Module 5.4 : Momentum based Gradient Descent

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- It takes a lot of time to navigate regions having a gentle slope
- This is because the gradient in these regions is very small
- Can we do something better ?
- Yes, let's take a look at 'Momentum based gradient descent'

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Update rule for momentum based gradient descent

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- In addition to the current update, also look at the history of updates.

$$update_t = \gamma \cdot update_{t-1} + \eta \nabla w_t$$
$$w_{t+1} = w_t - update_t$$

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$$update_1 = \gamma \cdot update_0 + \eta \nabla w_1 = \eta \nabla w_1$$
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$$update_3 = \gamma \cdot update_2 + \eta \nabla w_3 = \gamma(\gamma \cdot \eta \nabla w_1 + \eta \nabla w_2) + \eta \nabla w_3$$

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$$= \gamma \cdot update_2 + \eta \nabla w_3 = \gamma^2 \cdot \eta \nabla w_1 + \gamma \cdot \eta \nabla w_2 + \eta \nabla w_3$$

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update_3 &= \gamma \cdot update_2 + \eta \nabla w_3 = \gamma(\gamma \cdot \eta \nabla w_1 + \eta \nabla w_2) + \eta \nabla w_3 \\
&= \gamma \cdot update_2 + \eta \nabla w_3 = \gamma^2 \cdot \eta \nabla w_1 + \gamma \cdot \eta \nabla w_2 + \eta \nabla w_3 \\
update_4 &= \gamma \cdot update_3 + \eta \nabla w_4 = \gamma^3 \cdot \eta \nabla w_1 + \gamma^2 \cdot \eta \nabla w_2 + \gamma \cdot \eta \nabla w_3 + \eta \nabla w_4
\end{aligned}$$

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update_t &= \gamma \cdot update_{t-1} + \eta \nabla w_t = \gamma^{t-1} \cdot \eta \nabla w_1 + \gamma^{t-2} \cdot \eta \nabla w_1 + ... + \eta \nabla w_t
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```
def do_momentum_gradient_descent() :
    w, b, eta = init_w, init_b, 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = gamma * prev_v_w + eta* dw
        v_b = gamma * prev_v_b + eta* db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
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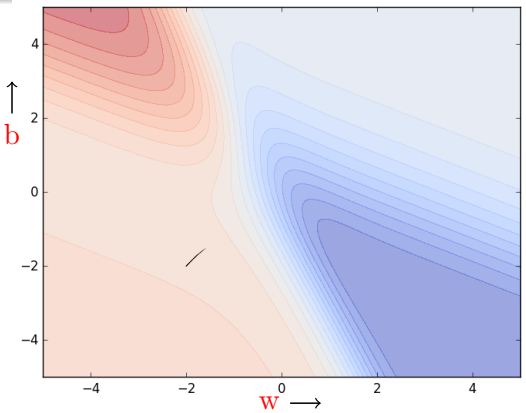
```



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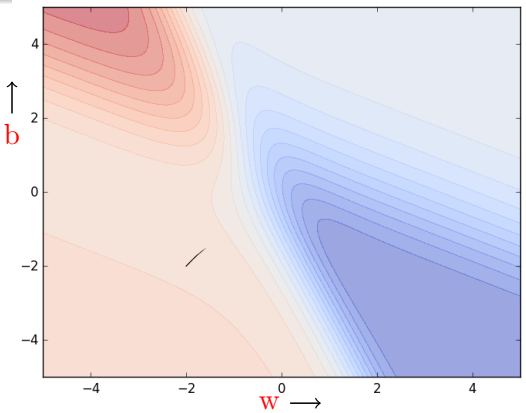

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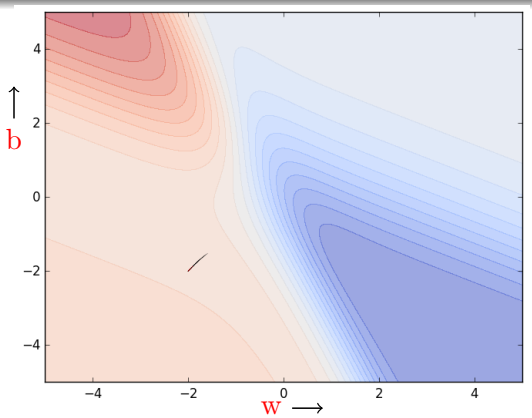

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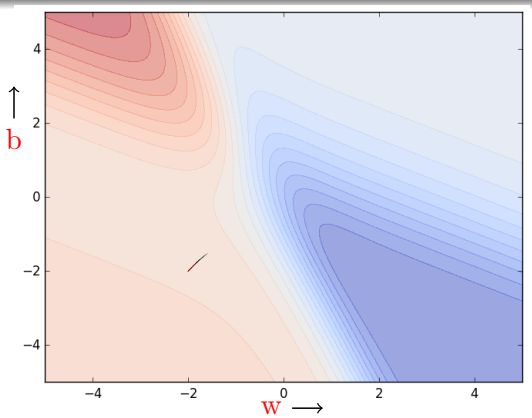

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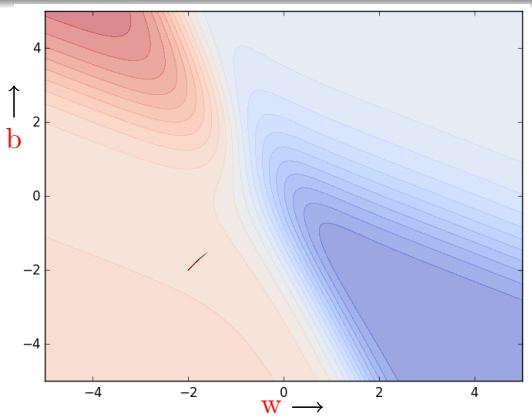

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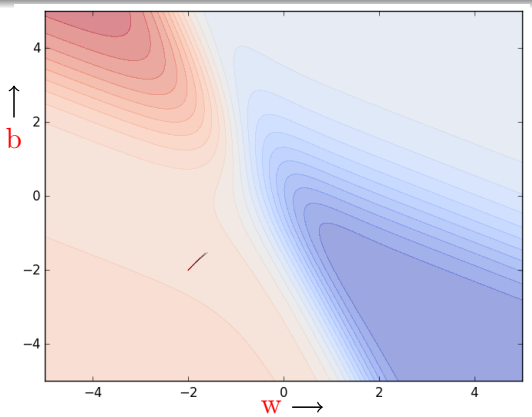

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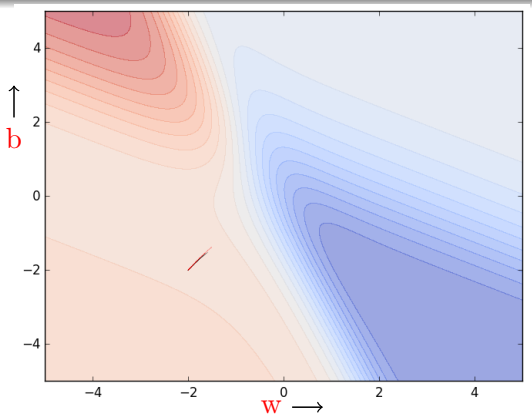

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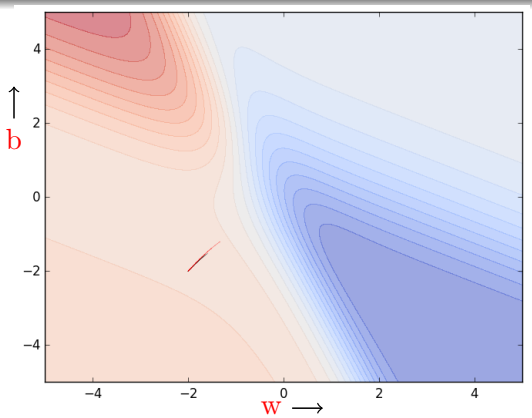

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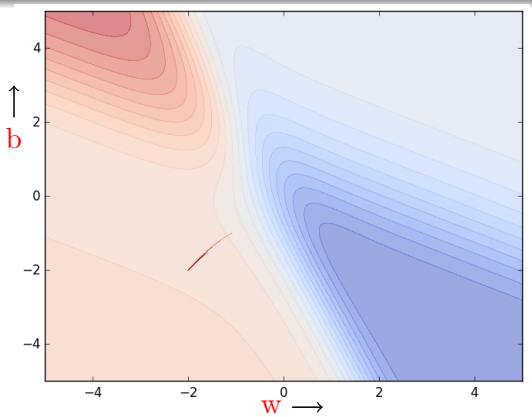

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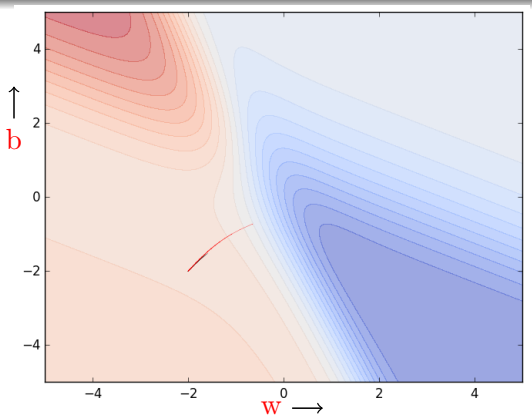

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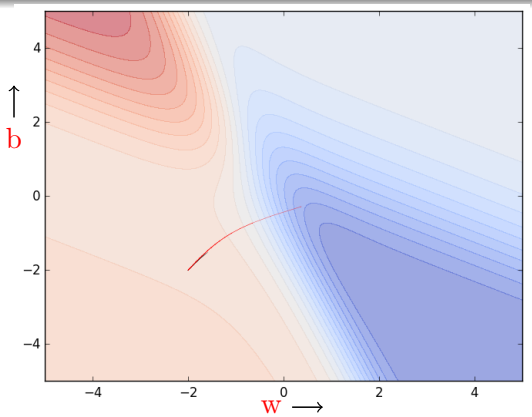

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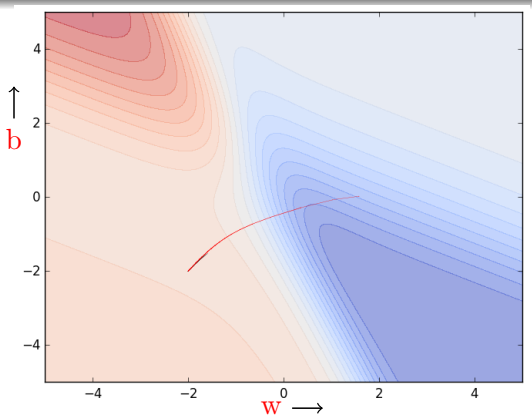

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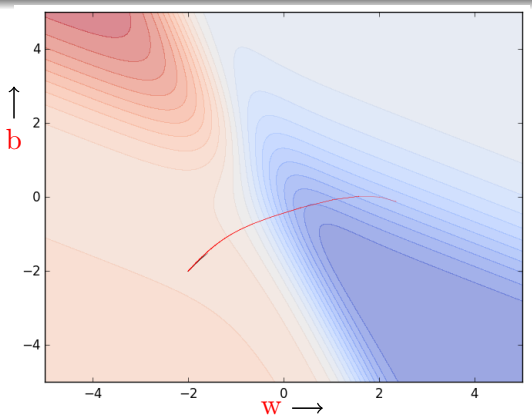

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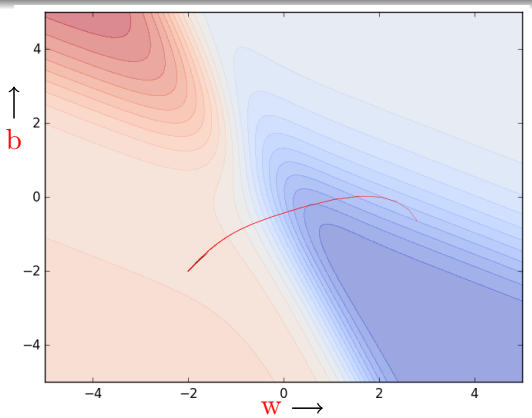

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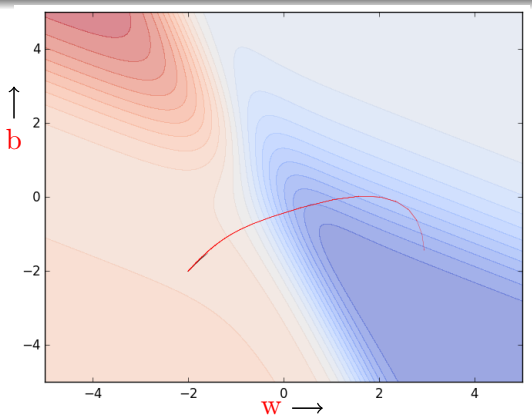

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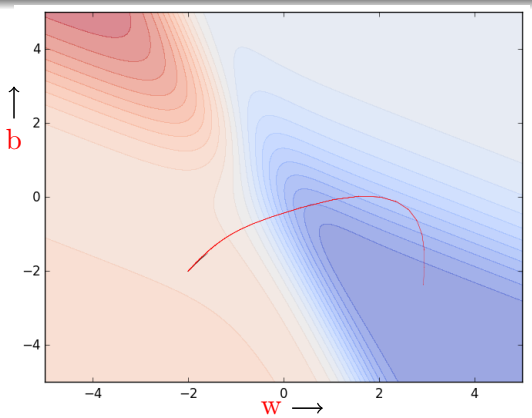

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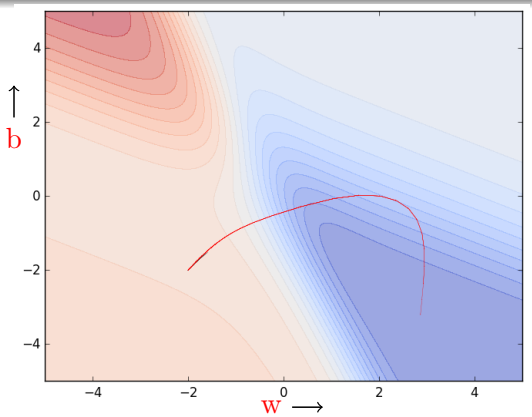

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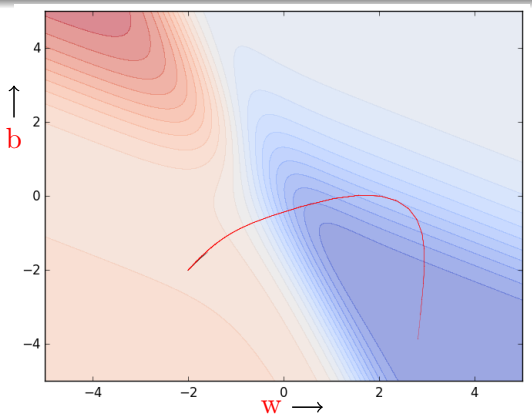
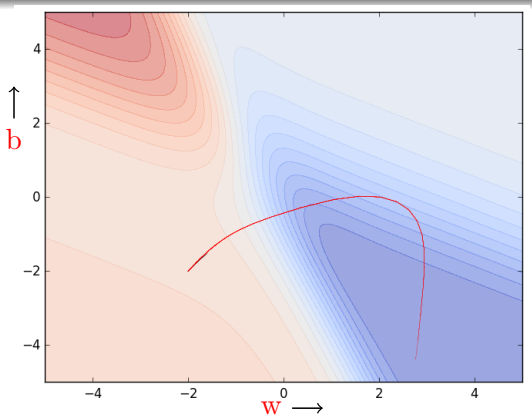
            db += grad_b(w, b, x, y)

        v_w = gamma * prev_v_w + eta* dw
        v_b = gamma * prev_v_b + eta* db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
        prev_v_b = v_b
```



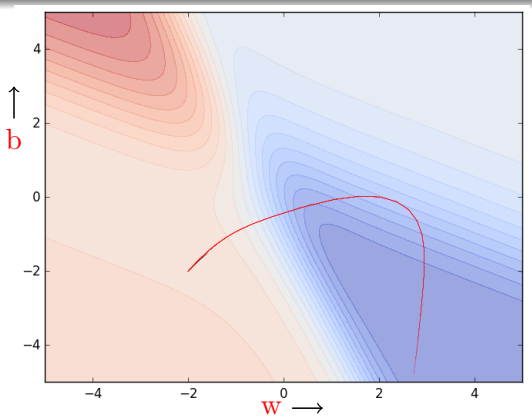
```python
def do_momentum_gradient_descent() :
    w, b, eta = init_w, init_b, 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = gamma * prev_v_w + eta* dw
        v_b = gamma * prev_v_b + eta* db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
        prev_v_b = v_b
```



```python
def do_momentum_gradient_descent() :
    w, b, eta = init_w, init_b, 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = gamma * prev_v_w + eta* dw
        v_b = gamma * prev_v_b + eta* db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
        prev_v_b = v_b
```



Some observations and questions

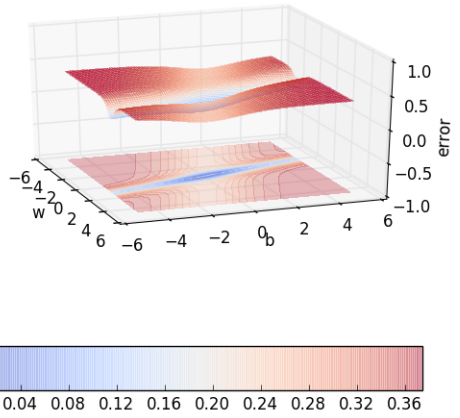
- Even in the regions having gentle slopes, momentum based gradient descent is able to take large steps because the momentum carries it along

Some observations and questions

- Even in the regions having gentle slopes, momentum based gradient descent is able to take large steps because the momentum carries it along
- Is moving fast always good? Would there be a situation where momentum would cause us to run pass our goal?

Some observations and questions

- Even in the regions having gentle slopes, momentum based gradient descent is able to take large steps because the momentum carries it along
- Is moving fast always good? Would there be a situation where momentum would cause us to run pass our goal?
- Let us change our input data so that we end up with a different error surface and then see what happens ...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6
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−2
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0.20
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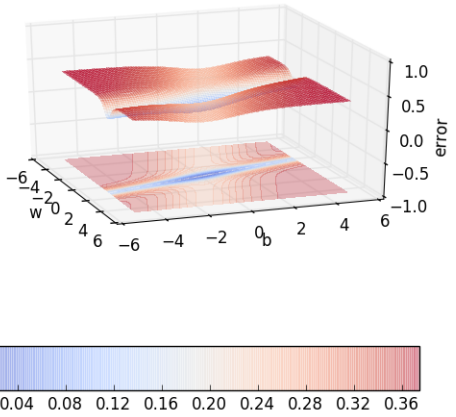
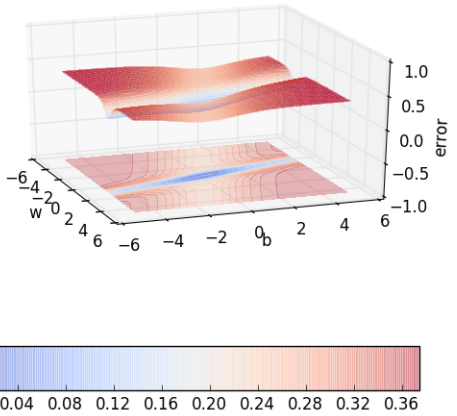
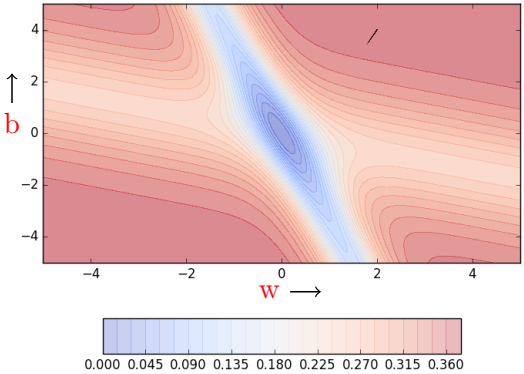
0.32
0.36

- In this case, the error is high on either side of the minima valley



- In this case, the error is high on either side of the minima valley
- Could momentum be detrimental in such cases... let's see....

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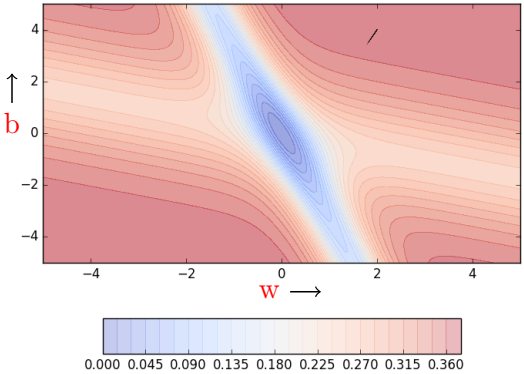
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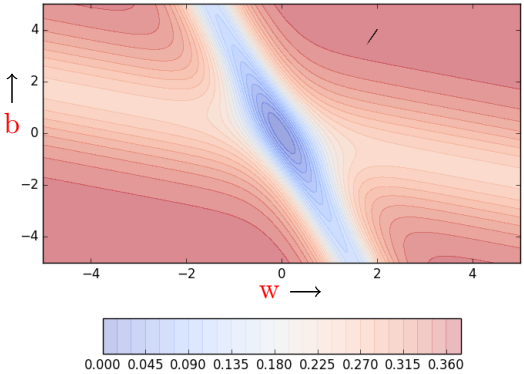
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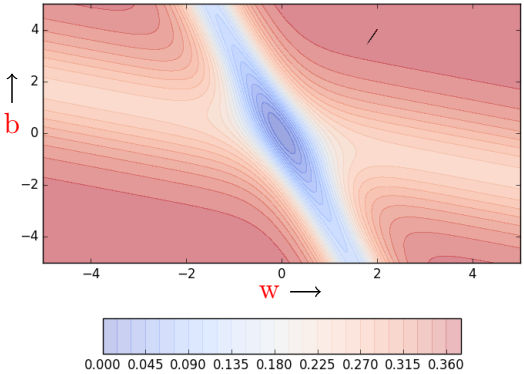
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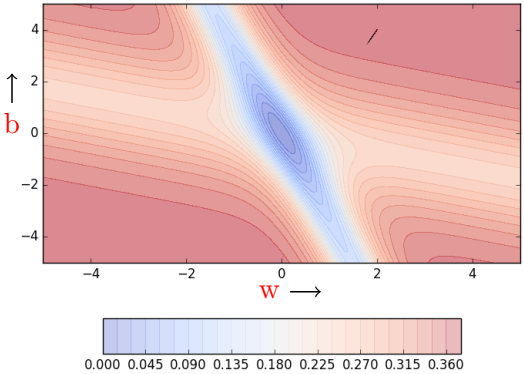
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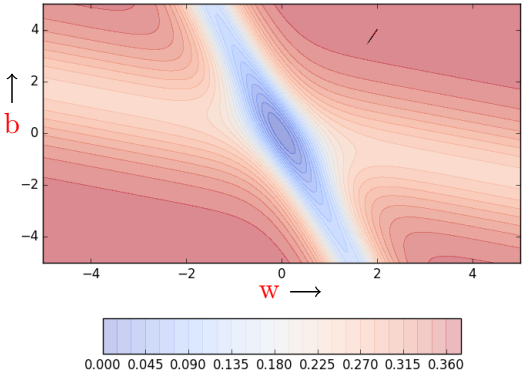
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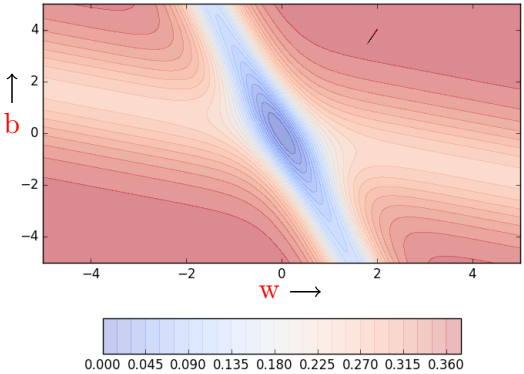
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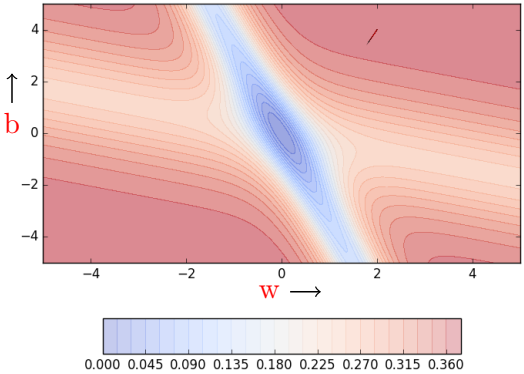
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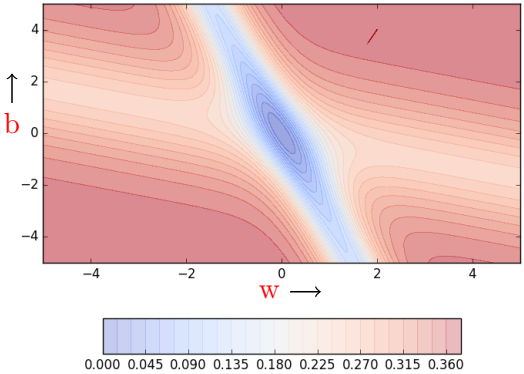
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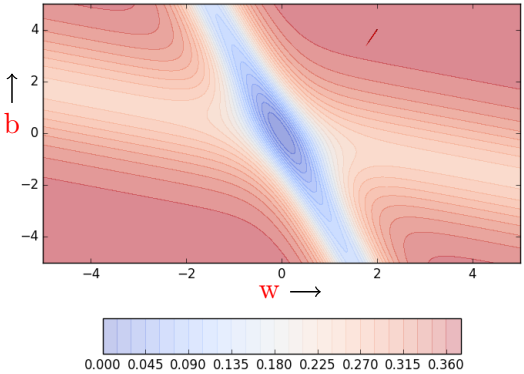
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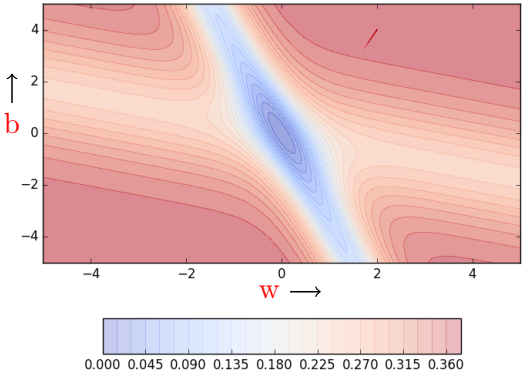
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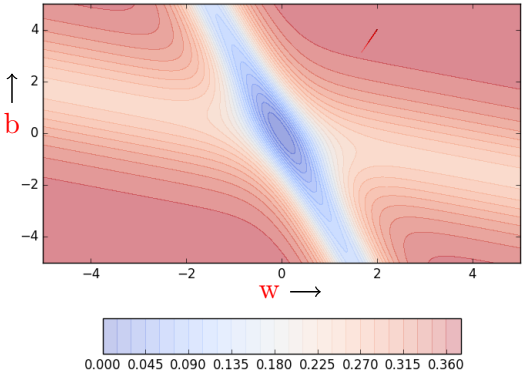
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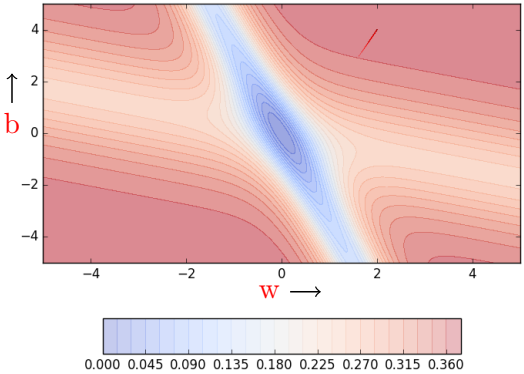
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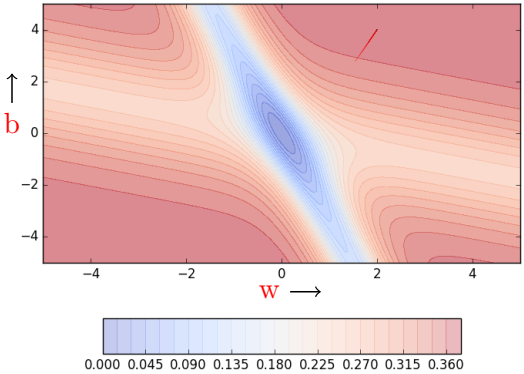
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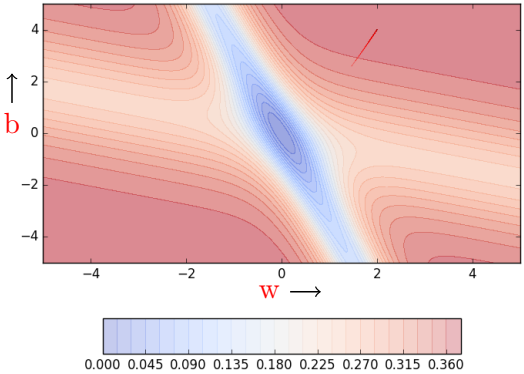
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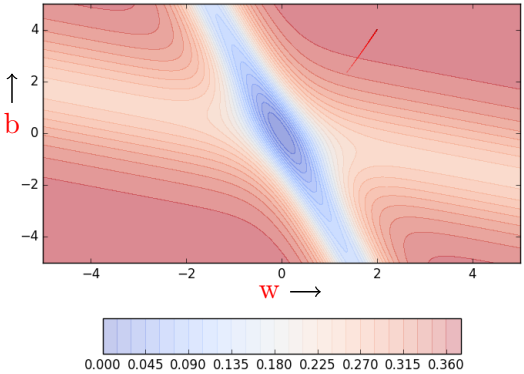
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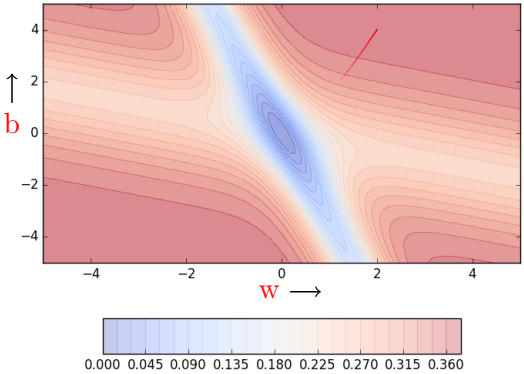
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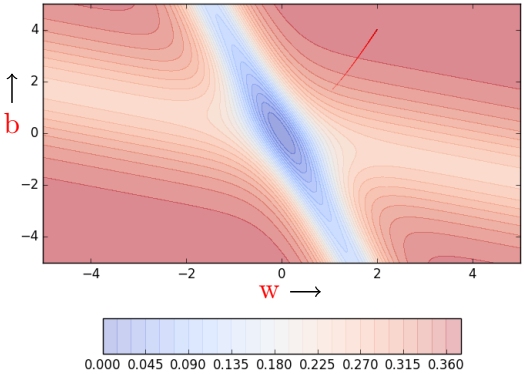
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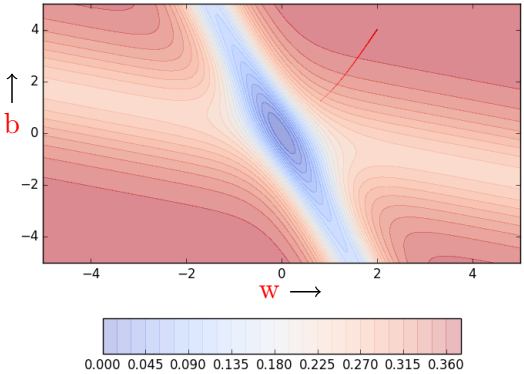
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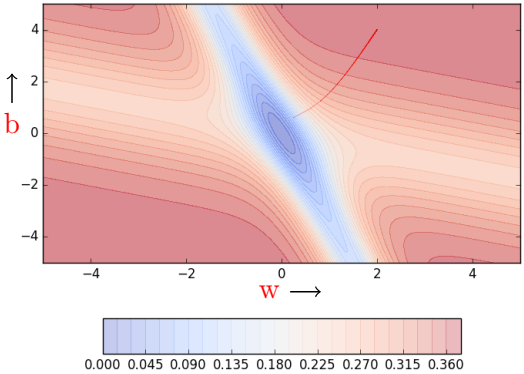
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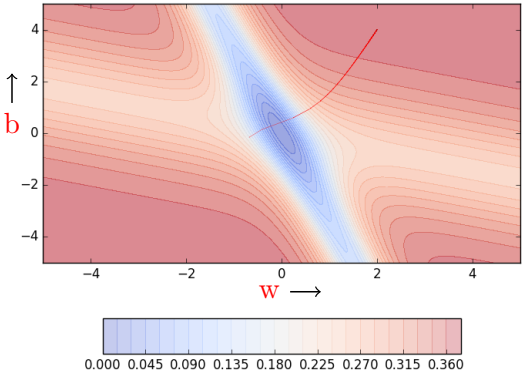
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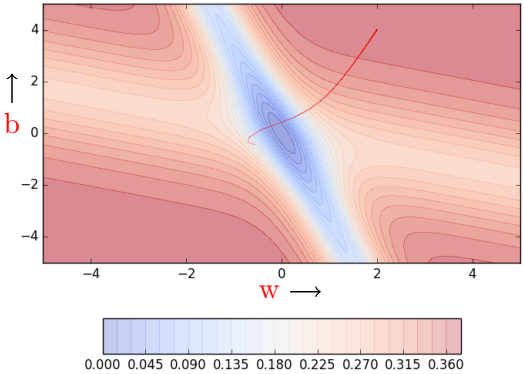
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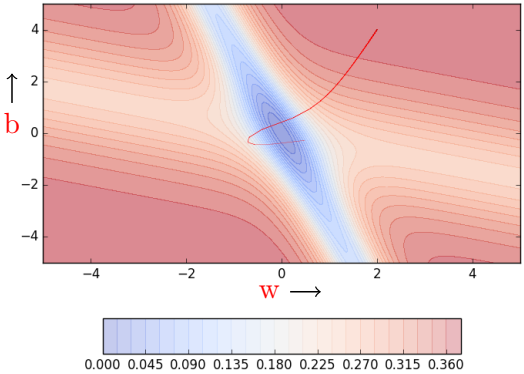
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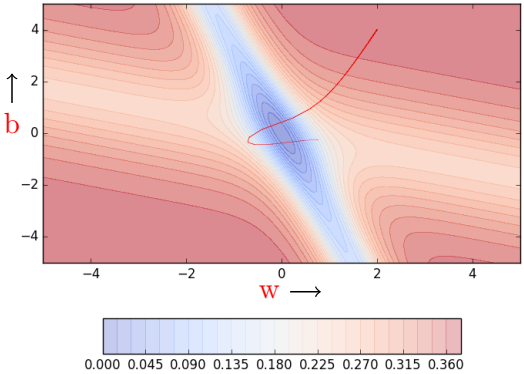
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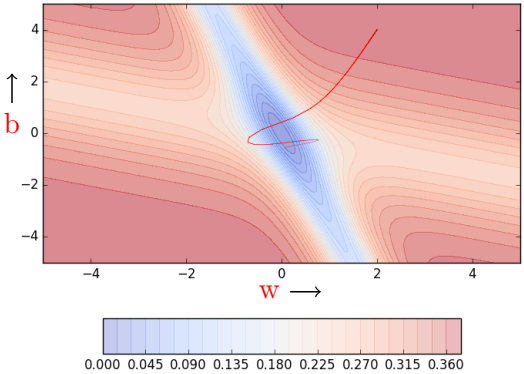
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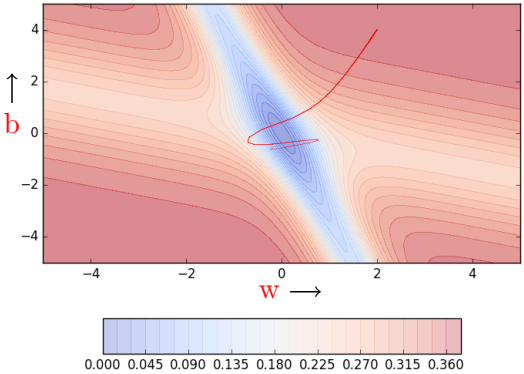
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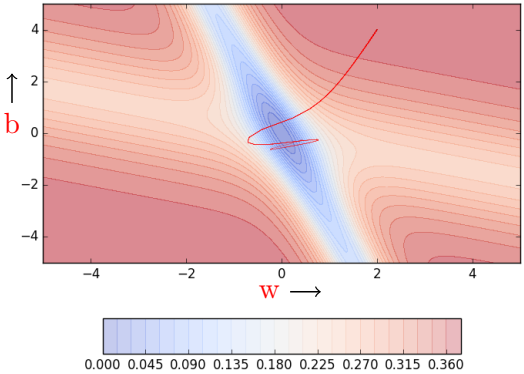
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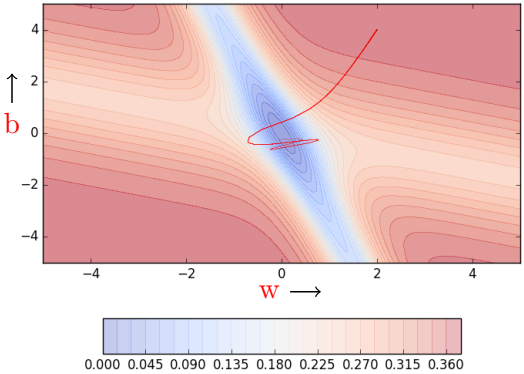
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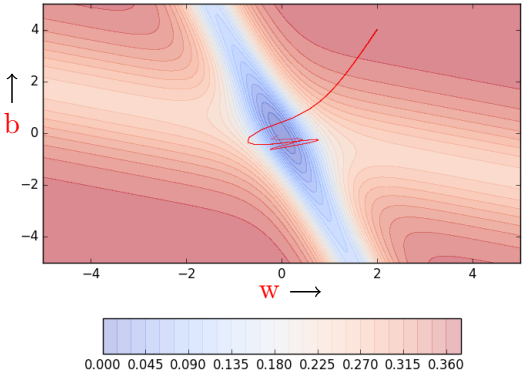
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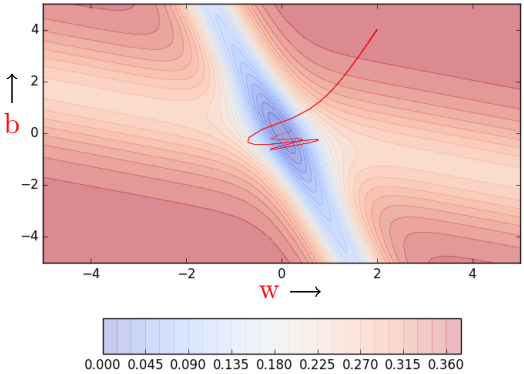
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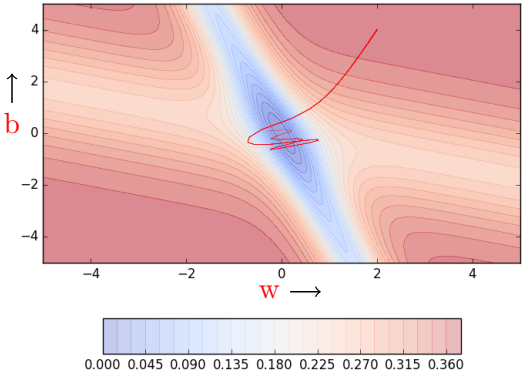
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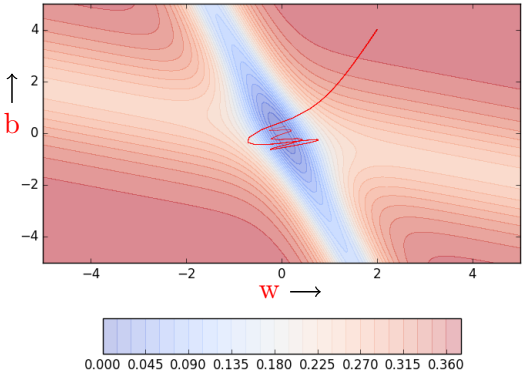
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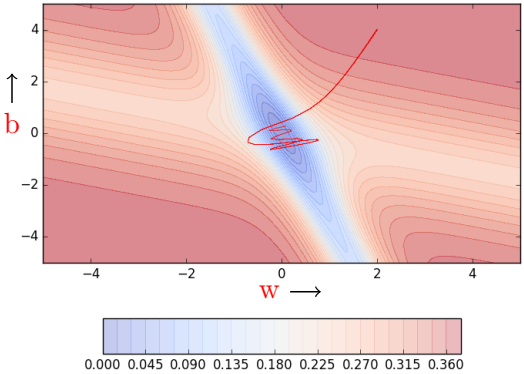
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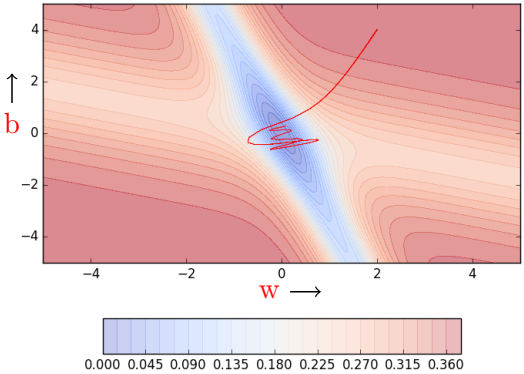
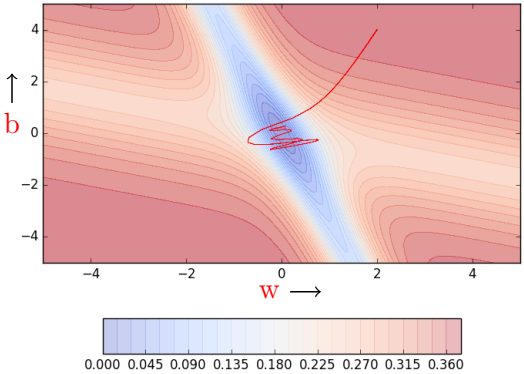
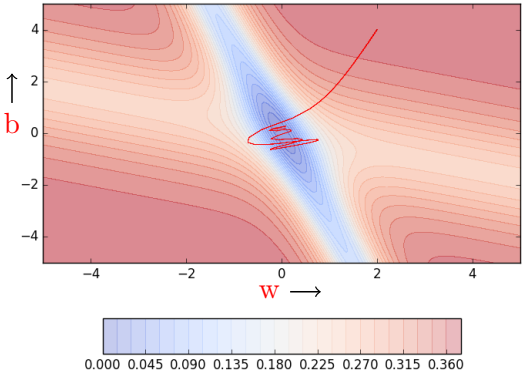
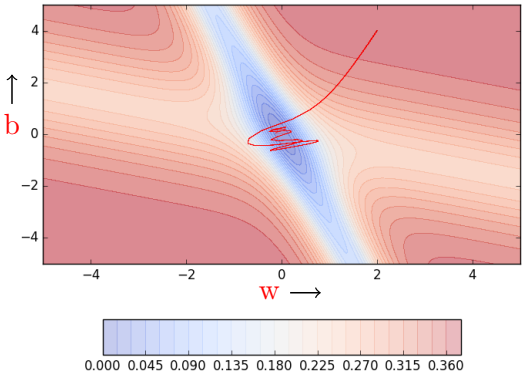
0.000 0.045 0.090 0.135 0.180 0.225 0.270 0.315 0.360


4
2
0
−2
−4
b
−4
−2
0
2
4
w
0.000 0.045 0.090 0.135 0.180 0.225 0.270 0.315 0.360

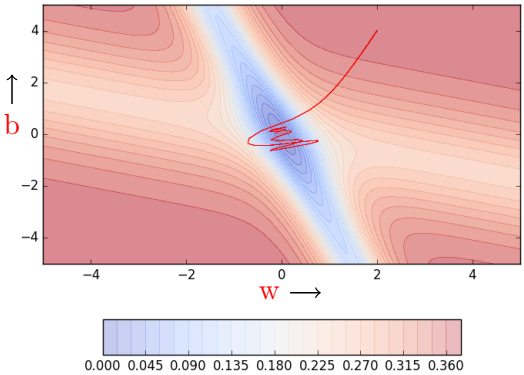

4
2
0
−2
−4
b
−4
−2
0
2
4
w
0.000 0.045 0.090 0.135 0.180 0.225 0.270 0.315 0.360


4
2
0
−2
−4
b
−4
−2
0
2
4
w
0.000 0.045 0.090 0.135 0.180 0.225 0.270 0.315 0.360

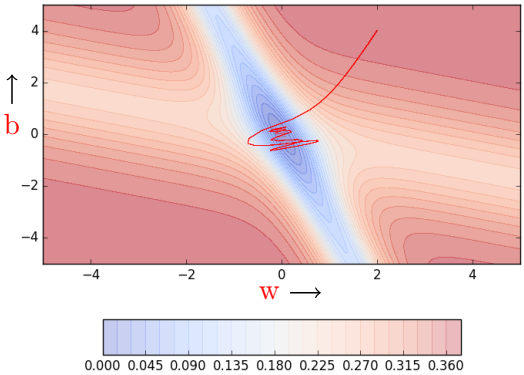
- Momentum based gradient descent oscillates in and out of the minima valley as the momentum carries it out of the valley



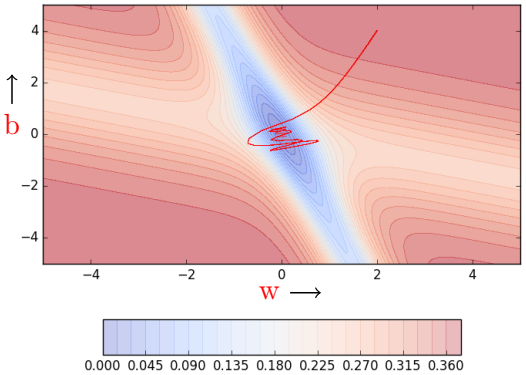
- Momentum based gradient descent oscillates in and out of the minima valley as the momentum carries it out of the valley
- Takes a lot of u-turns before finally converging



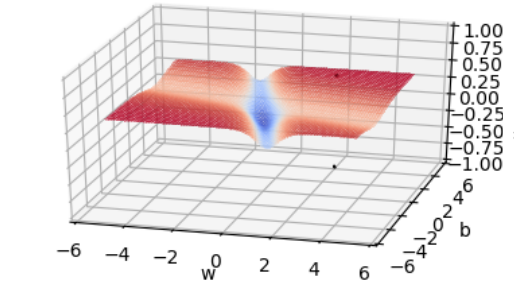
- Momentum based gradient descent oscillates in and out of the minima valley as the momentum carries it out of the valley
- Takes a lot of u-turns before finally converging
- Despite these u-turns it still converges faster than vanilla gradient descent



- Momentum based gradient descent oscillates in and out of the minima valley as the momentum carries it out of the valley
- Takes a lot of u-turns before finally converging
- Despite these u-turns it still converges faster than vanilla gradient descent
- After 100 iterations momentum based method has reached an error of 0.00001 whereas vanilla gradient descent is still stuck at an error of 0.36



Let's look at a 3d visualization and a different geometric perspective of the same thing...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0.75
0.50
0.25
0.00
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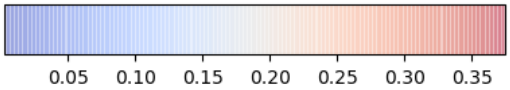
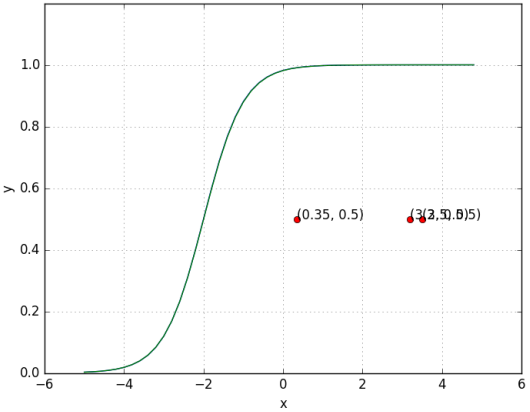
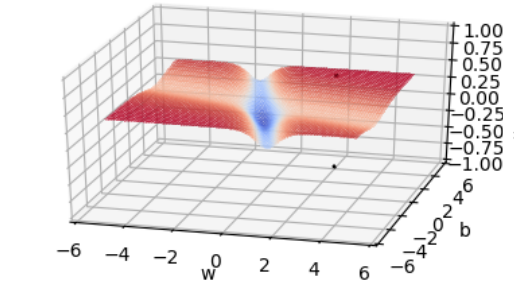
−0.50
−0.75
−1.00
−6
−4
−2
0
2
4
6
w
b


0.05
0.10
0.15
0.20
0.25
0.30
0.35


1.0
0.8
0.6
0.4
0.2
0.0
y
−6
−4
−2
0
2
4
6
x
(0.35, 0.5)
(3.5, 0.5)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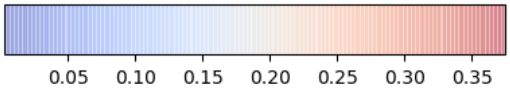
0
2
4
6
w
b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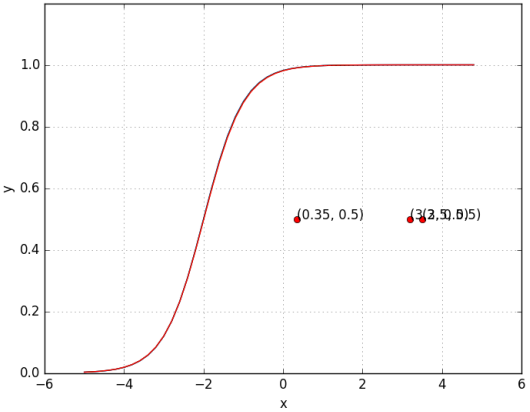
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y
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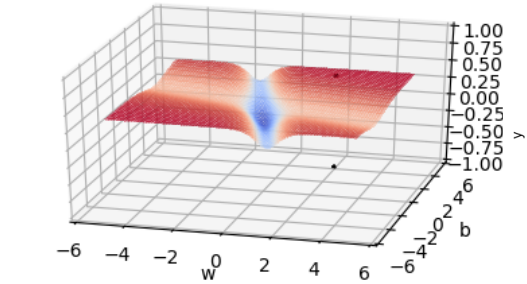
x
(0.35, 0.5)
(3.5, 0.5)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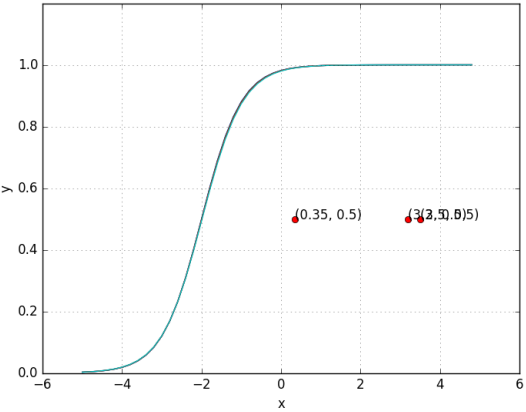
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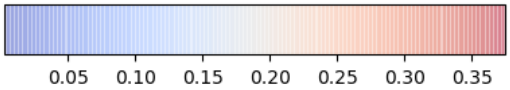
x
(0.35, 0.5)
(3.5, 0.5)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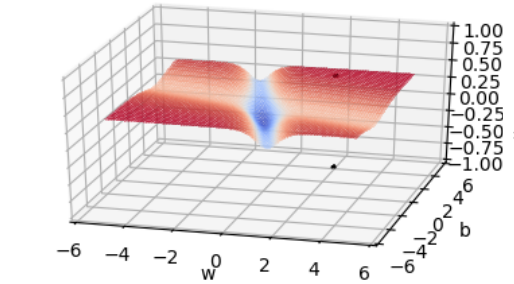
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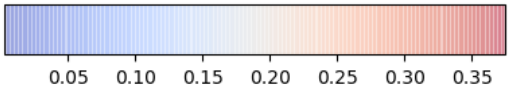
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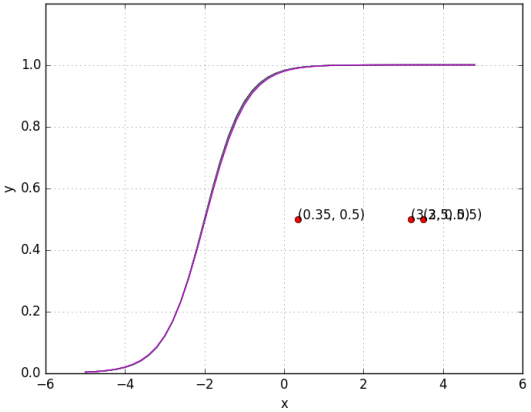
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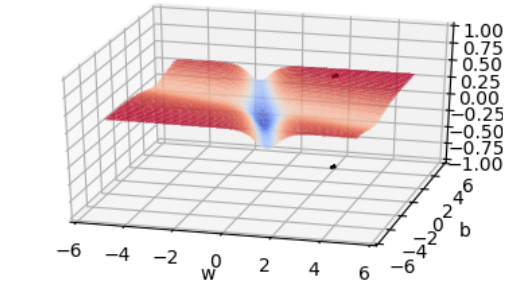
x
(0.35, 0.5)
(3.5, 0.5)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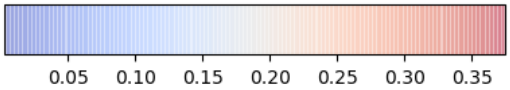
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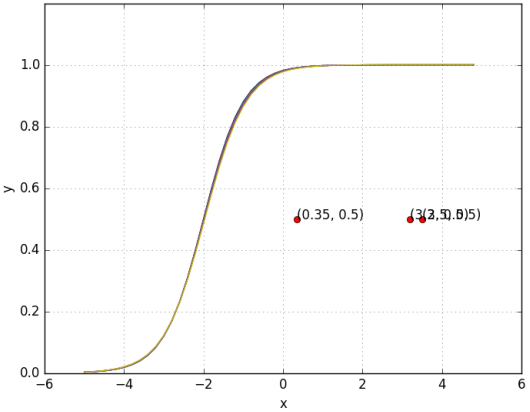
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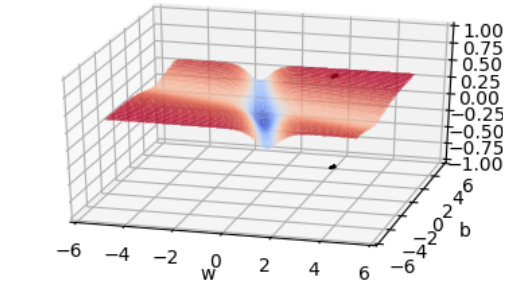
(3.5, 0.5)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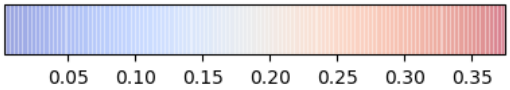
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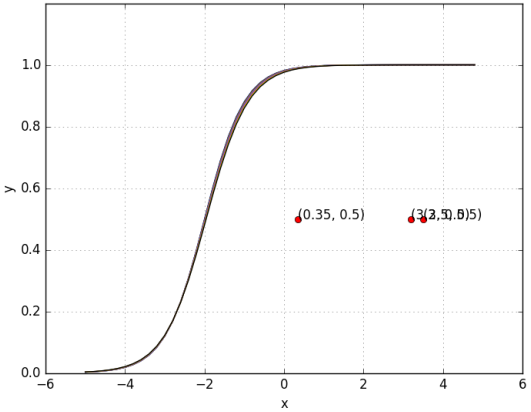
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x
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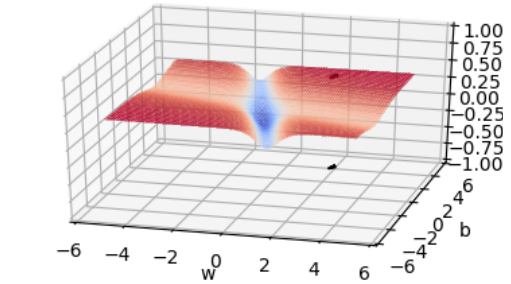
(3.15, 0.5)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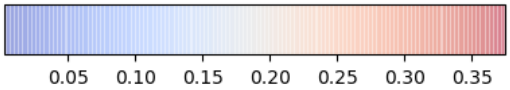
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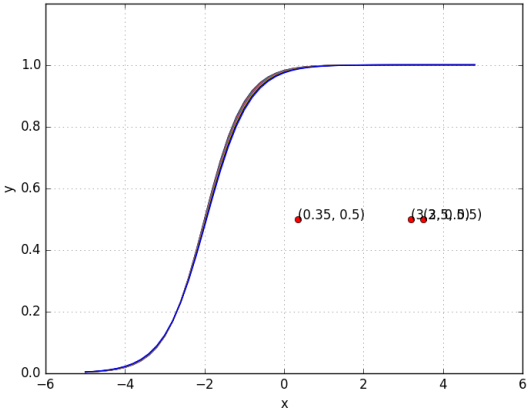
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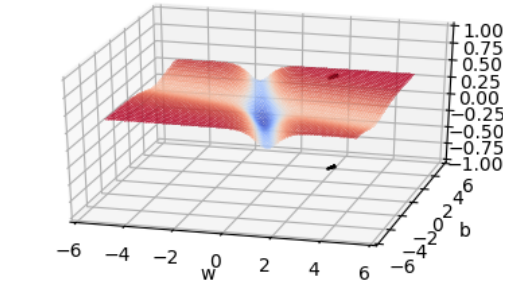
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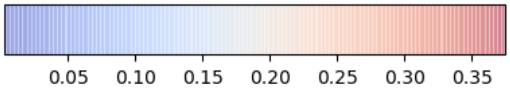
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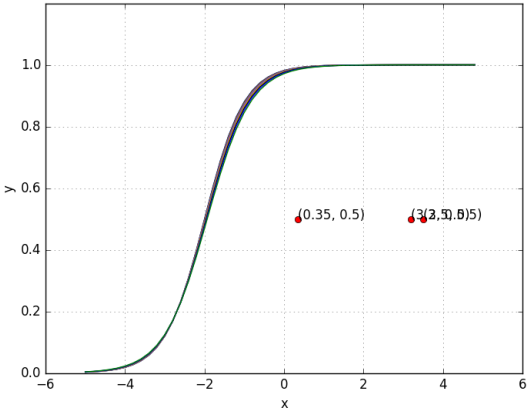
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y
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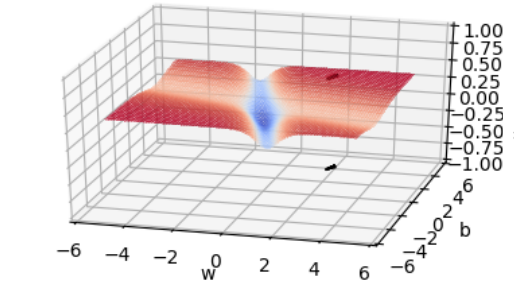
x
(0.35, 0.5)
(3.5, 0.5)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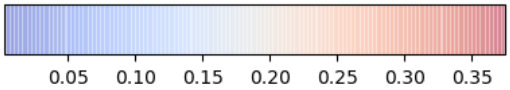
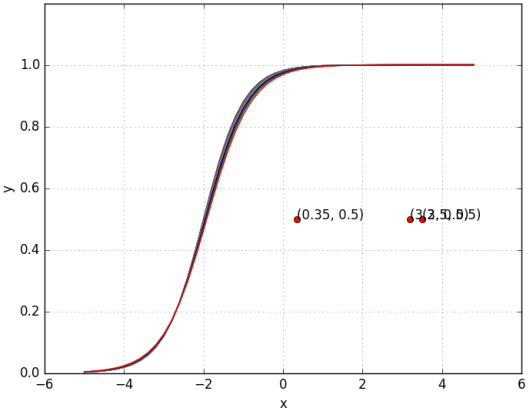
−6 −4 −2 0 2 4 6
w
−6 −4 −2 0 2 4 6
b


0.05 0.10 0.15 0.20 0.25 0.30 0.35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y
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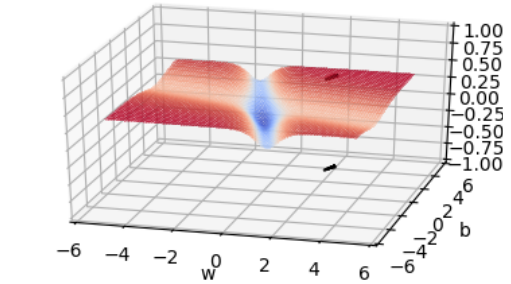
x
(0.35, 0.5)
(3.5, 0.5)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−6 −4 −2 0 2 4 6
w
−6 −4 −2 0 2 4 6
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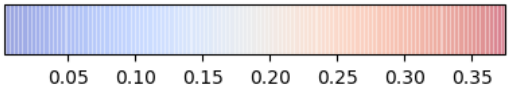
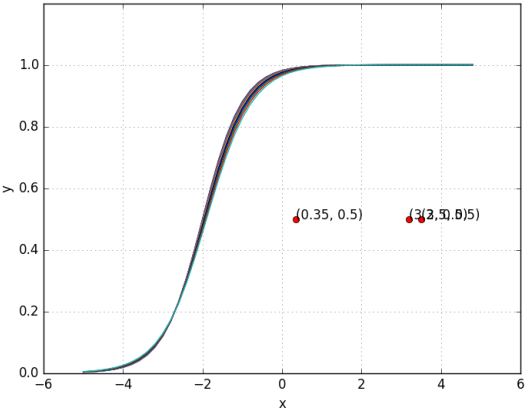

0.05 0.10 0.15 0.20 0.25 0.30 0.35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x
(0.35, 0.5)
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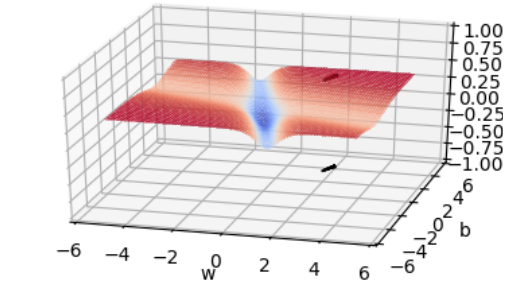
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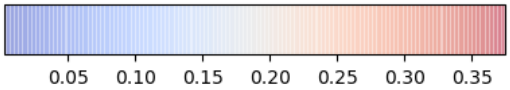
0.05
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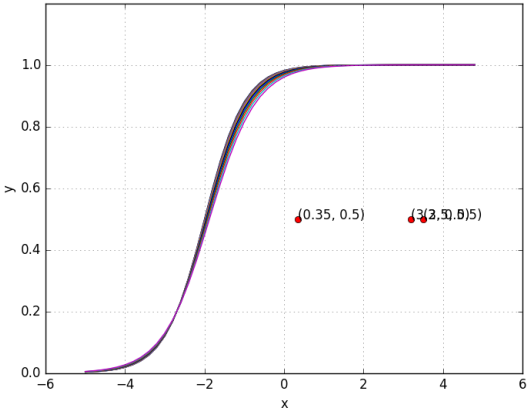
1.0
0.8
0.6
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y
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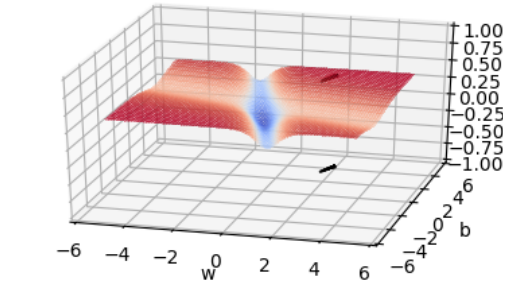
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−6 −4 −2 0 2 4 6
w
−6 −4 −2 0 2 4 6
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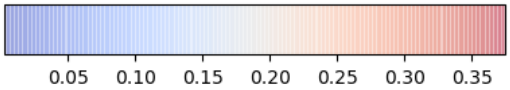
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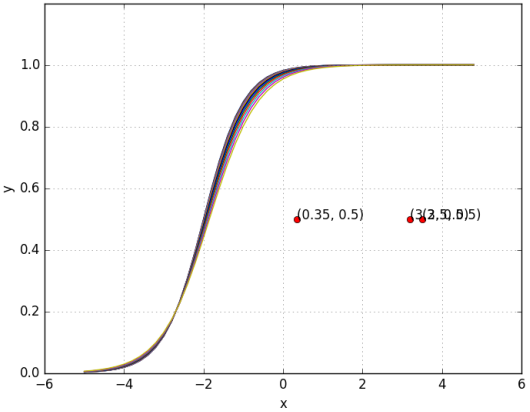
1.0
0.8
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y
−6 −4 −2 0 2 4 6
x
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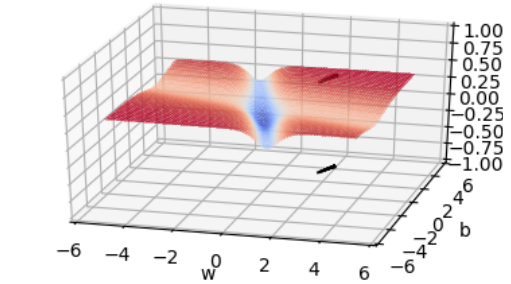
(3.5, 0.5)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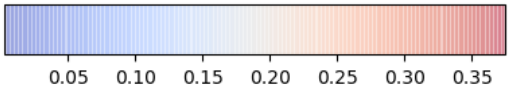
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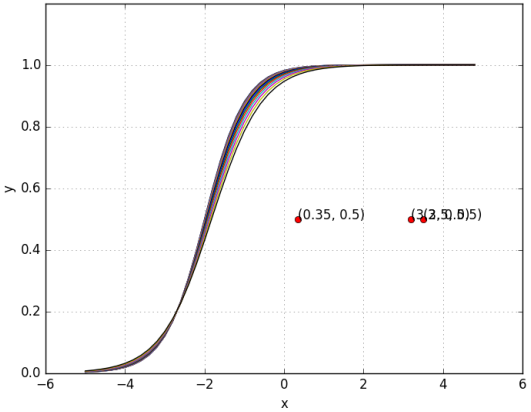
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x
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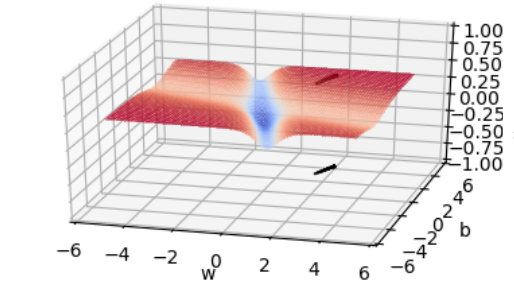
(3.5, 0.5)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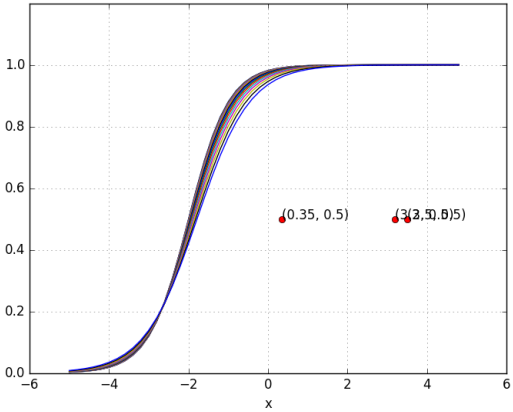
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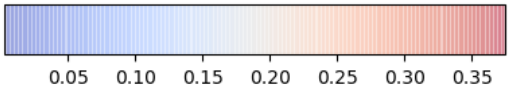
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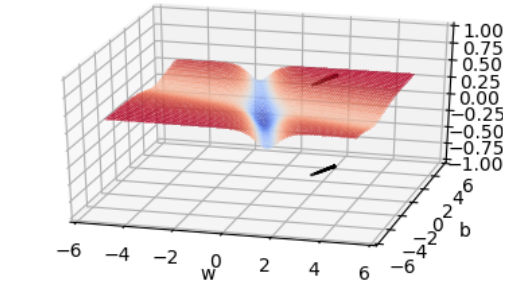
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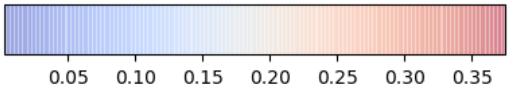
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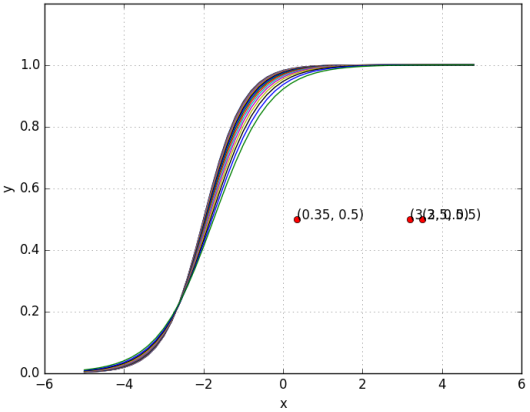
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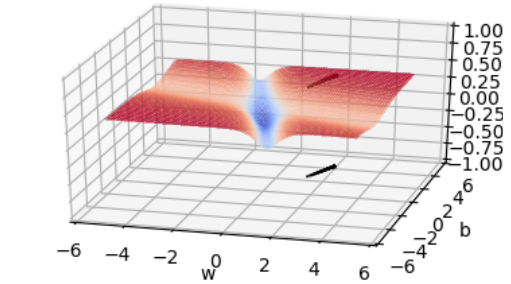
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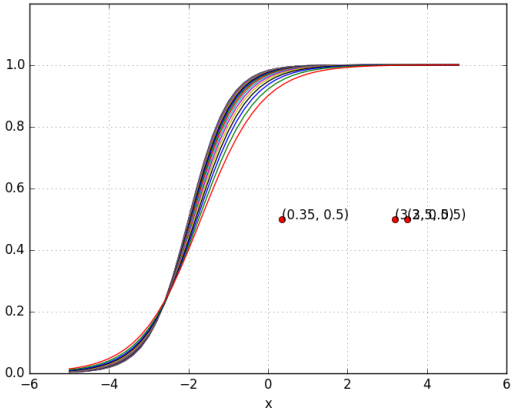
b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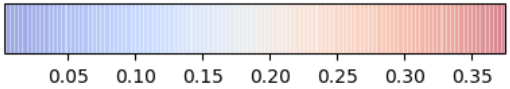
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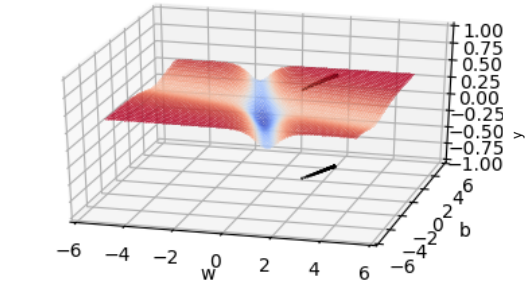
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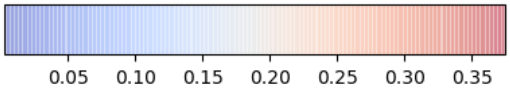
w
−6 −4 −2 0 2 4 6
b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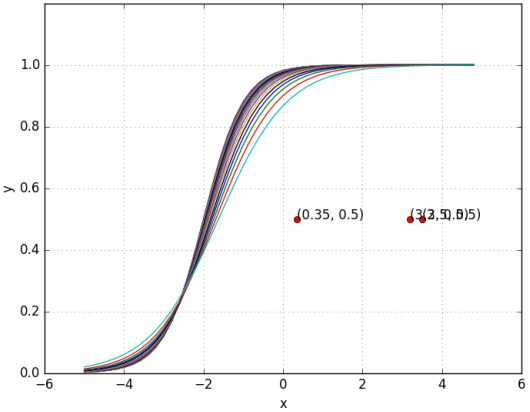
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x
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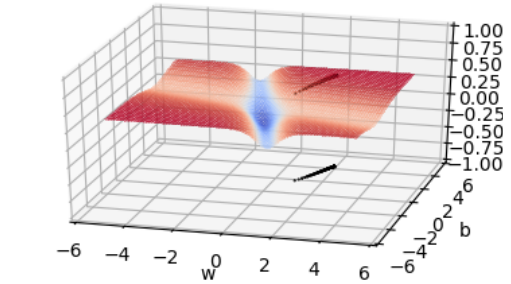
(3.5, 0.5)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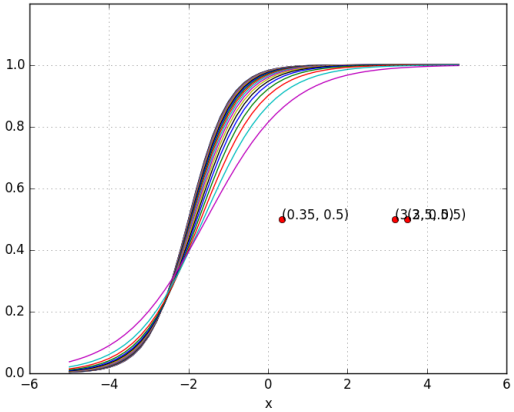
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x
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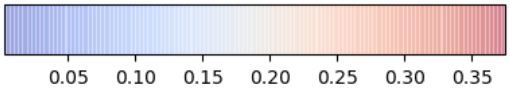
(3.5, 0.5)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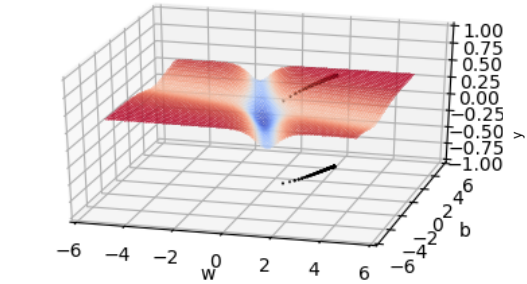
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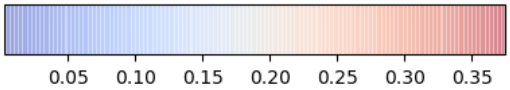
w
b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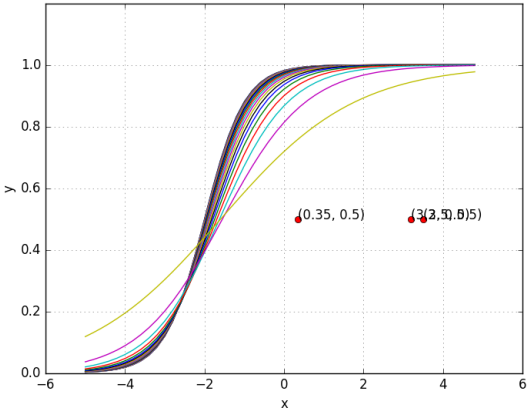
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x
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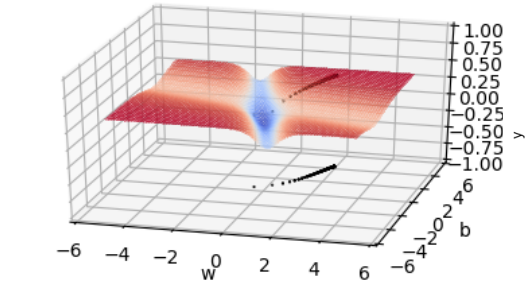
(3.5, 0.5)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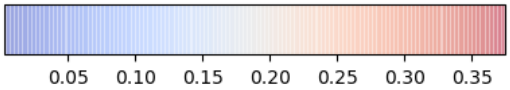
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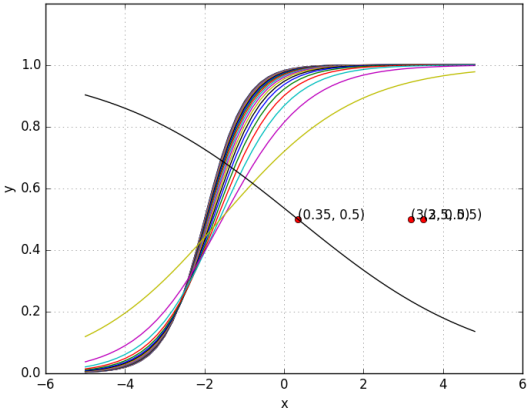
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(3(3.5, 0.5)5)
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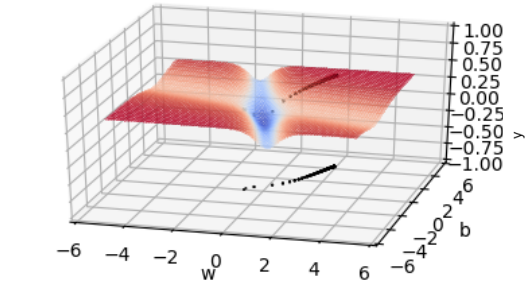
x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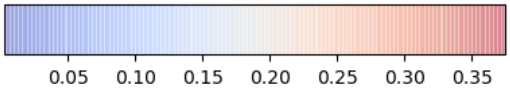
b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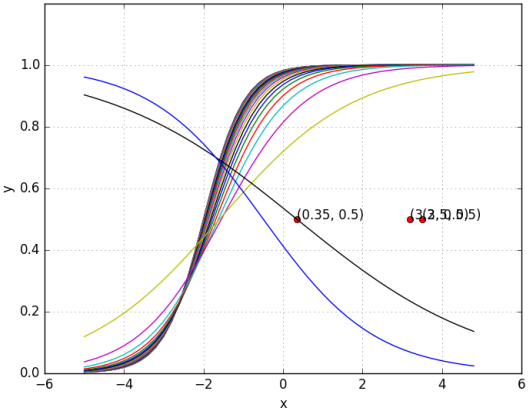
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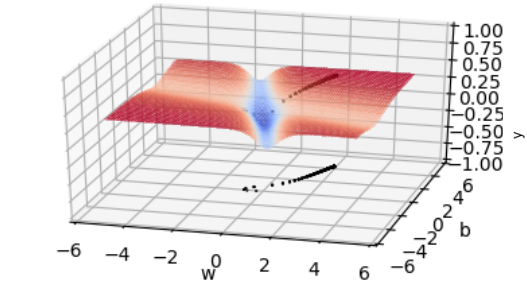
(3.5, 0.5)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w
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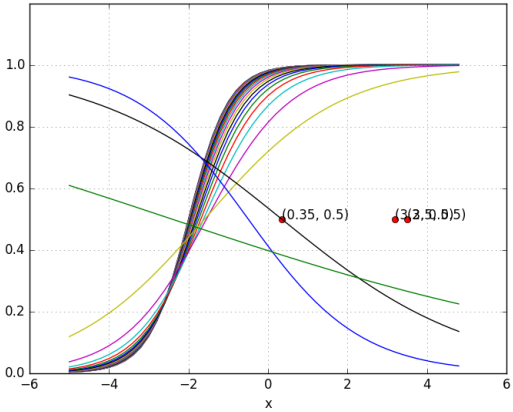
b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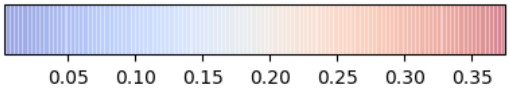
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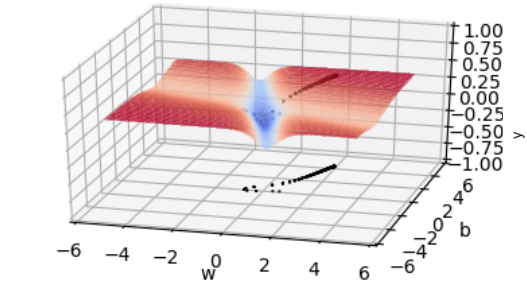
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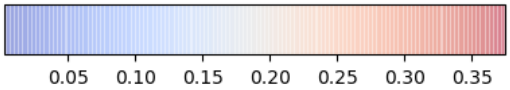
b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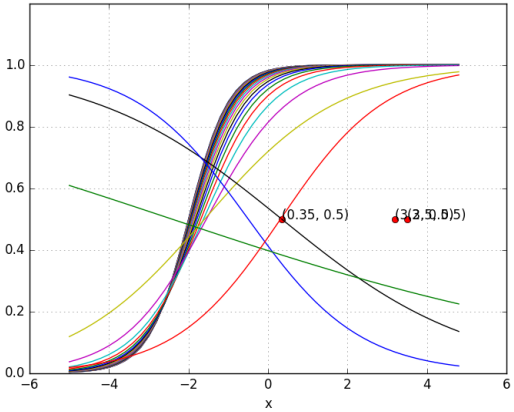
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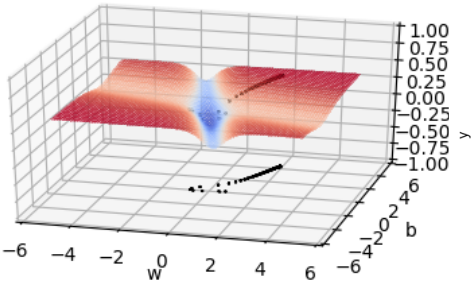
x
(0.35, 0.5)
(3(3.5, 0.5)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w
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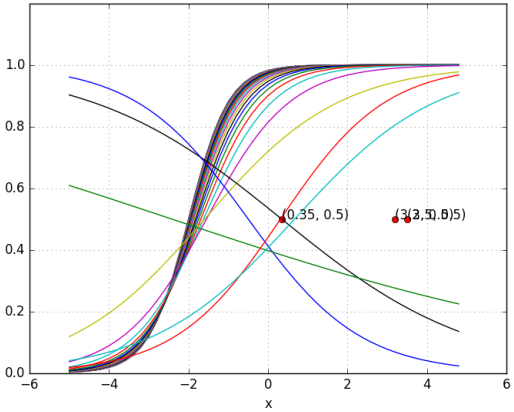
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x
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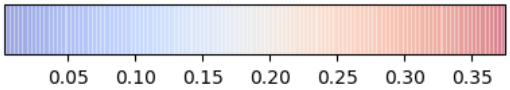
(3.5, 0.5)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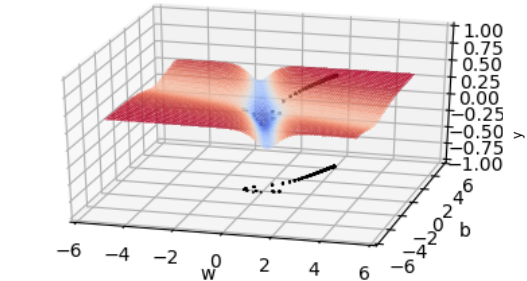
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w
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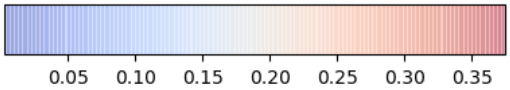
b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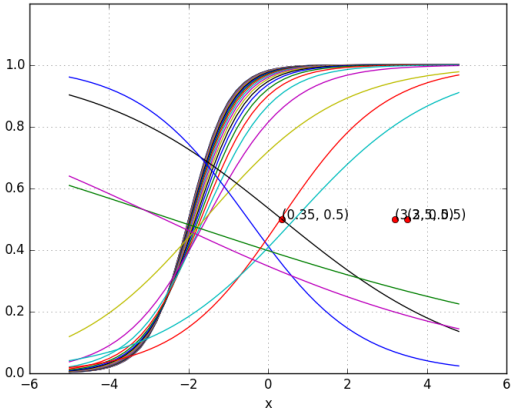
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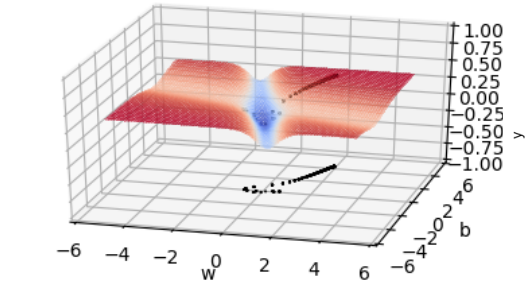
x
(0.35, 0.5)
(3.5, 0.5)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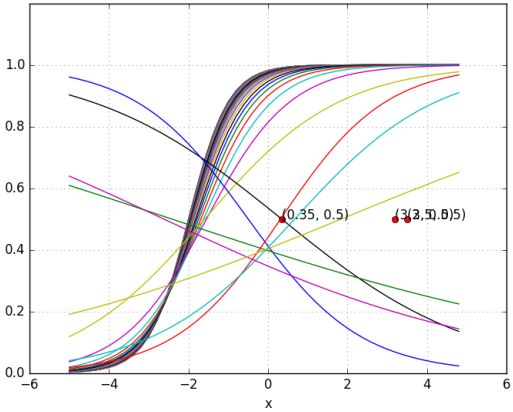
−6 −4 −2 0 2 4 6
w
−6 −4 −2 0 2 4 6
b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x
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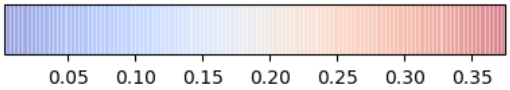
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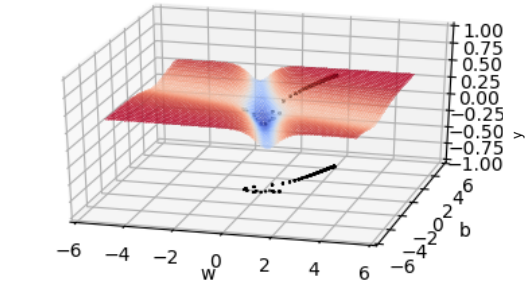
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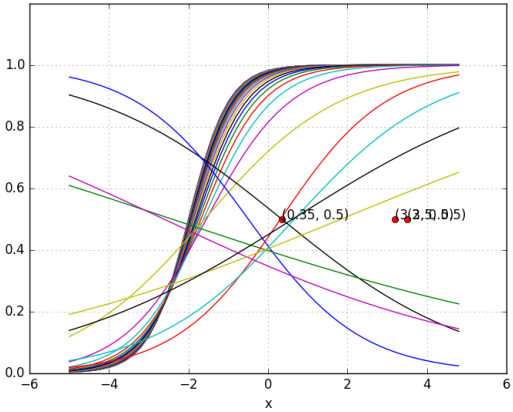
w
b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0.4
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0.0
y
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−2
0
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4
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x
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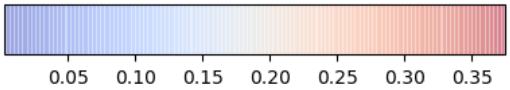
(3.25, 0.5)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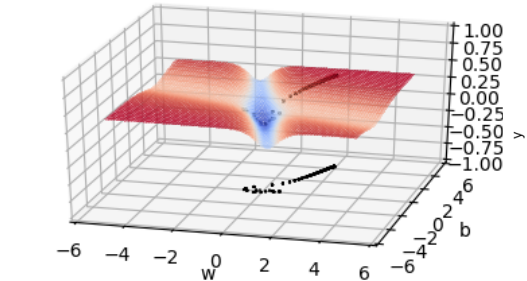
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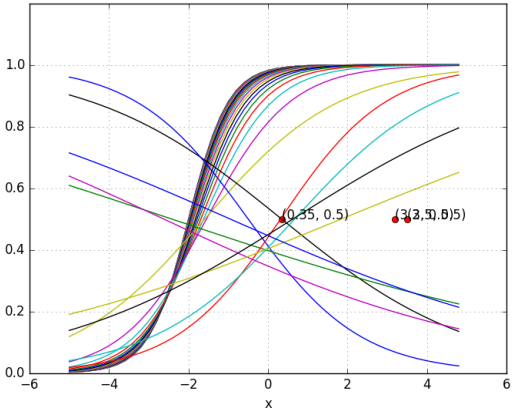
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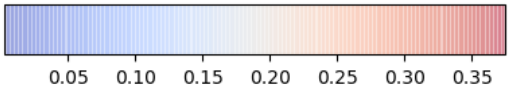
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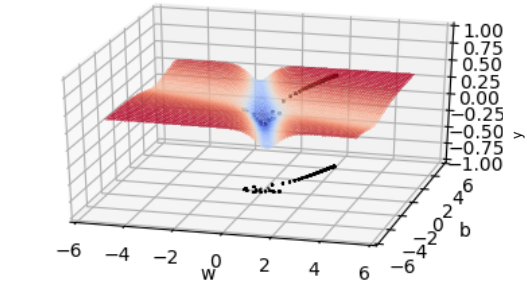
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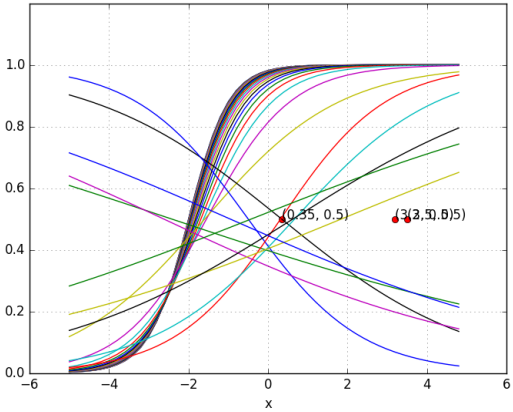
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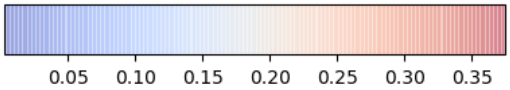
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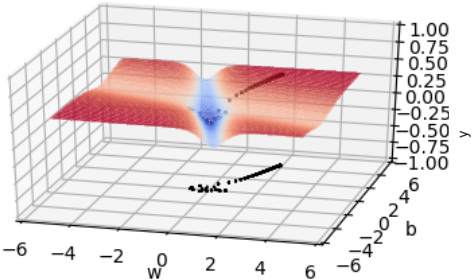
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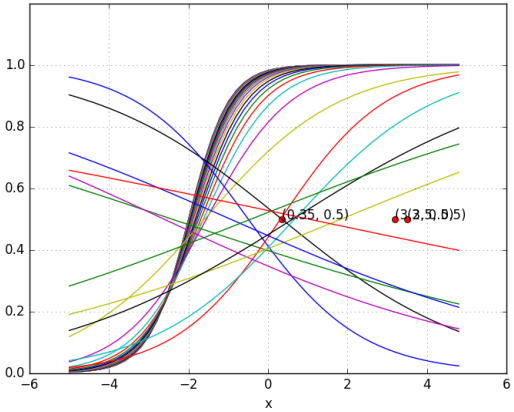
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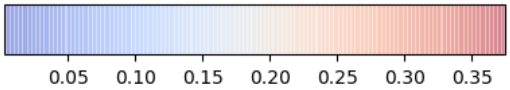
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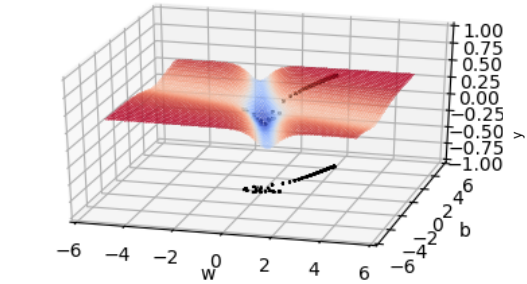
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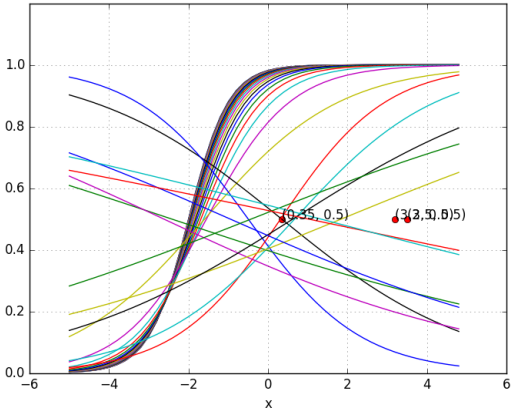
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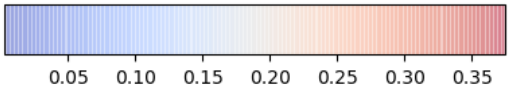
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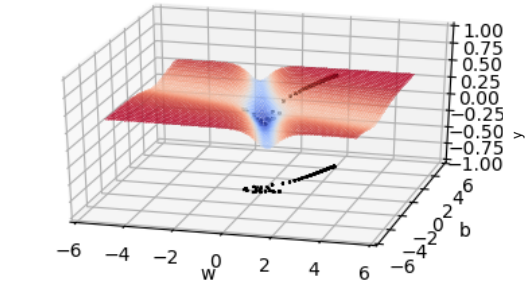
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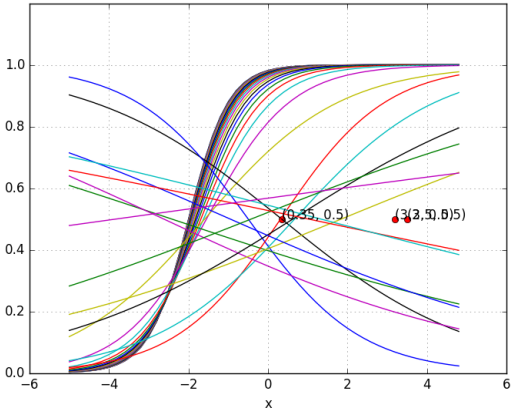
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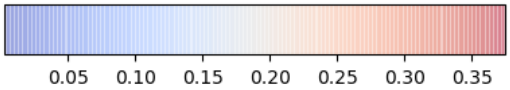
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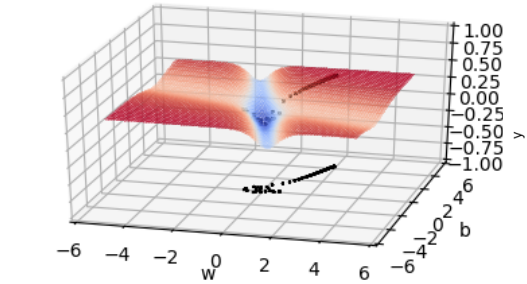
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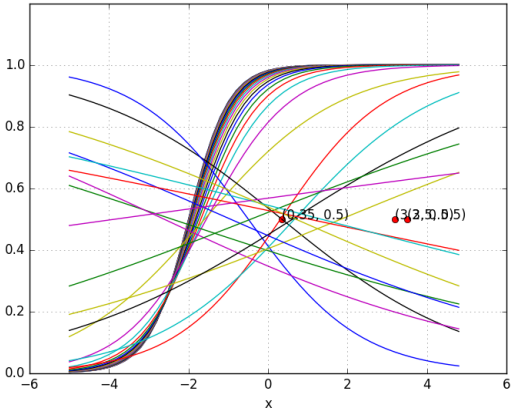
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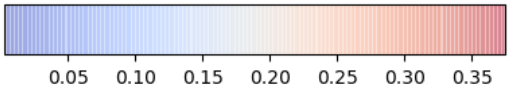
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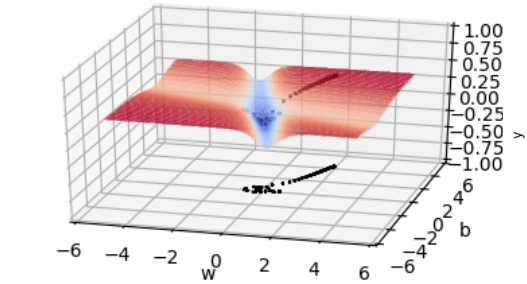
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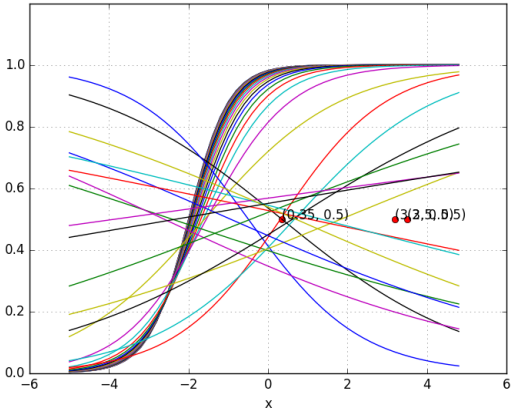
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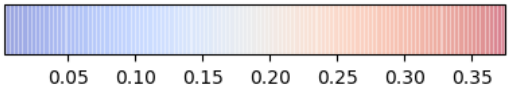
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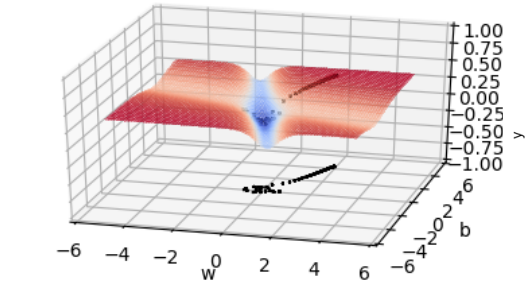
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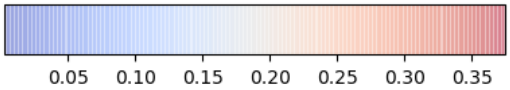
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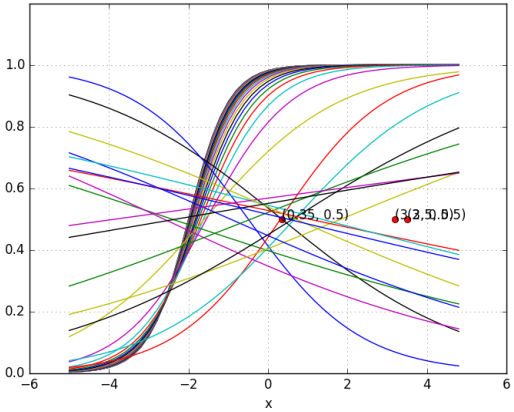
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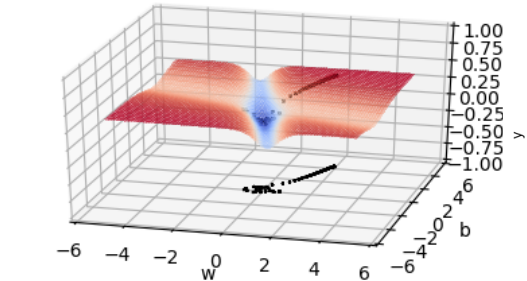
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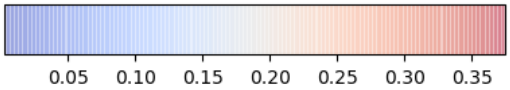
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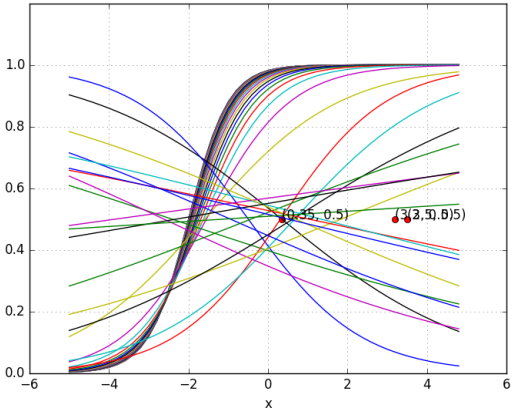
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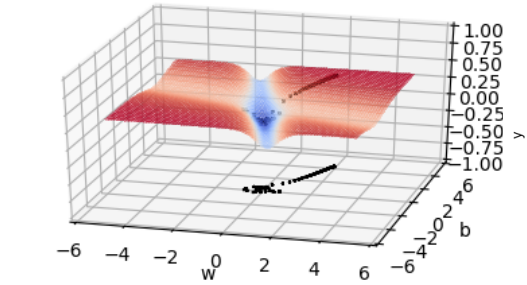
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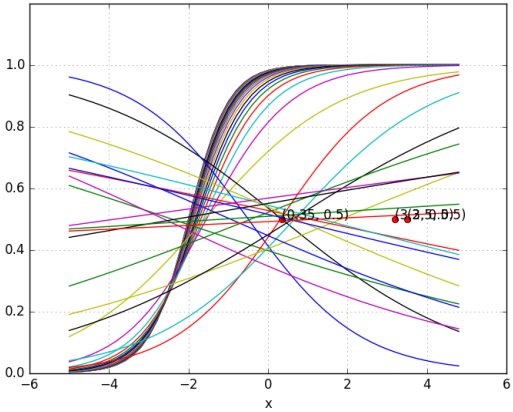
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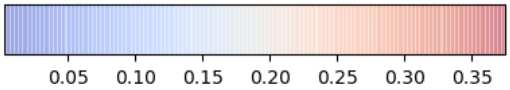
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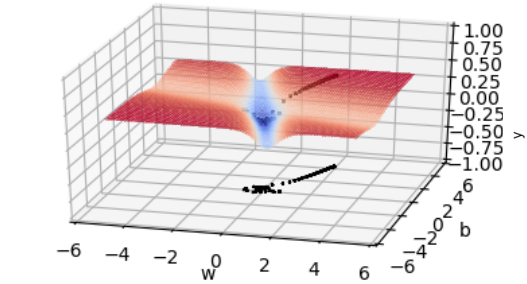
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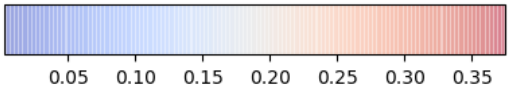
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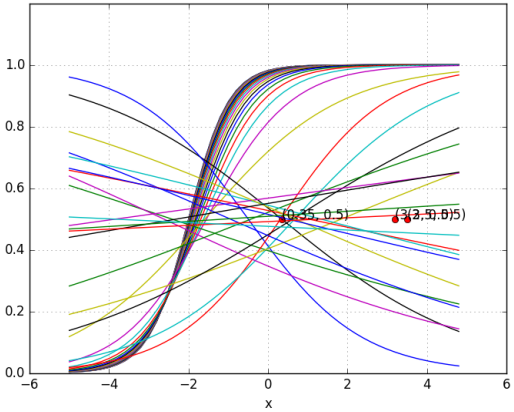
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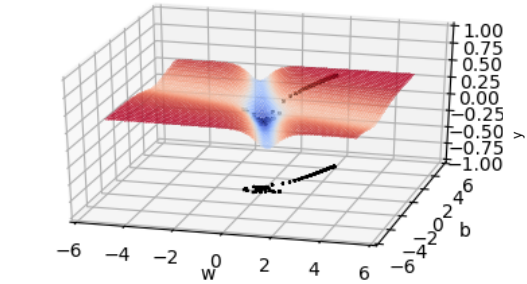
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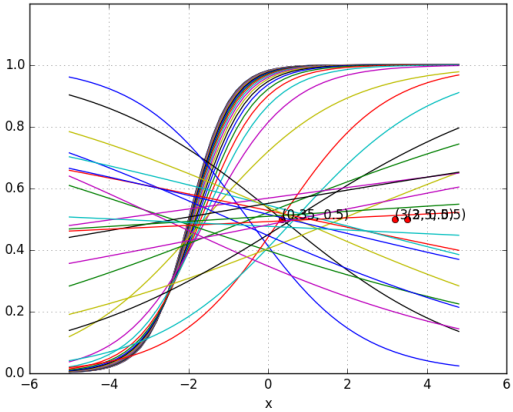
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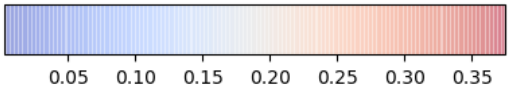
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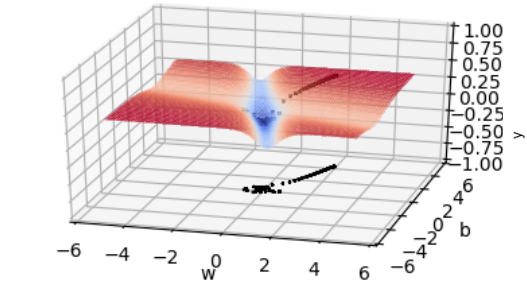
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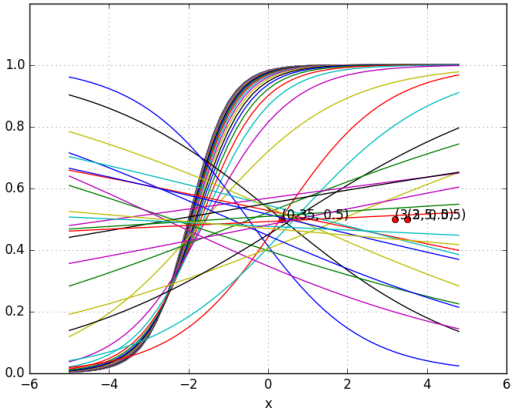
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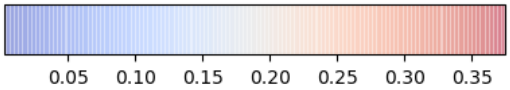
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