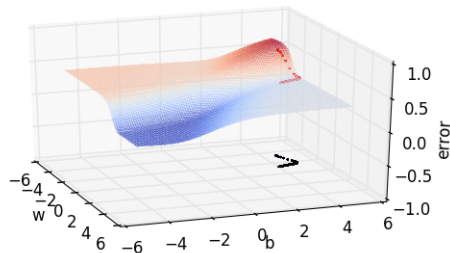
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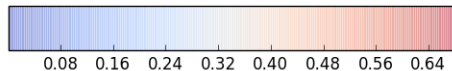
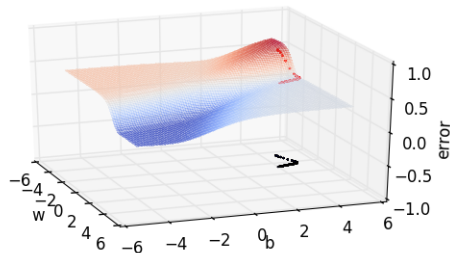
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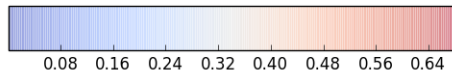
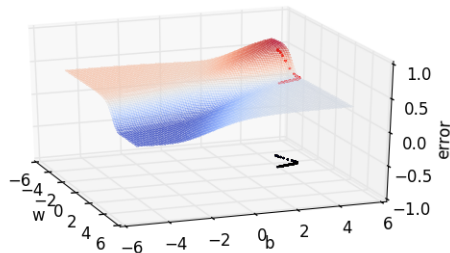
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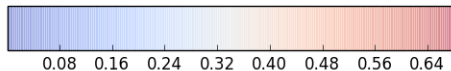
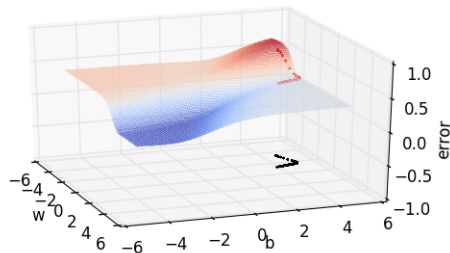
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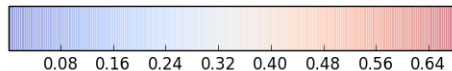
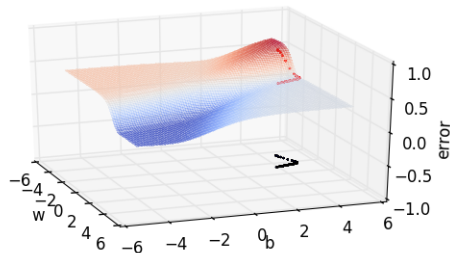
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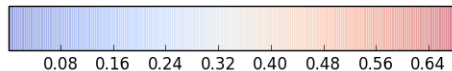
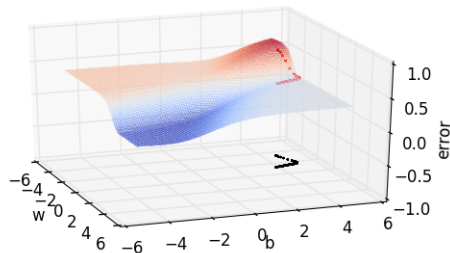
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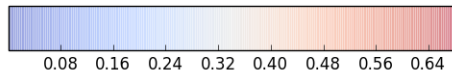
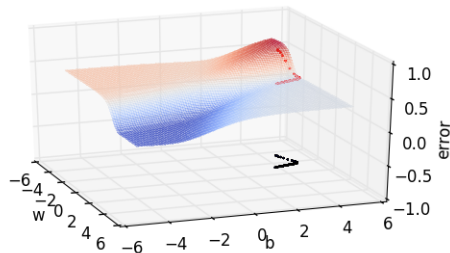
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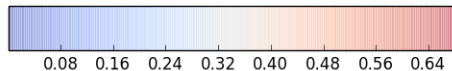
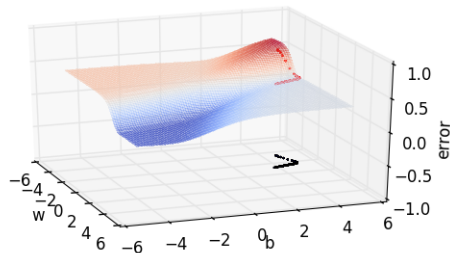
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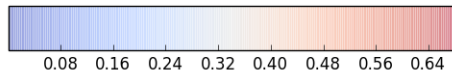
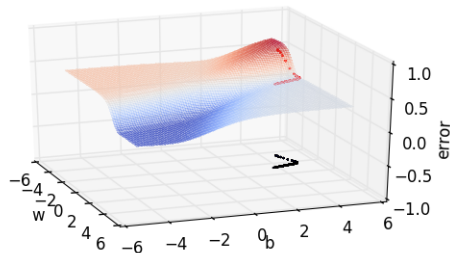
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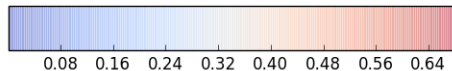
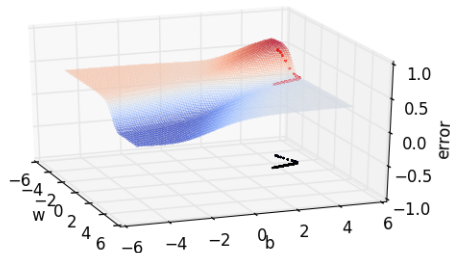
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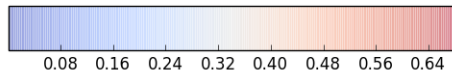
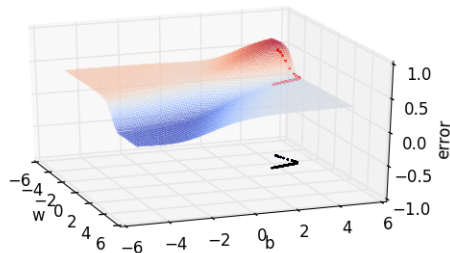
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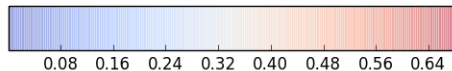
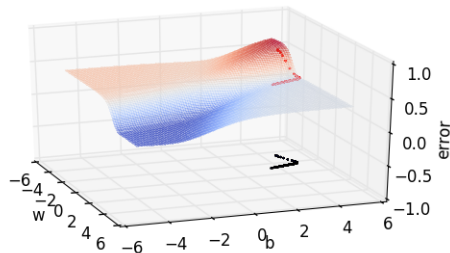
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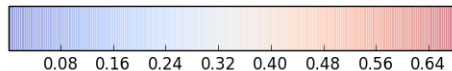
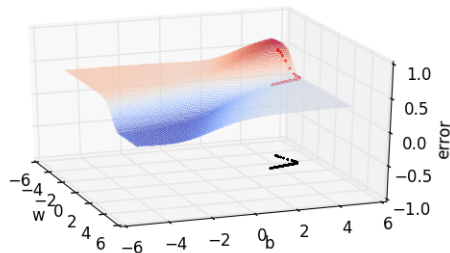
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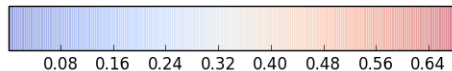
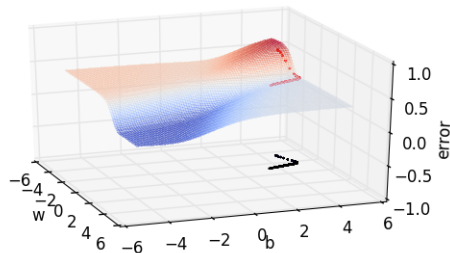
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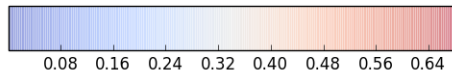
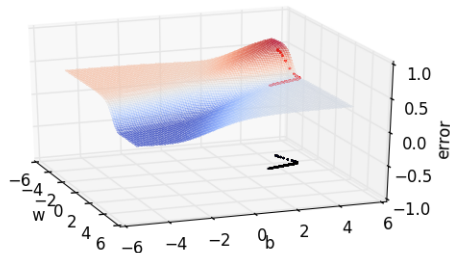
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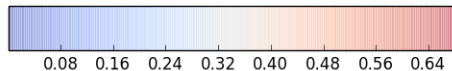
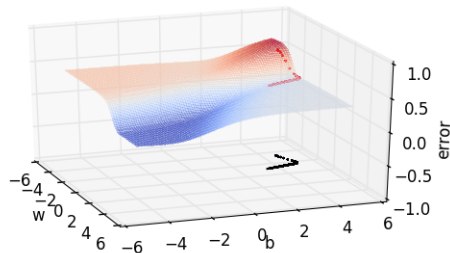
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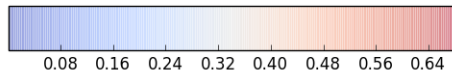
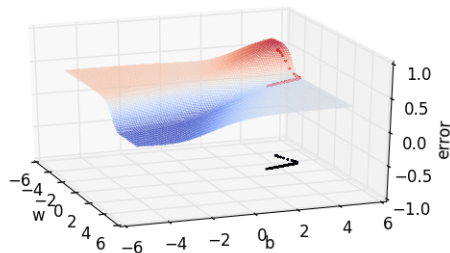
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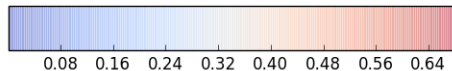
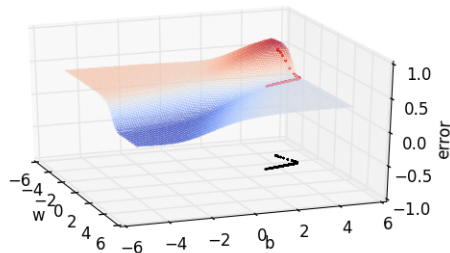
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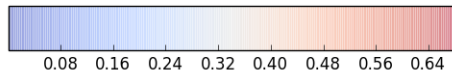
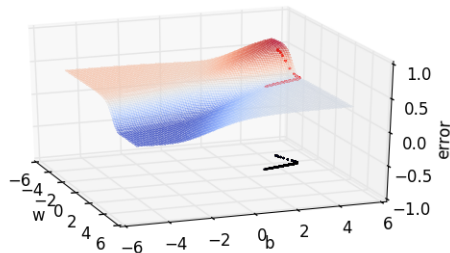
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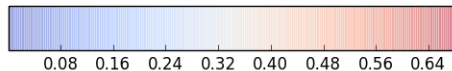
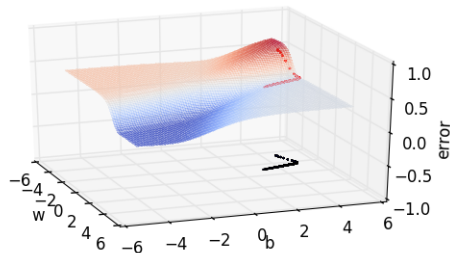
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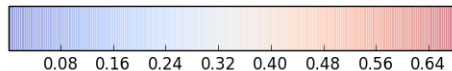
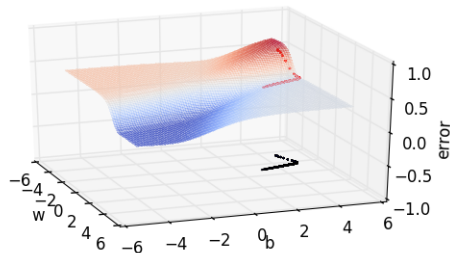
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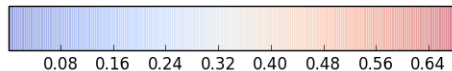
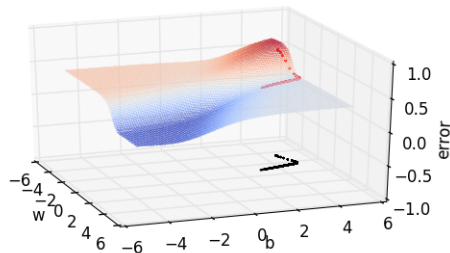
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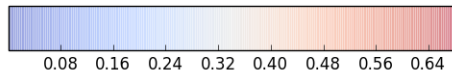
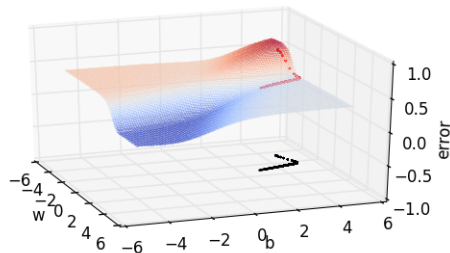
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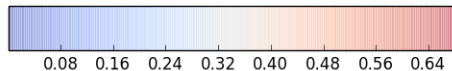
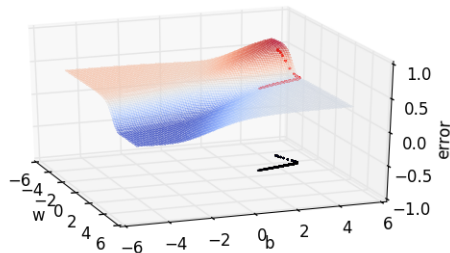
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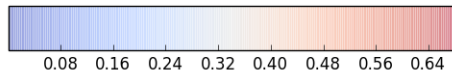
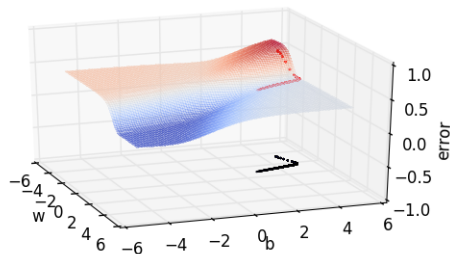
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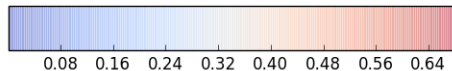
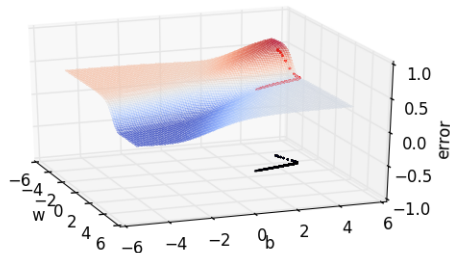
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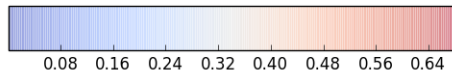
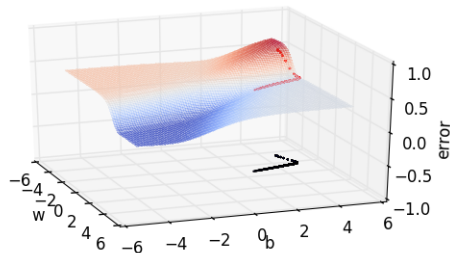
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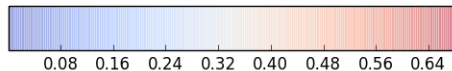
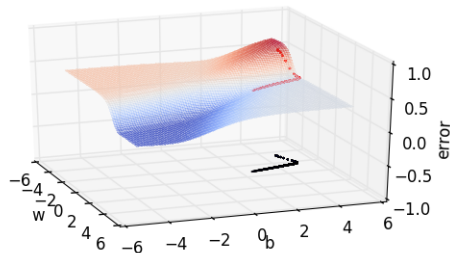
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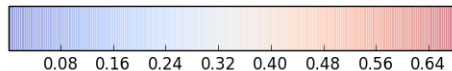
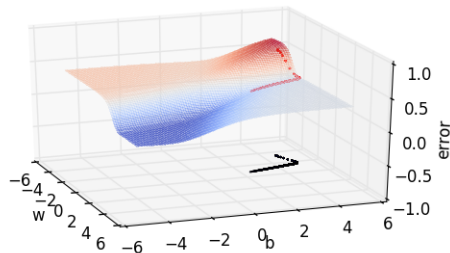
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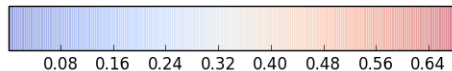
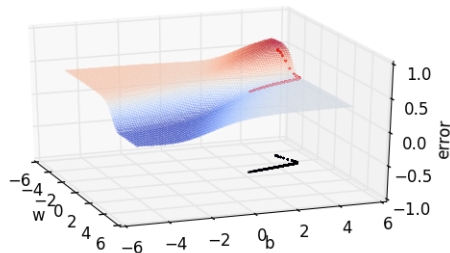
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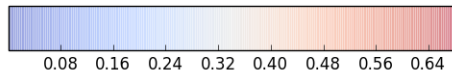
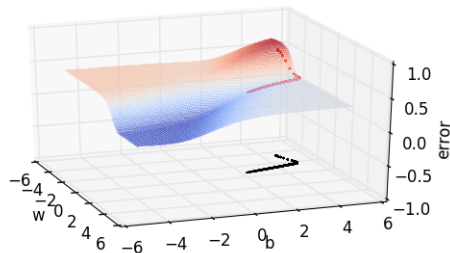
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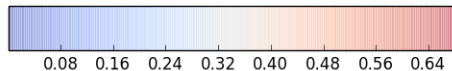
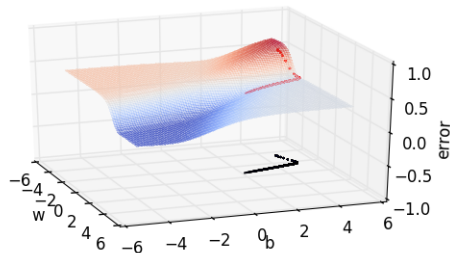
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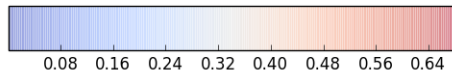
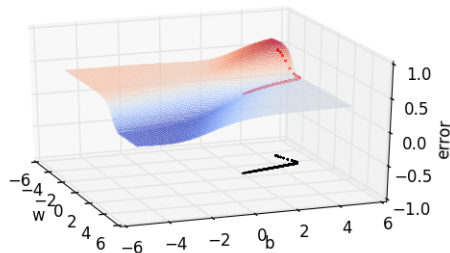
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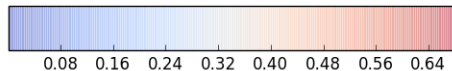
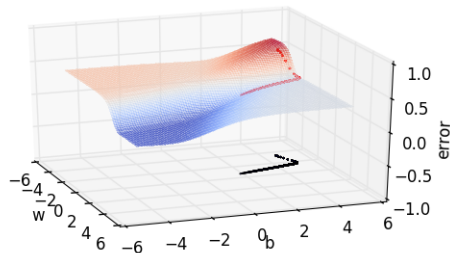
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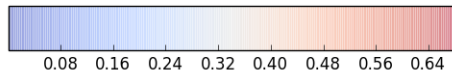
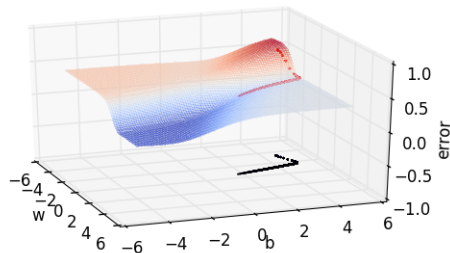
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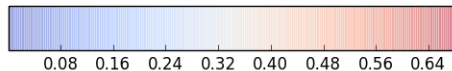
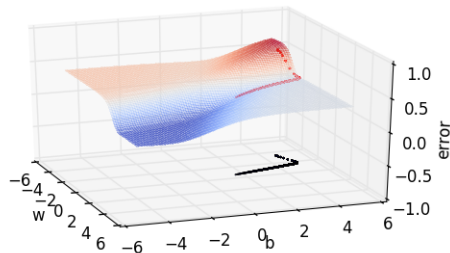
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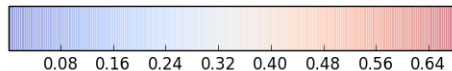
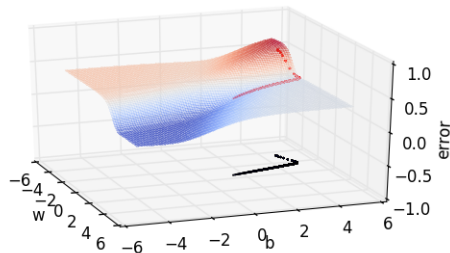
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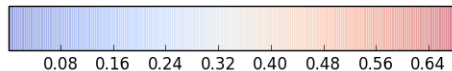
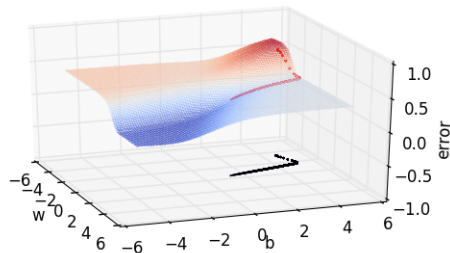
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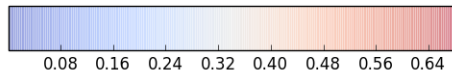
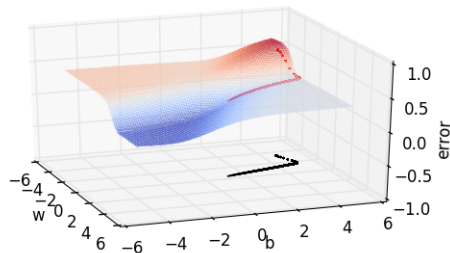
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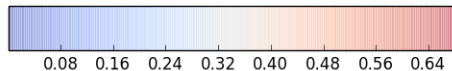
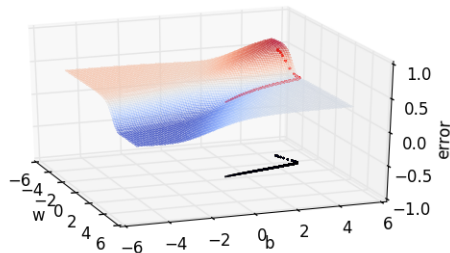
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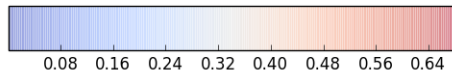
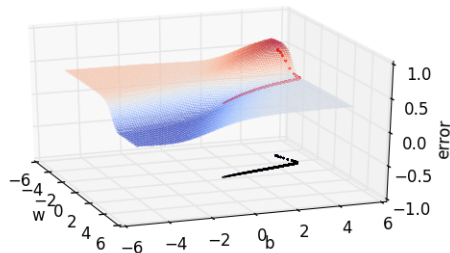
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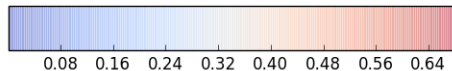
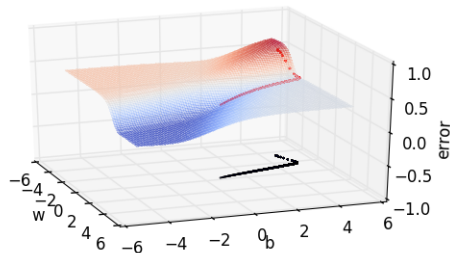
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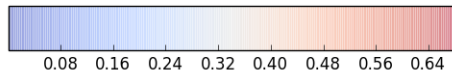
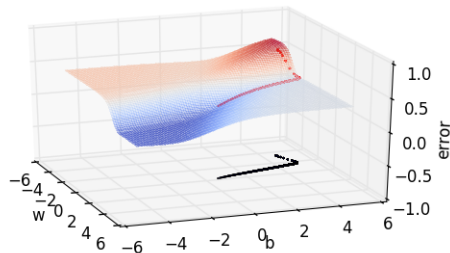
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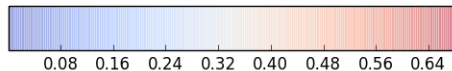
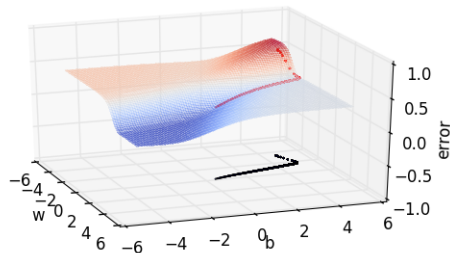
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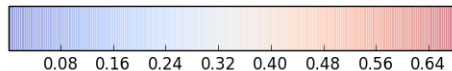
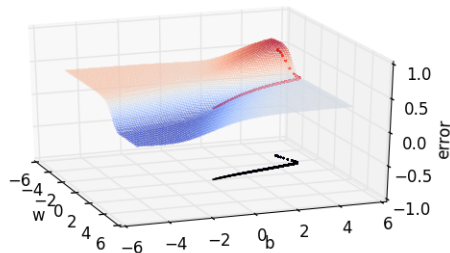
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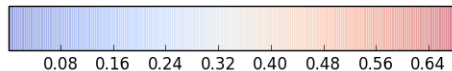
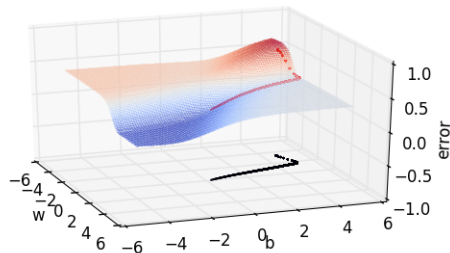
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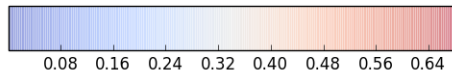
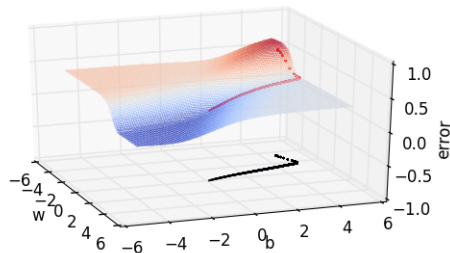
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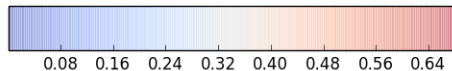
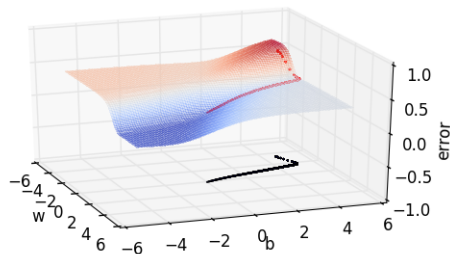
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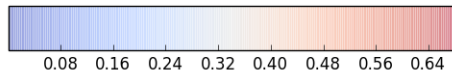
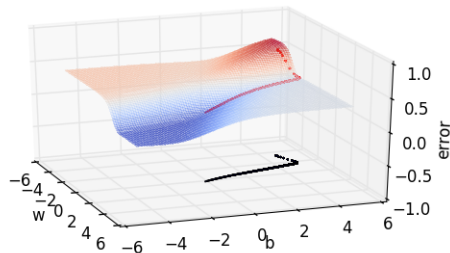
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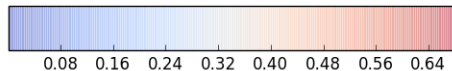
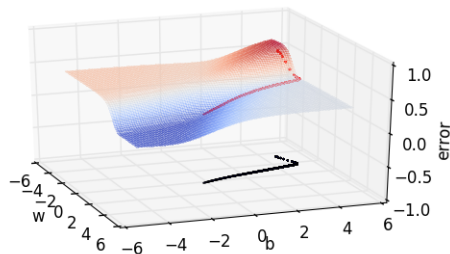
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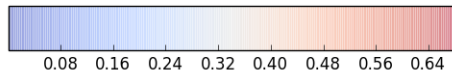
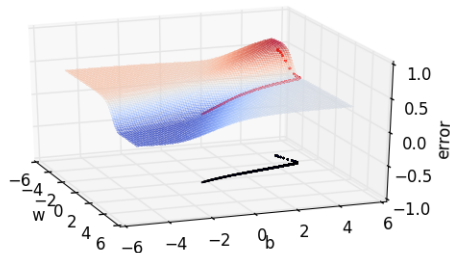
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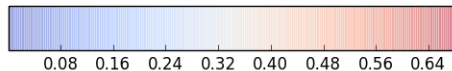
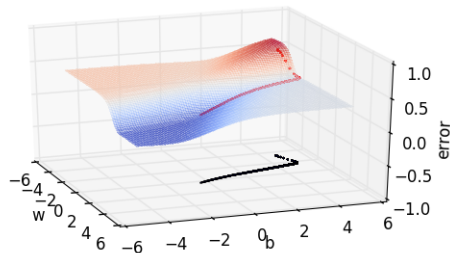
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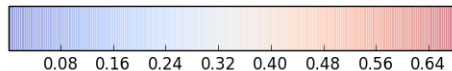
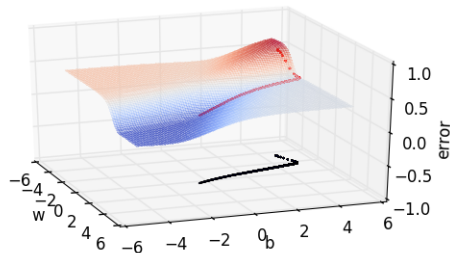
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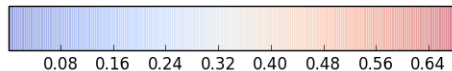
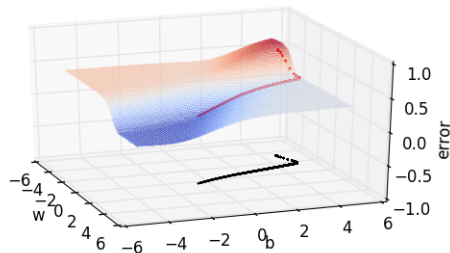
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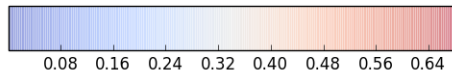
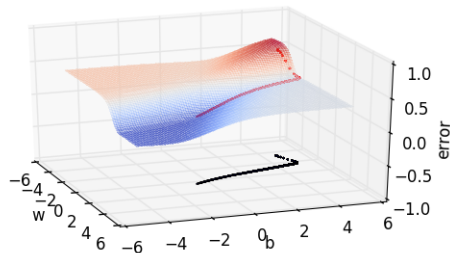
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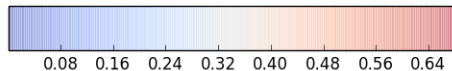
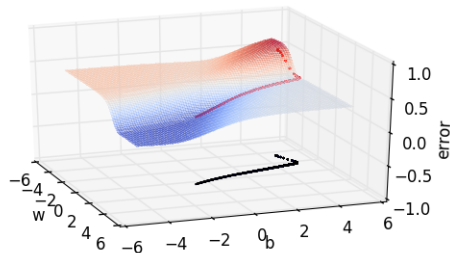
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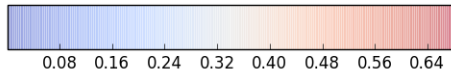
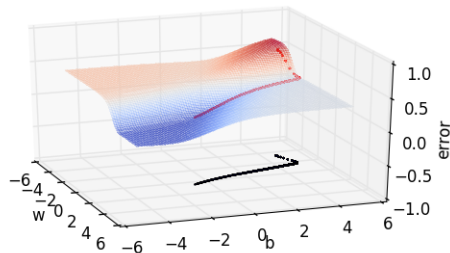
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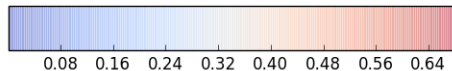
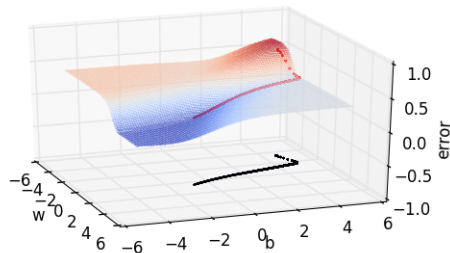
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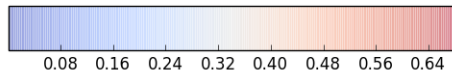
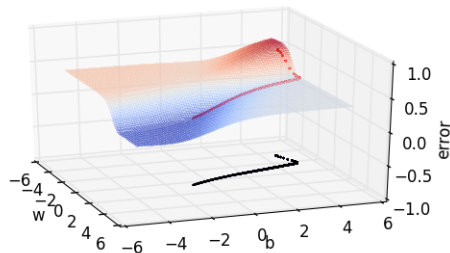
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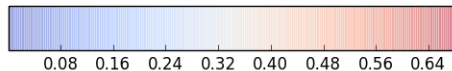
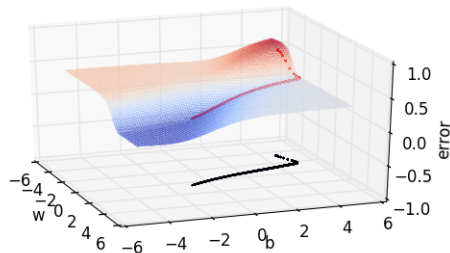
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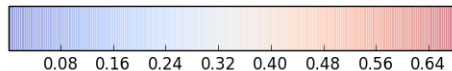
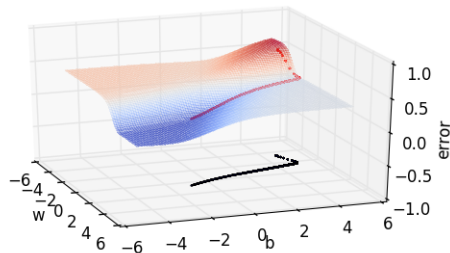
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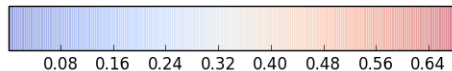
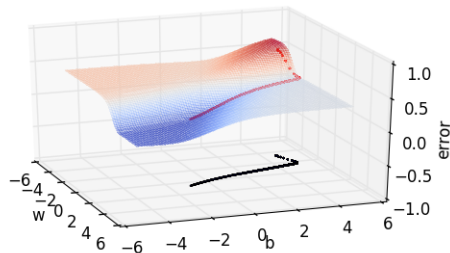
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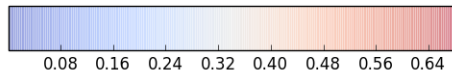
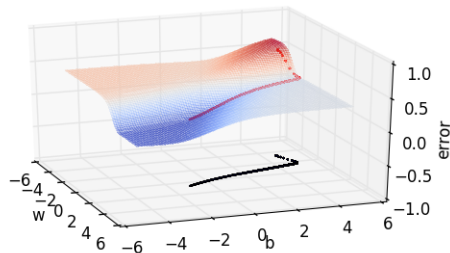
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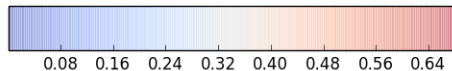
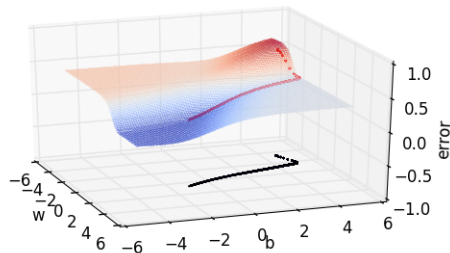
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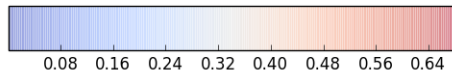
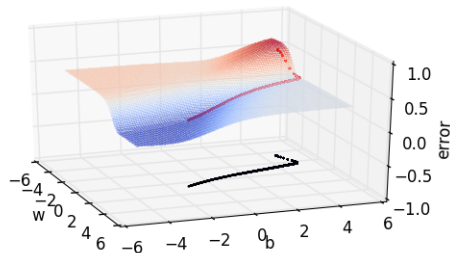
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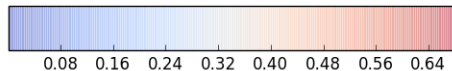
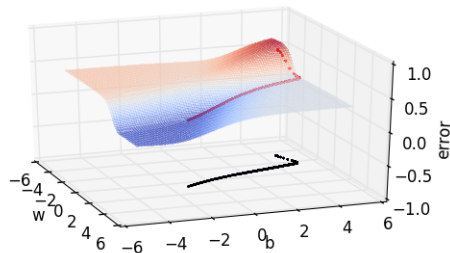
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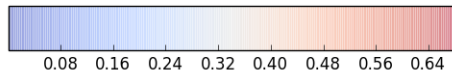
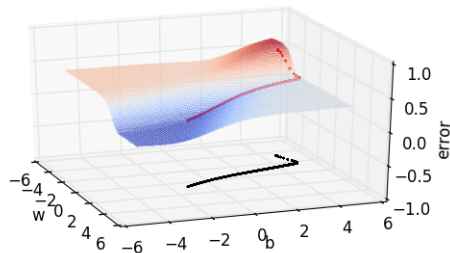
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