Module 5.5 : Nesterov Accelerated Gradient Descent

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- Yes, let's look at Nesterov accelerated gradient

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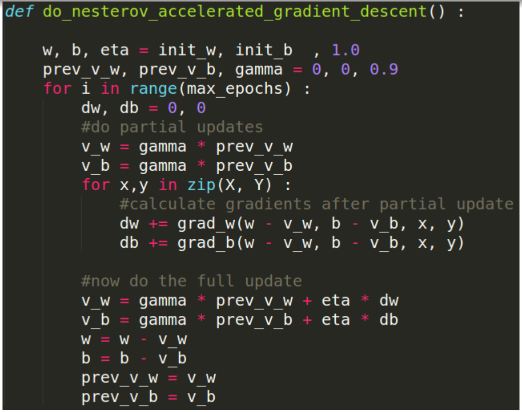
Update rule for NAG

$$\begin{aligned} w_{look\_ahead} &= w_t - \gamma \cdot update_{t-1} \\ update_t &= \gamma \cdot update_{t-1} + \eta \nabla w_{look\_ahead} \\ w_{t+1} &= w_t - update_t \end{aligned}$$

We will have similar update rule for b_t



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    for i in range(max_epochs) :
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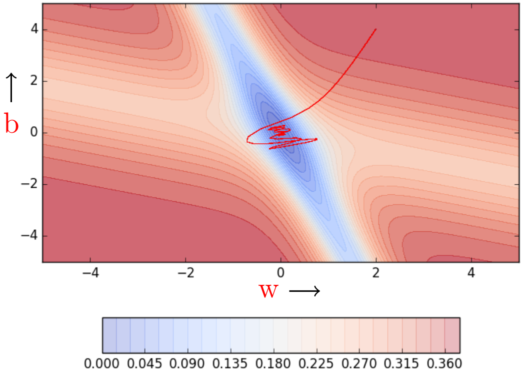

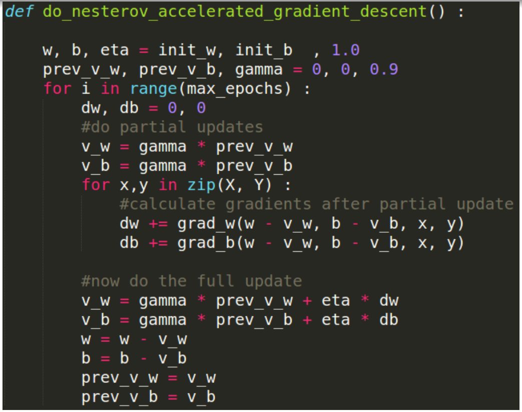

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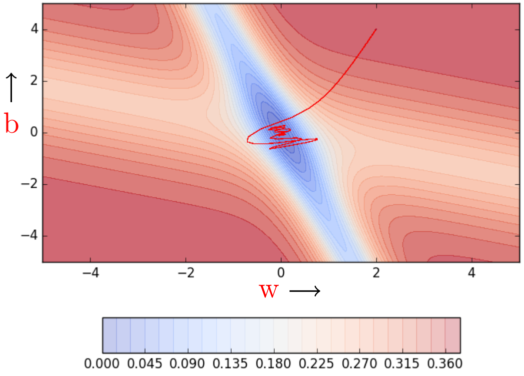

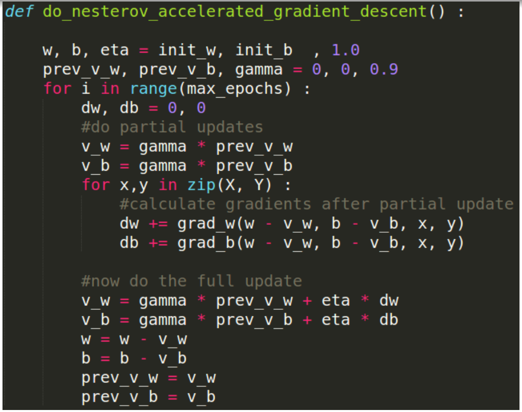

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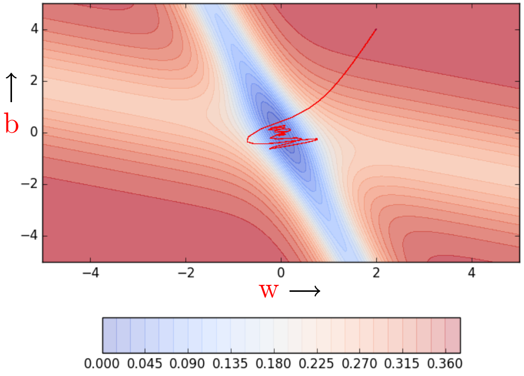

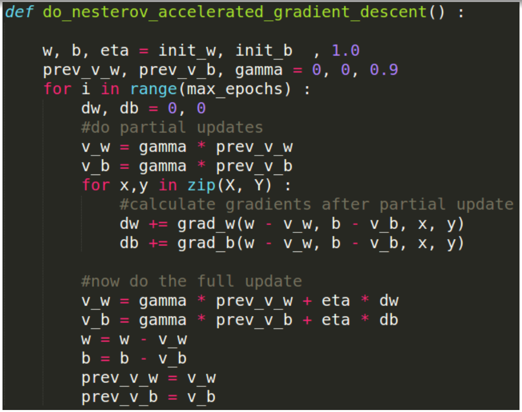

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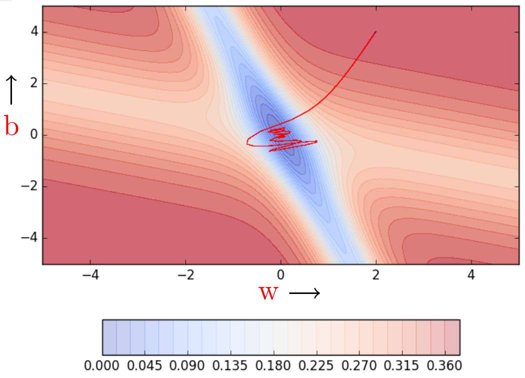

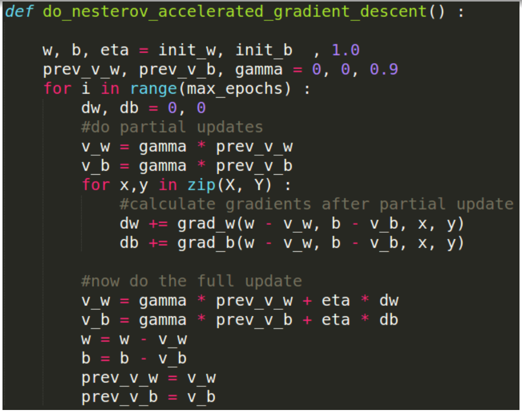

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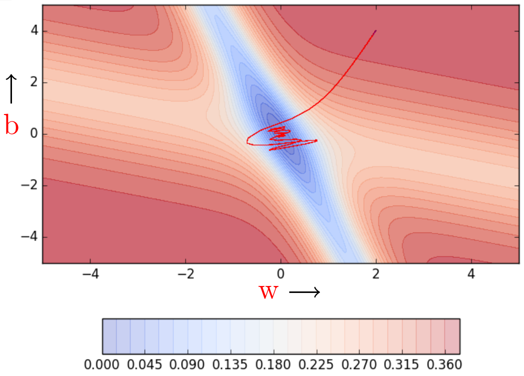

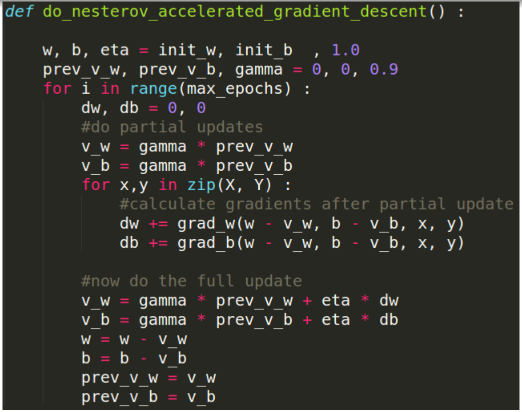

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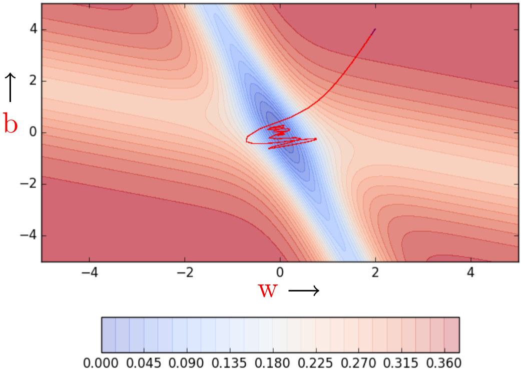

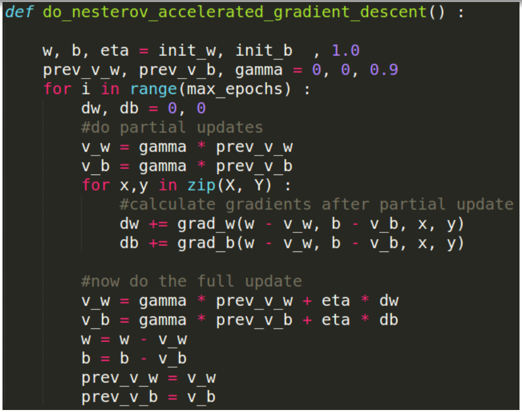

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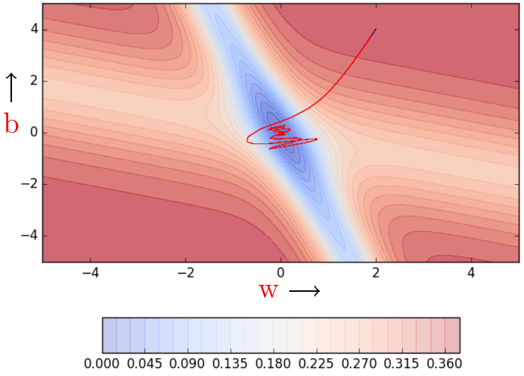

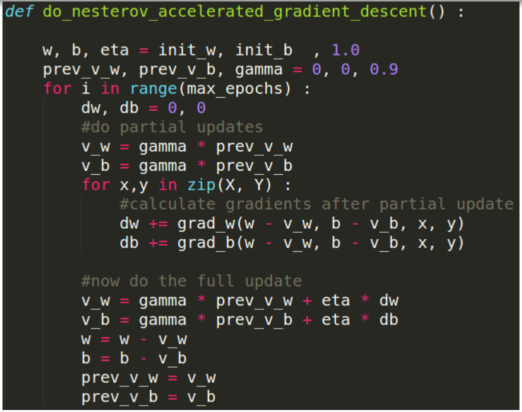

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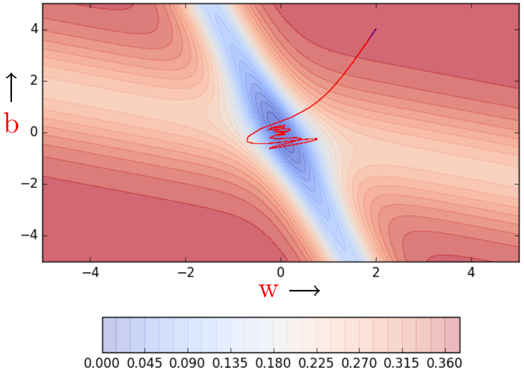

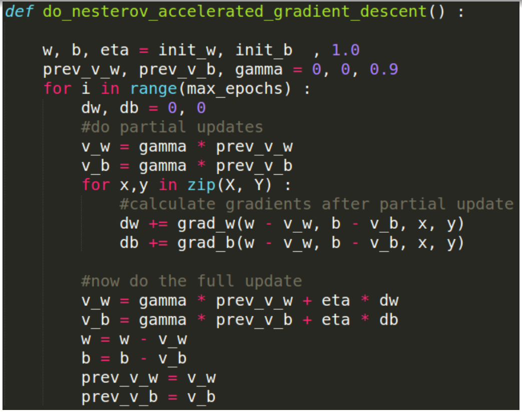

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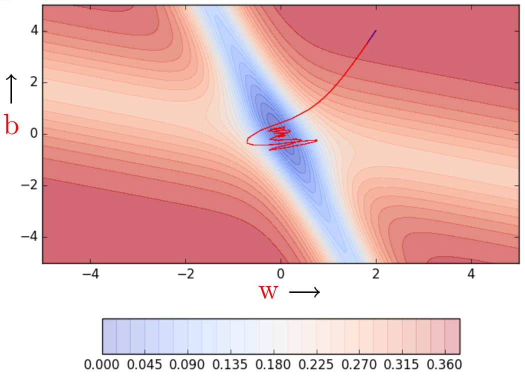

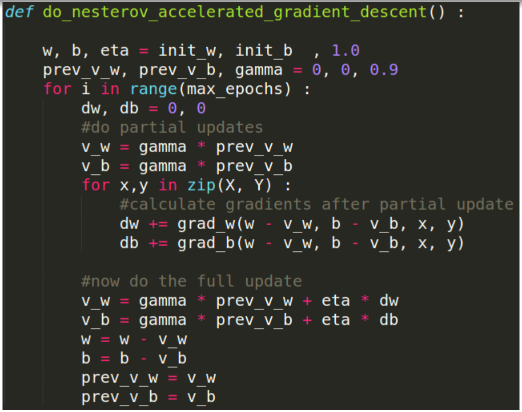

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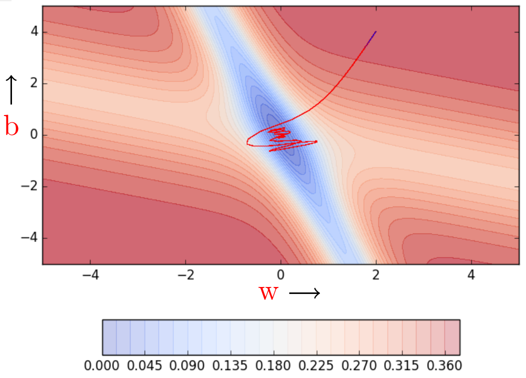

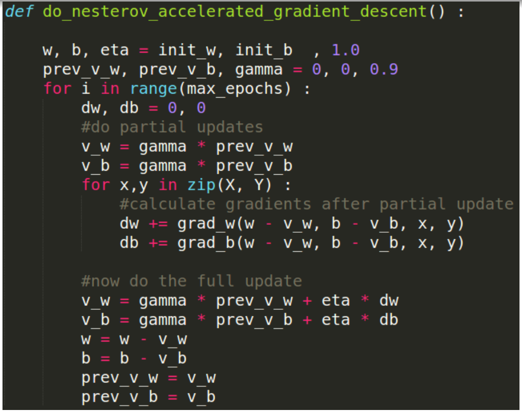

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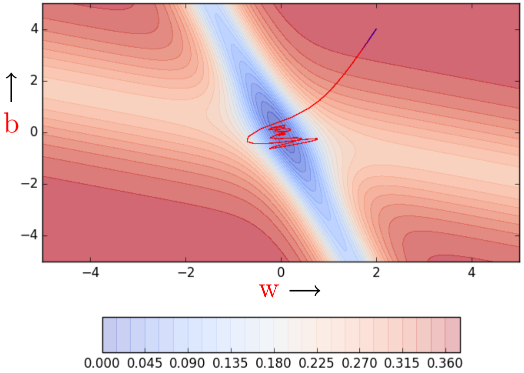

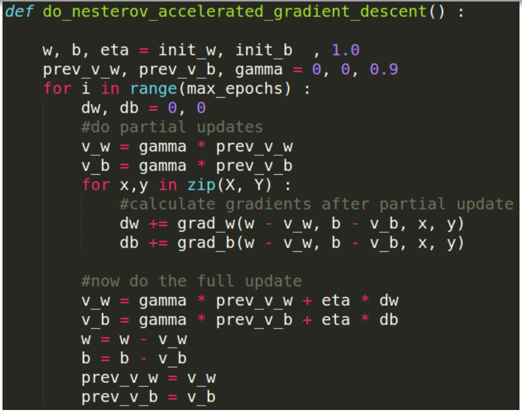

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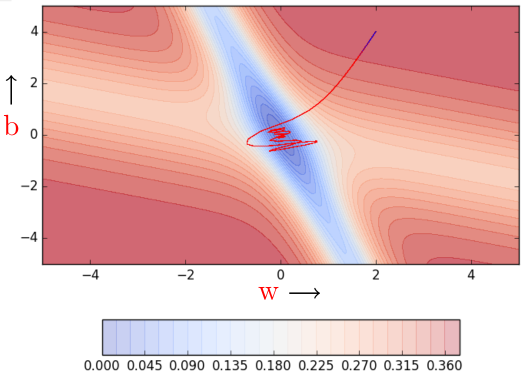

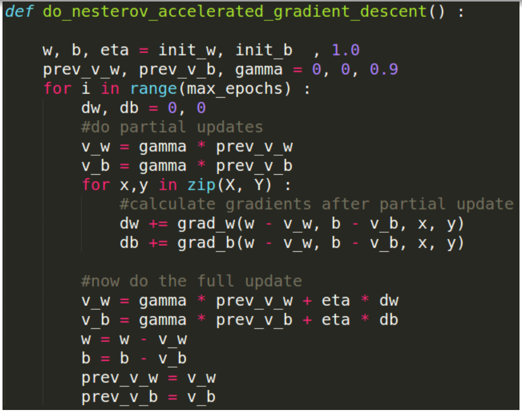

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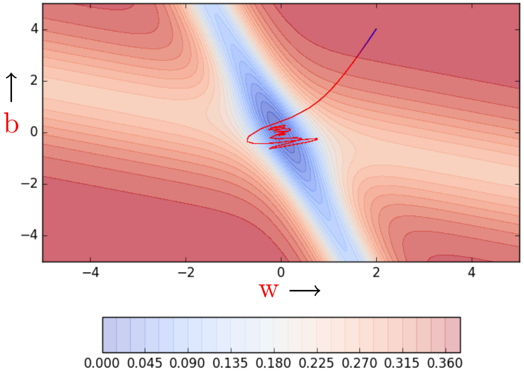

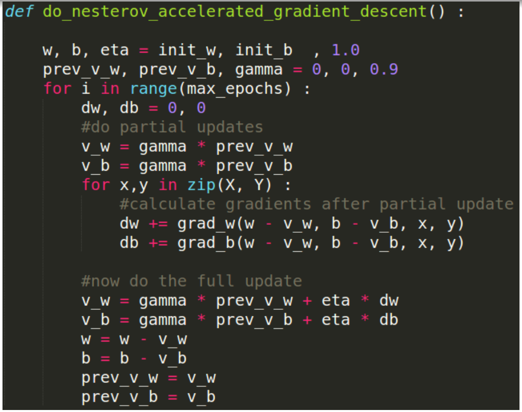

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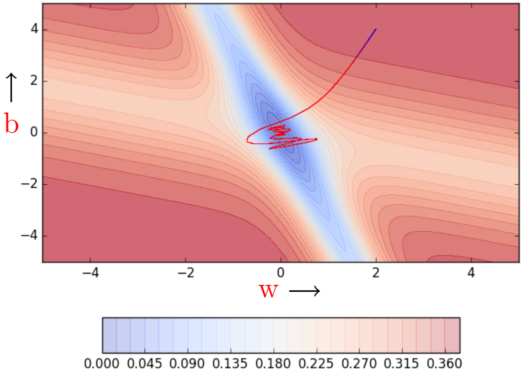

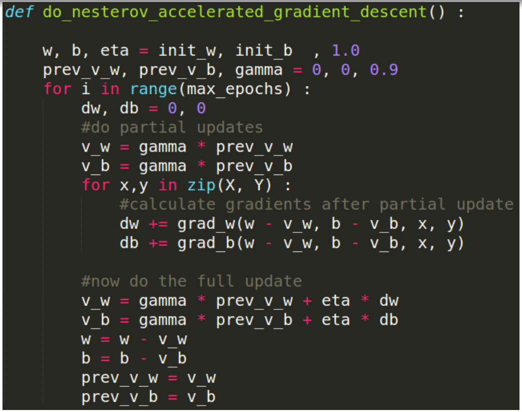

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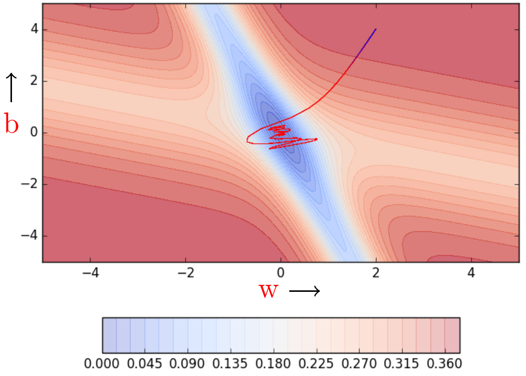

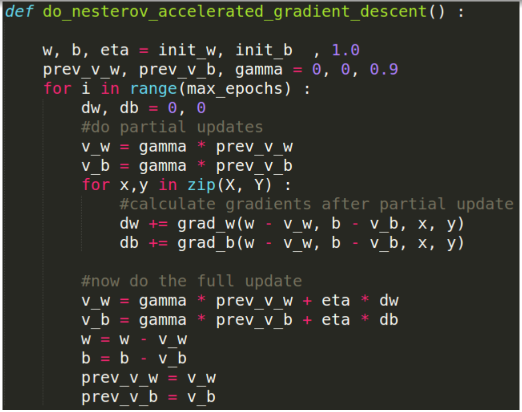

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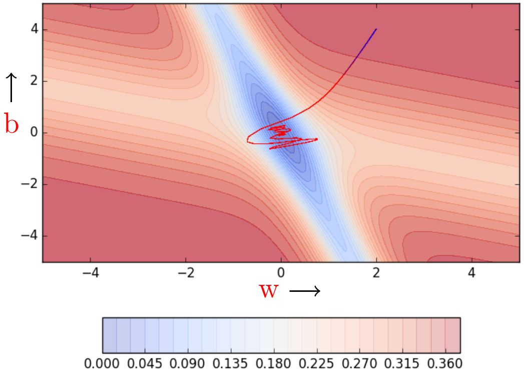

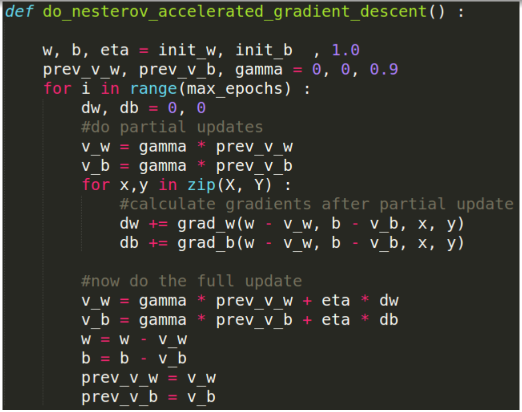

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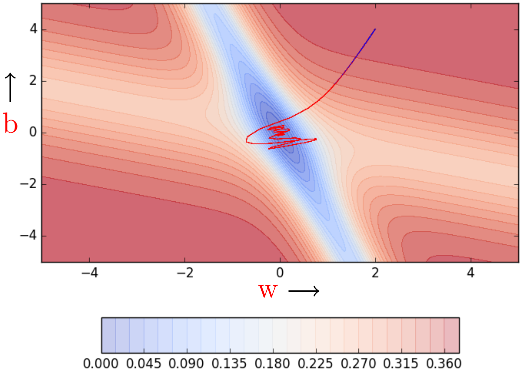

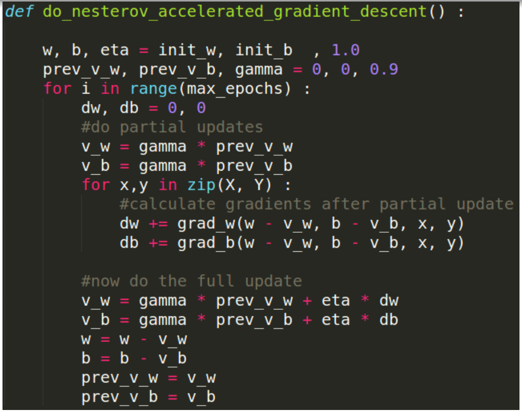

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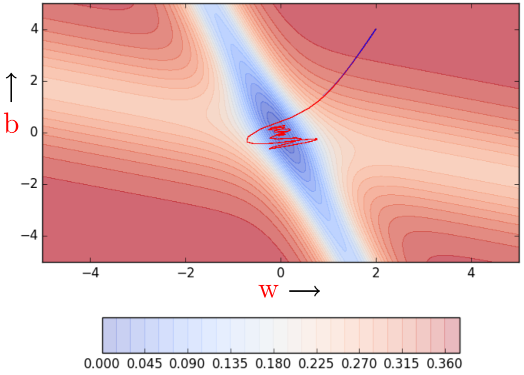

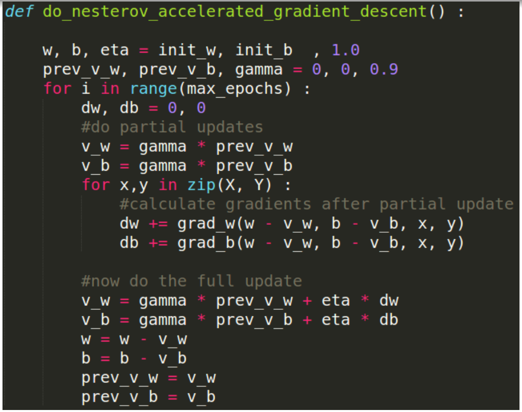
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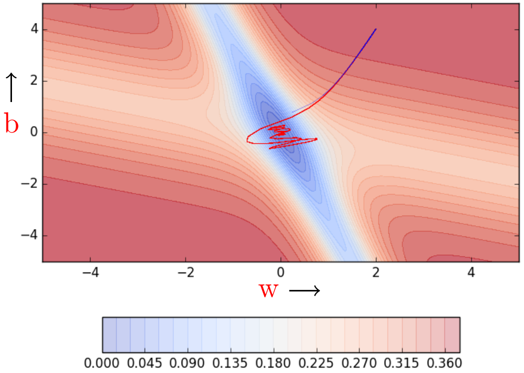

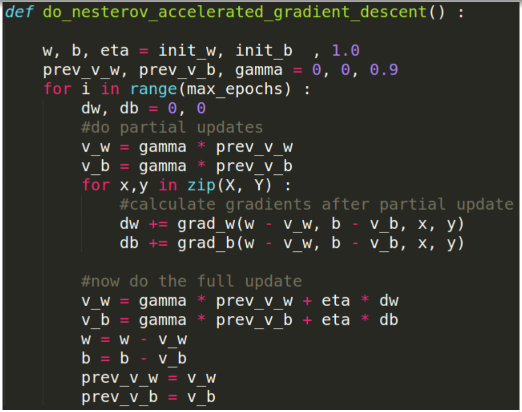

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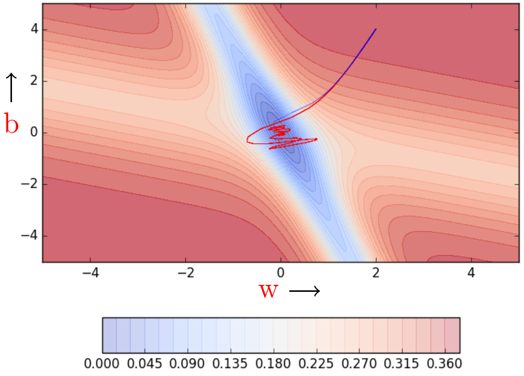

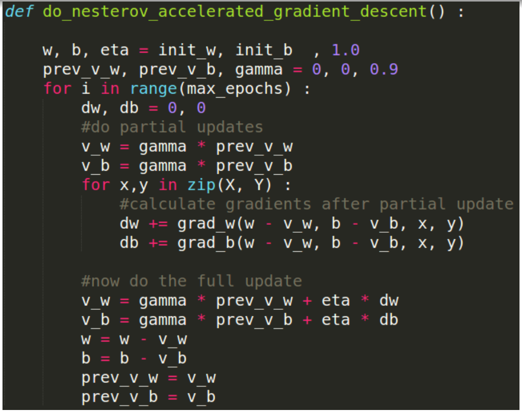

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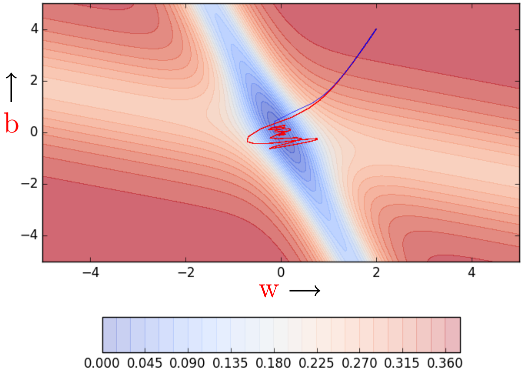

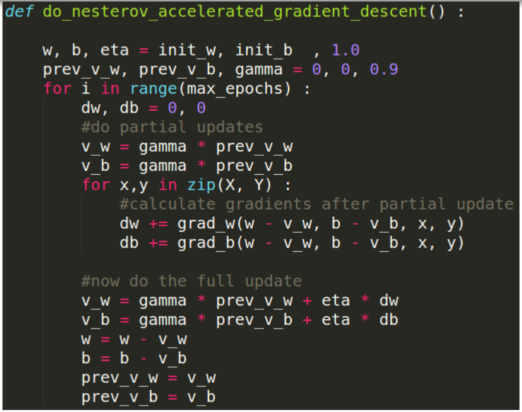

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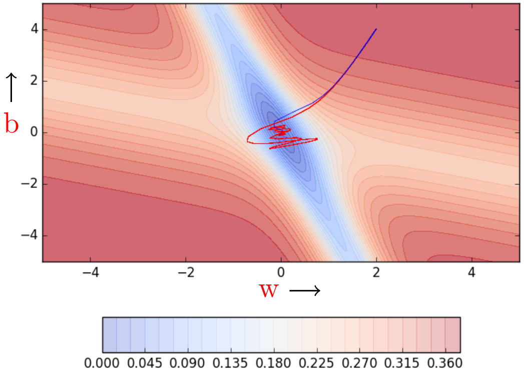

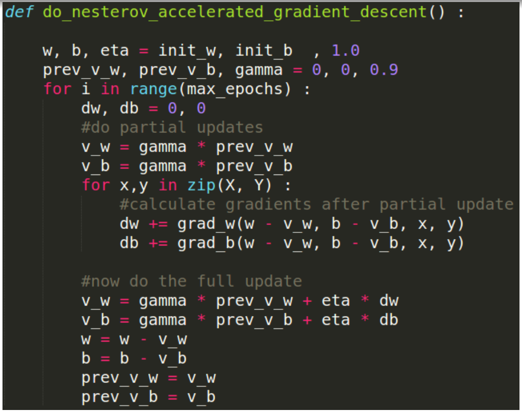

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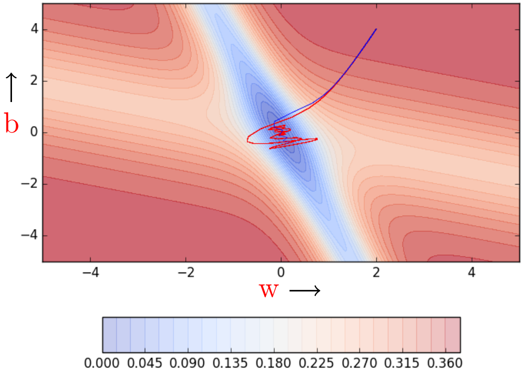

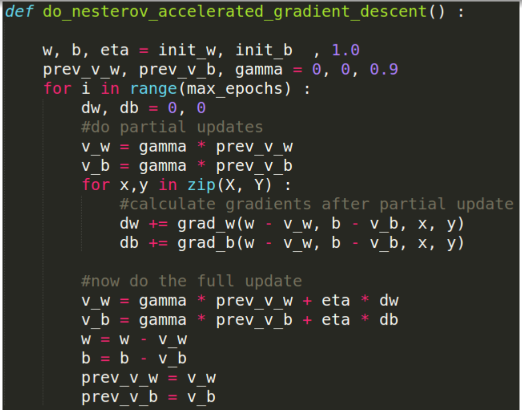

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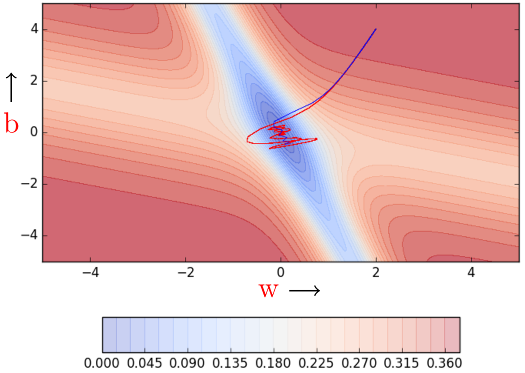

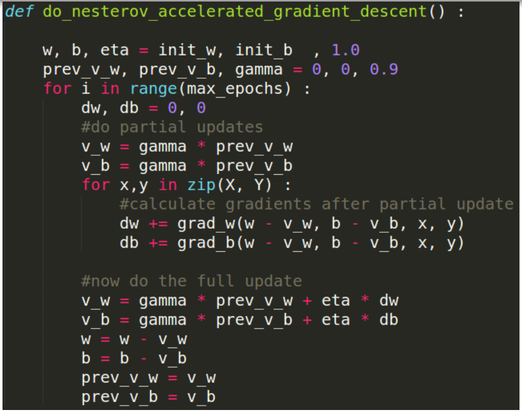

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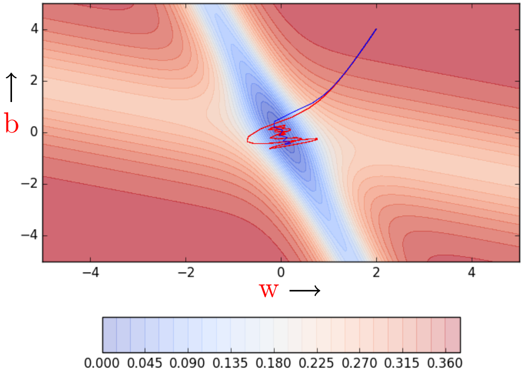

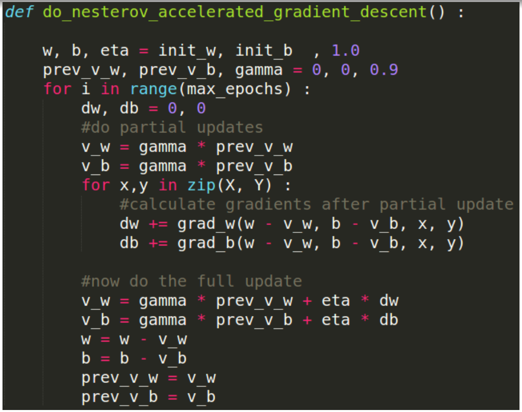

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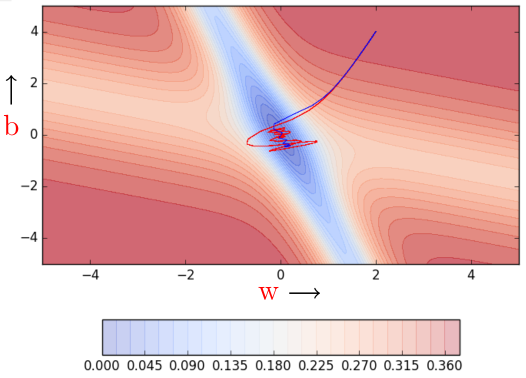

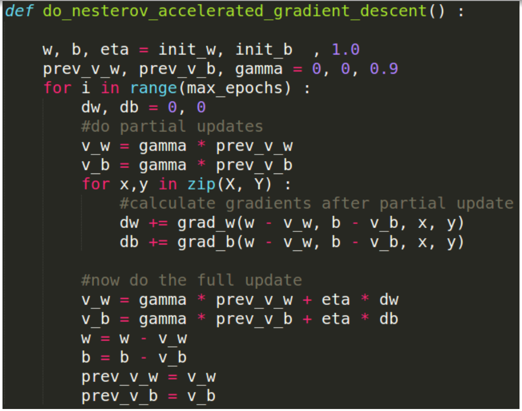

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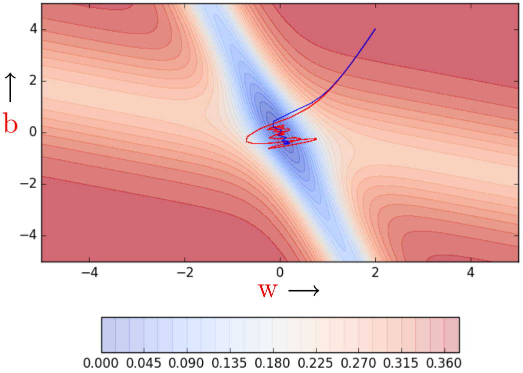

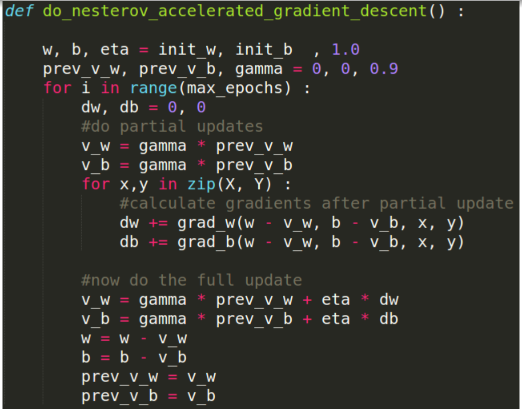

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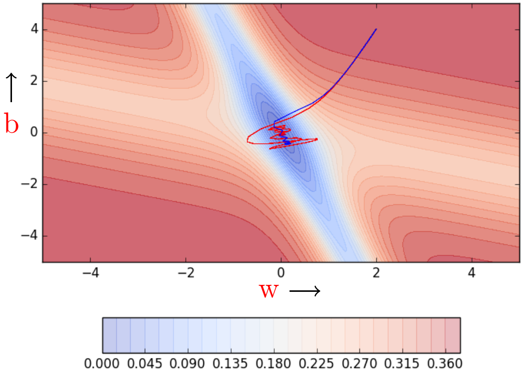

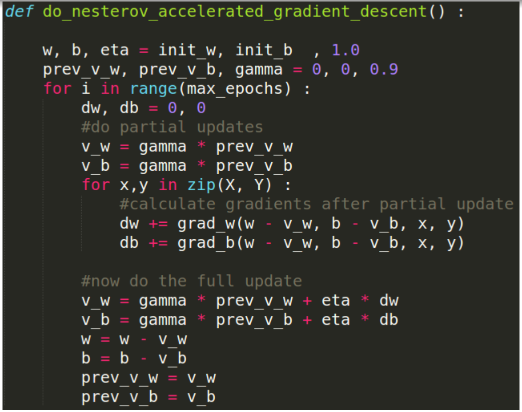

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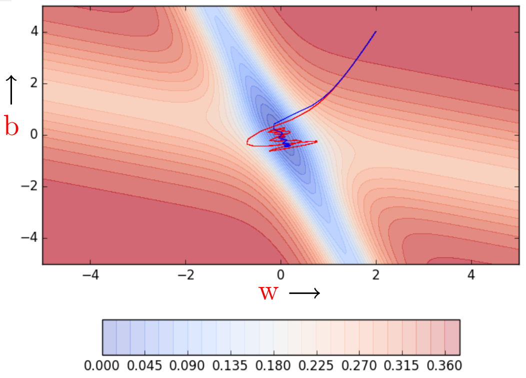

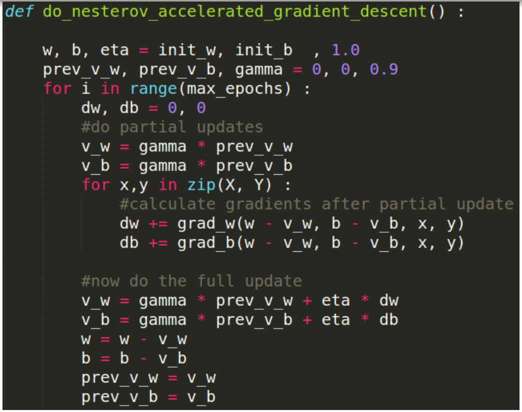
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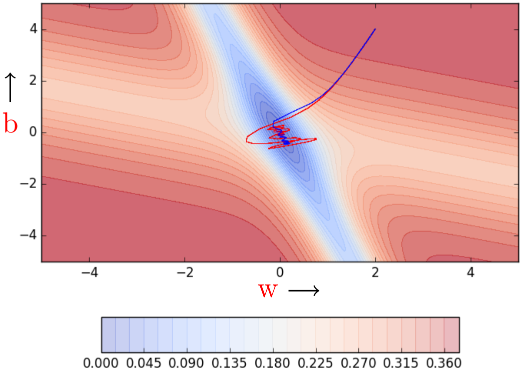

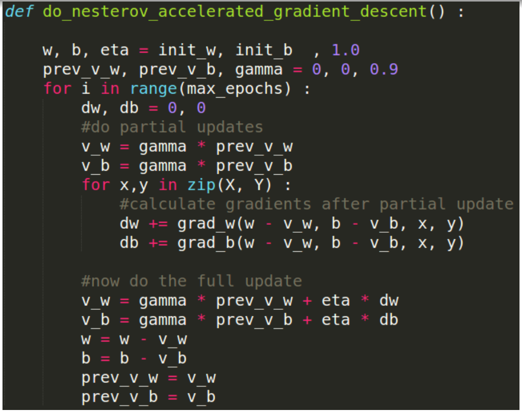

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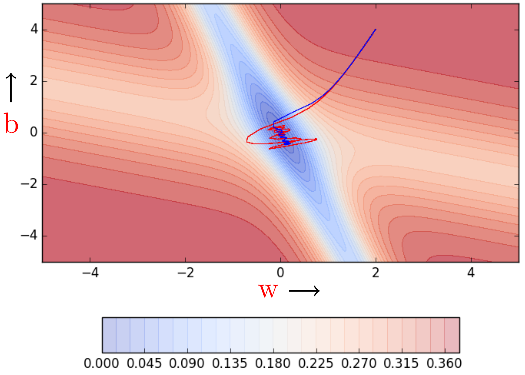

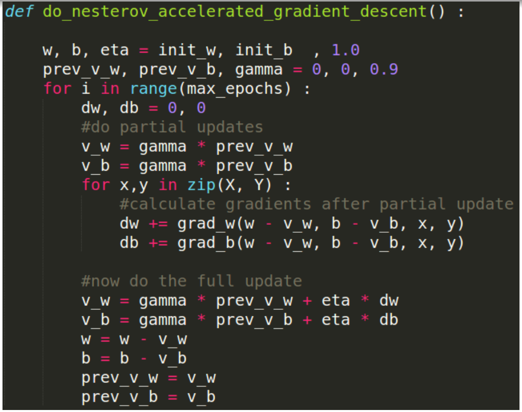

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def do_nesterov_accelerated_gradient_descent() :

    w, b, eta = init_w, init_b  , 1.0
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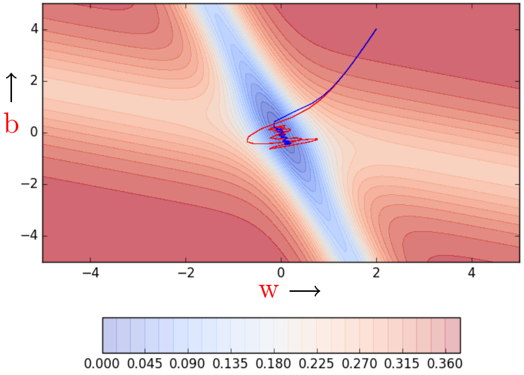

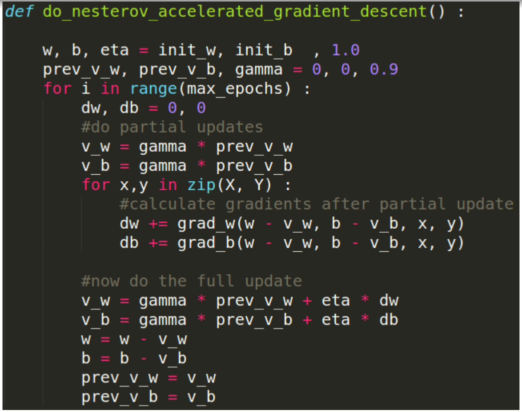

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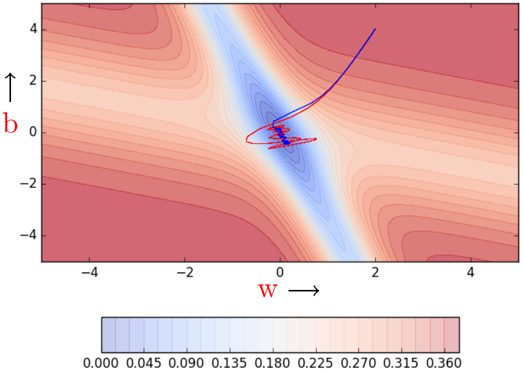

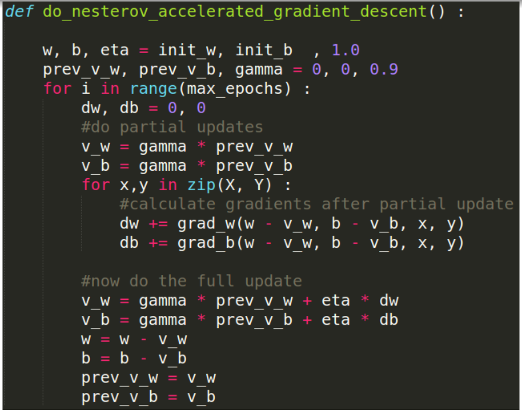

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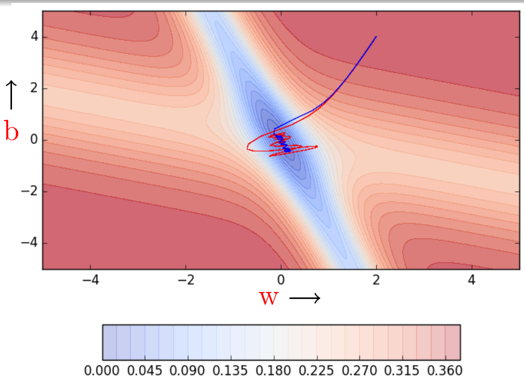

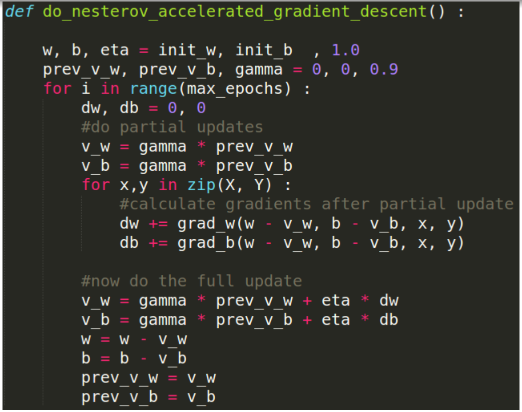

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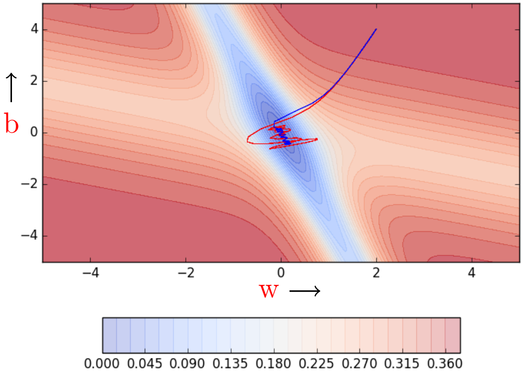

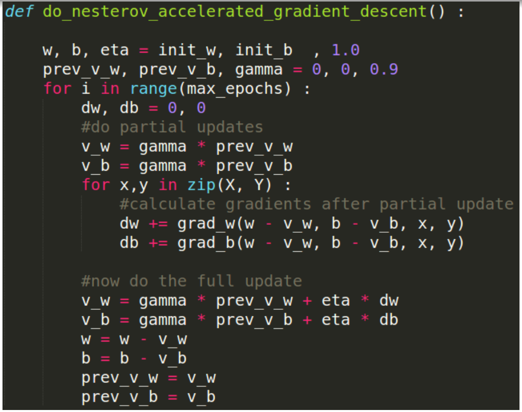

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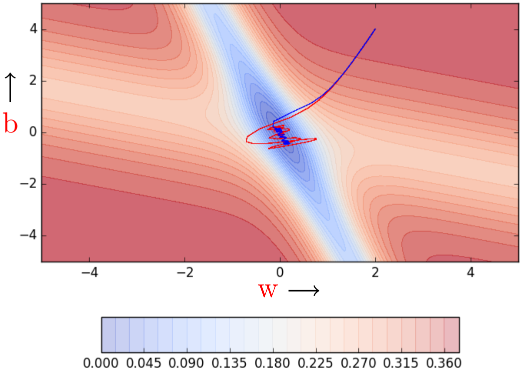

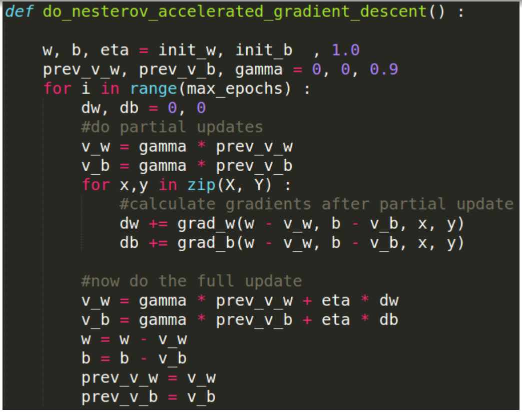

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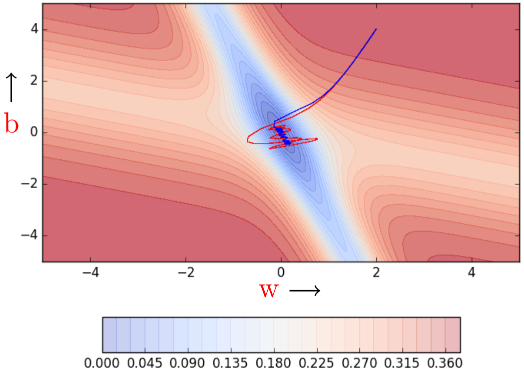

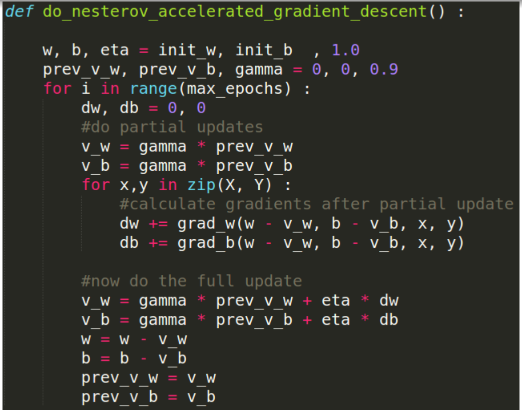

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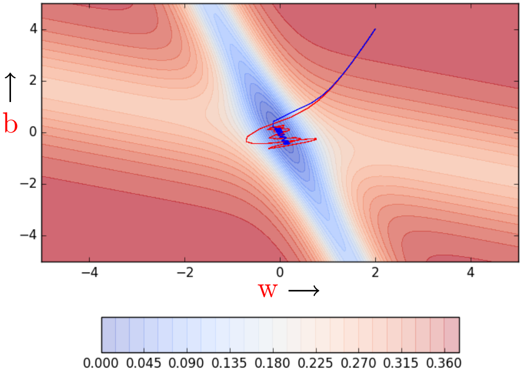

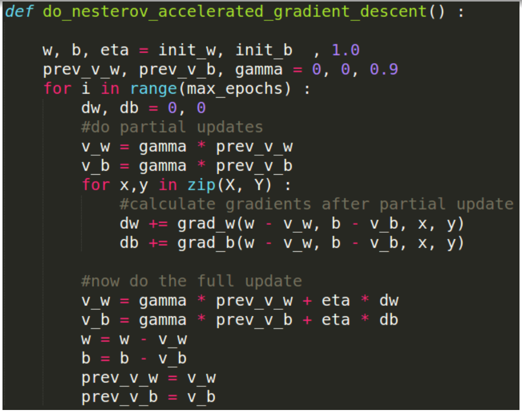

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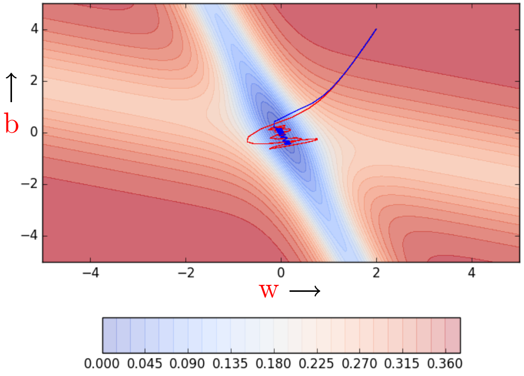

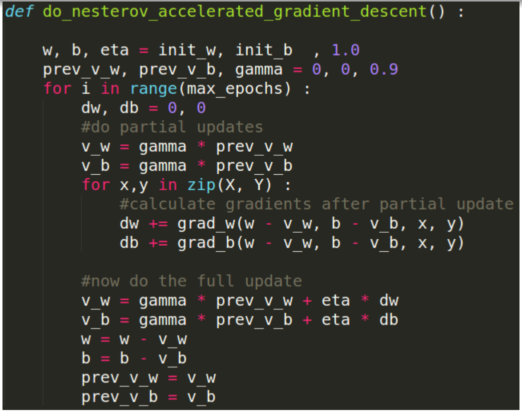

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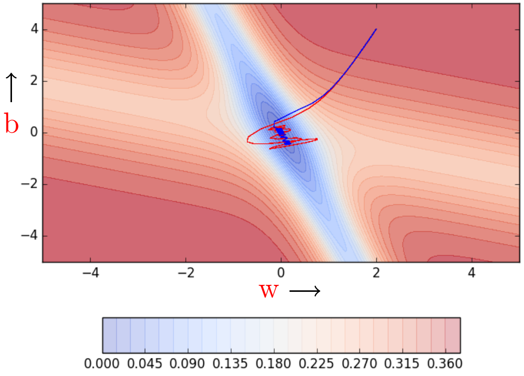

Observations about NAG

- Looking ahead helps NAG in correcting its course quicker than momentum based gradient descent

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- Looking ahead helps NAG in correcting its course quicker than momentum based gradient descent
- Hence the oscillations are smaller and the chances of escaping the minima valley also smaller