

Module 5.6 : Stochastic And Mini-Batch Gradient Descent

Let's digress a bit and talk about the stochastic version of these algorithms...

```

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Y = [0.2, 0.9]

def f(w, b, x): #sigmoid with parameters w,b
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    for i in range(max_epochs):
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        for x, y in zip(X, Y):
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        w = w - eta * dw
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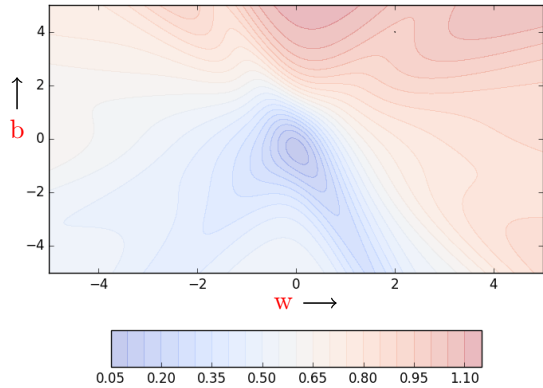
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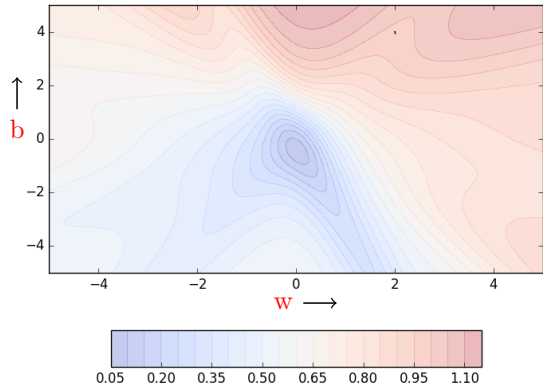
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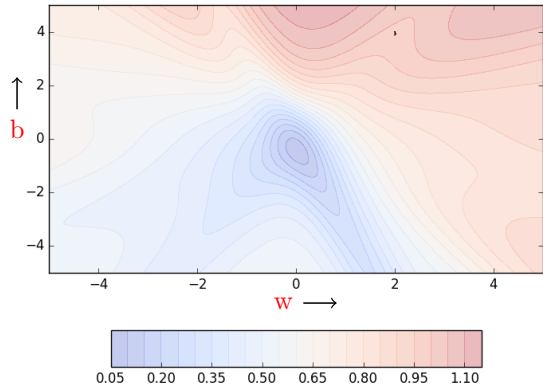
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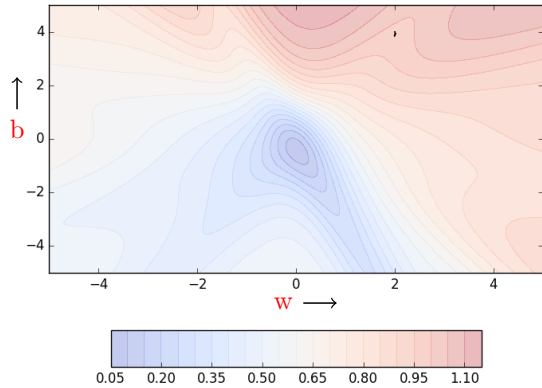
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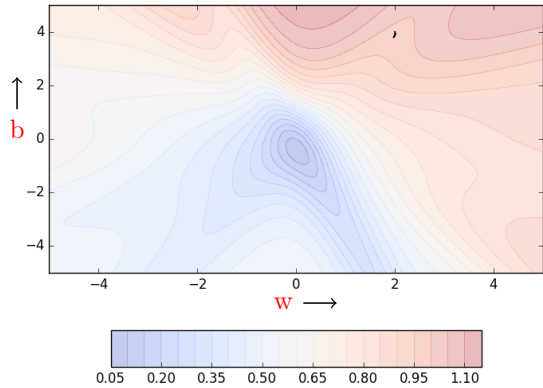
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- Let's see this algorithm in action when we have a few data points

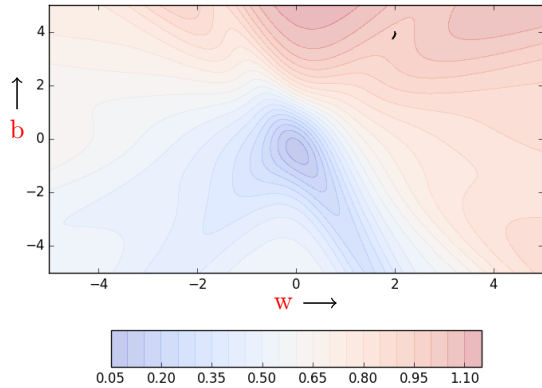


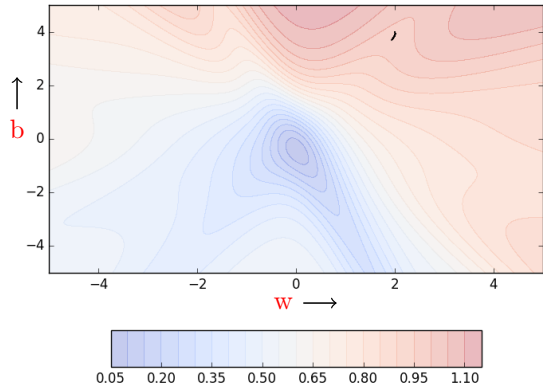


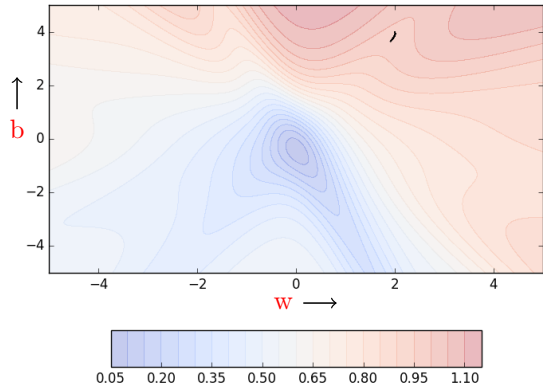


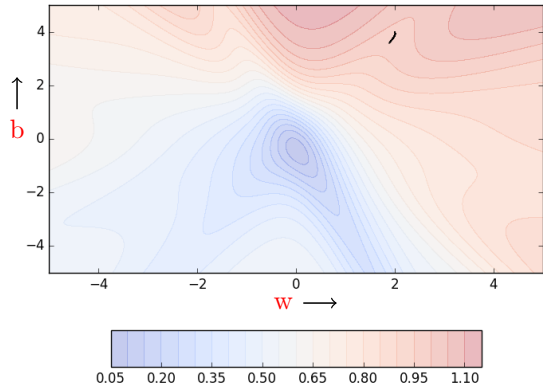


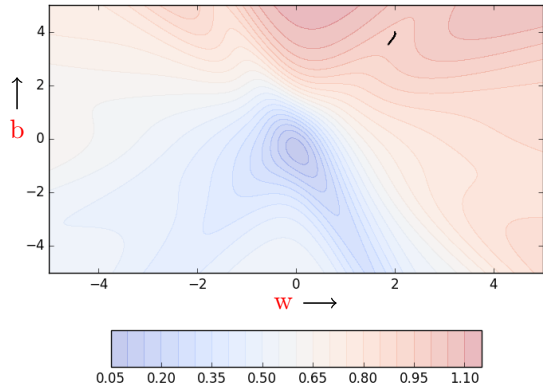


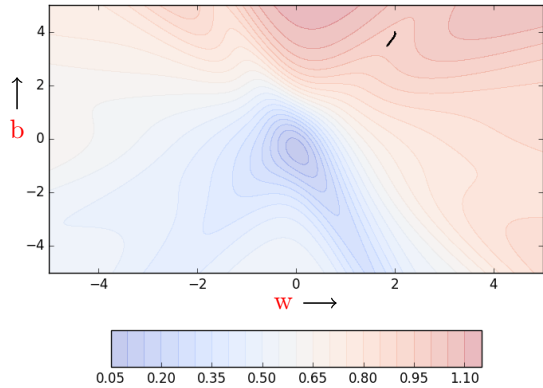


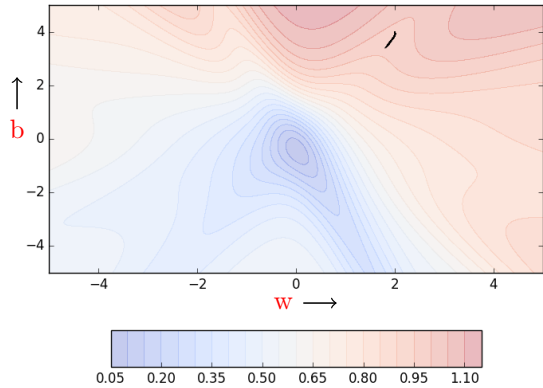


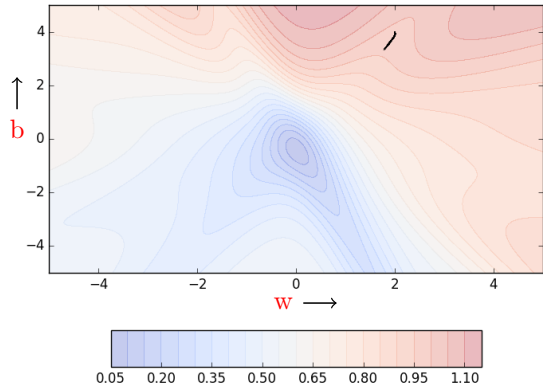


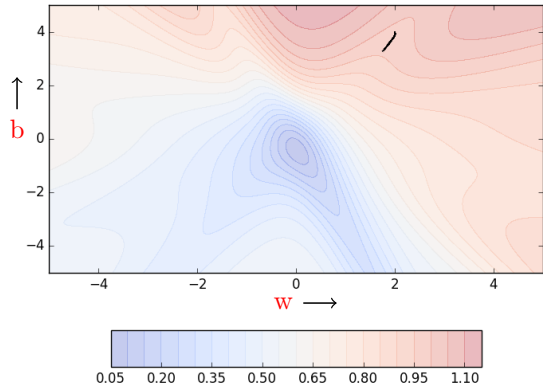


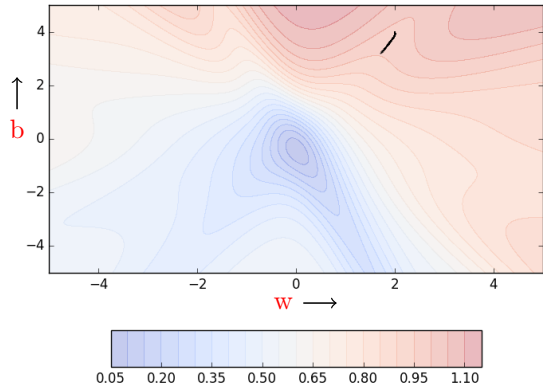


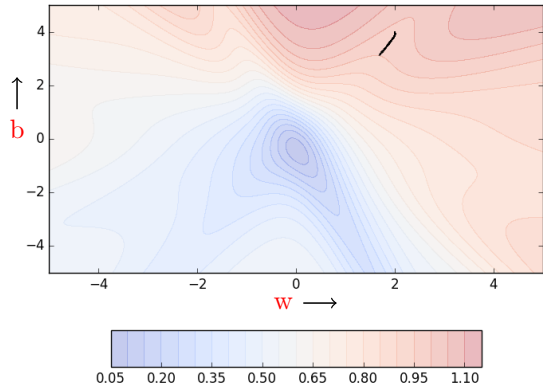


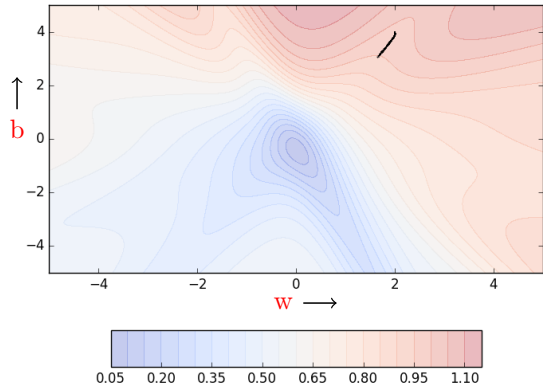


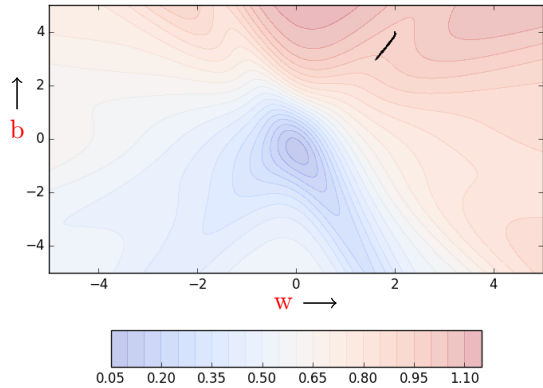


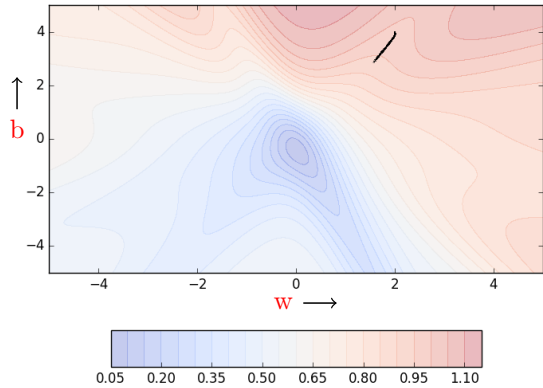


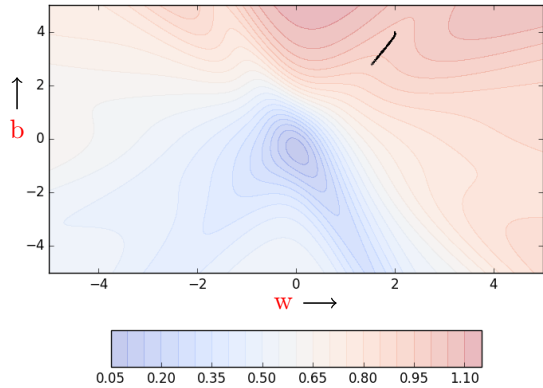


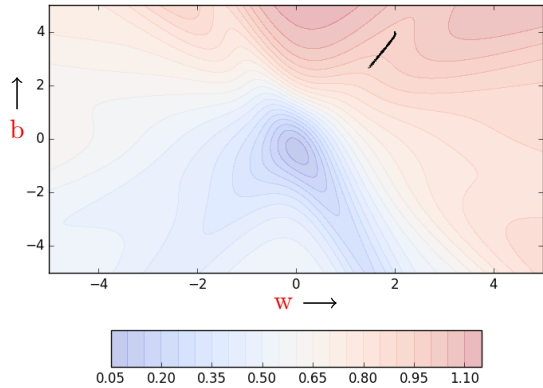


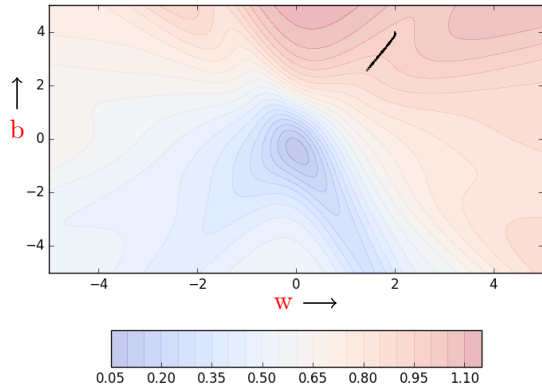


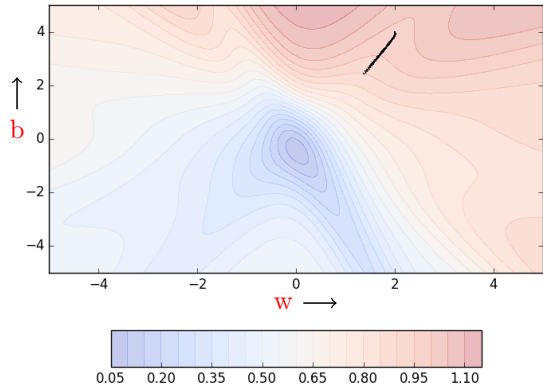


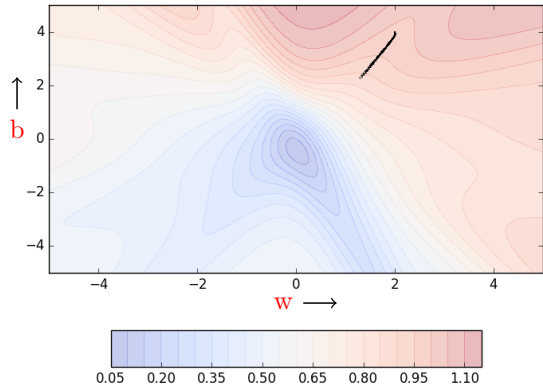


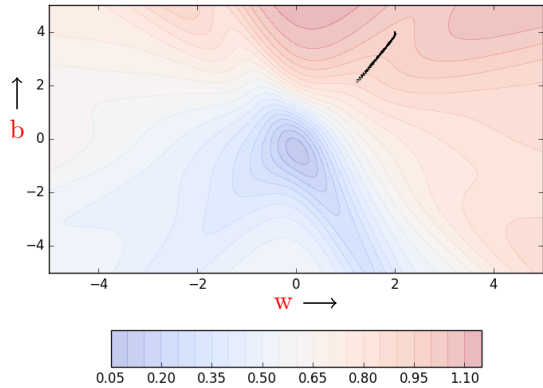


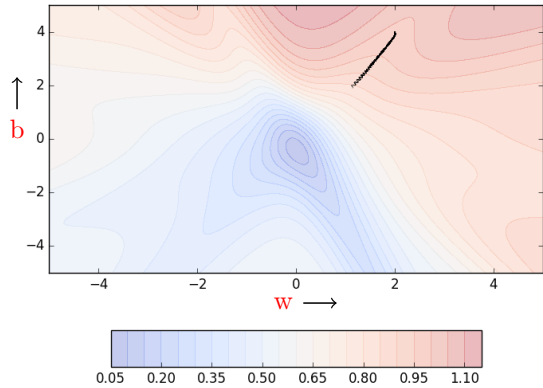


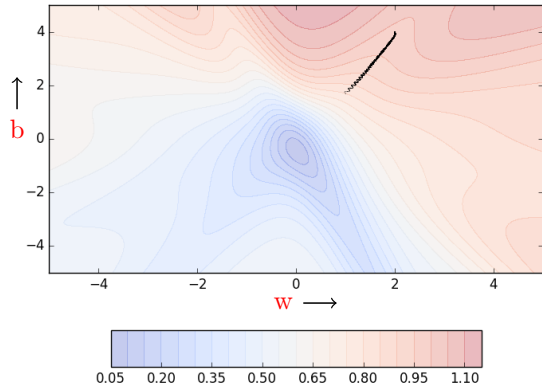


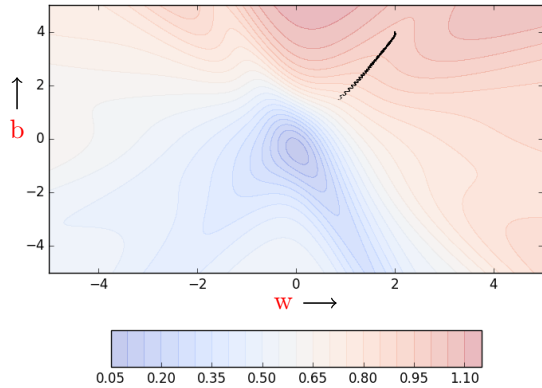


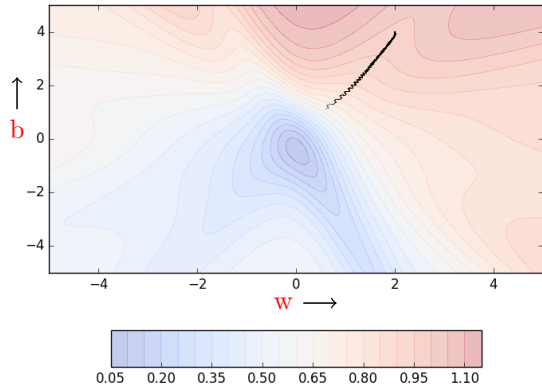


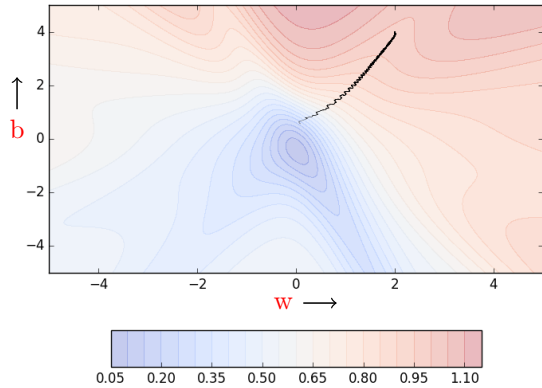


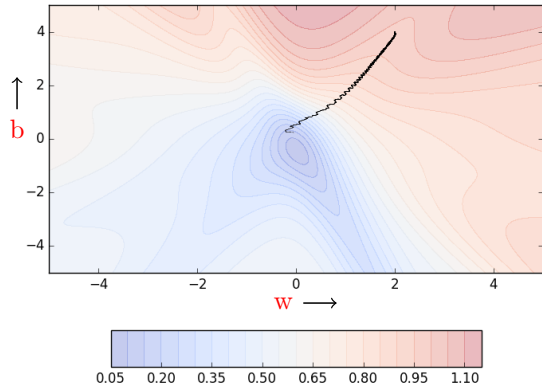


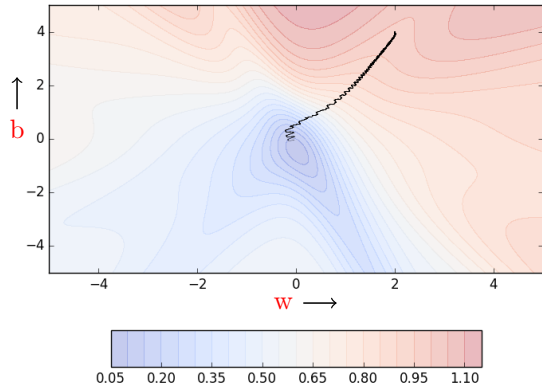


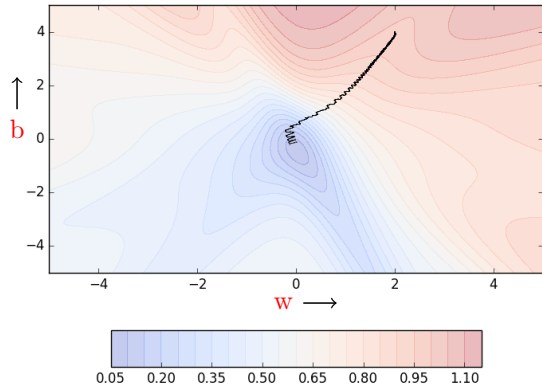


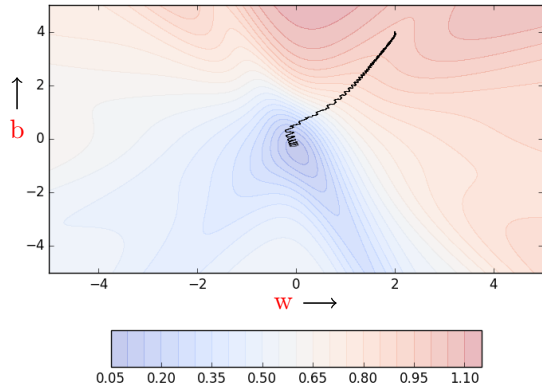


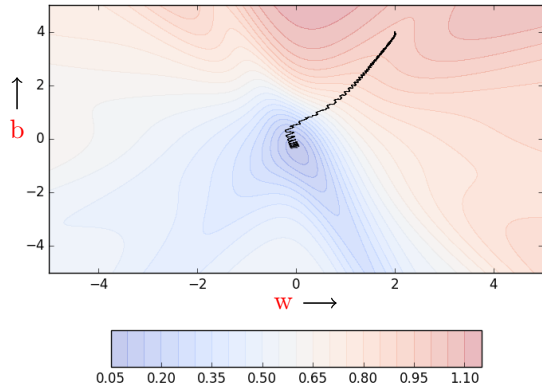


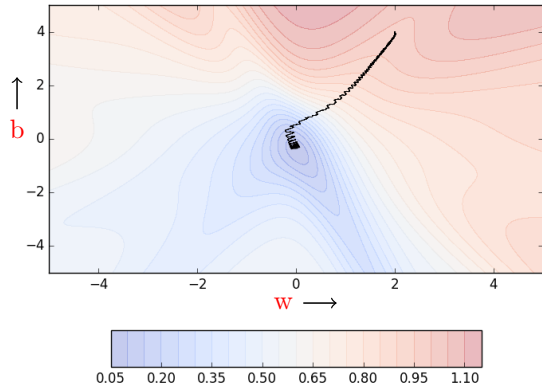




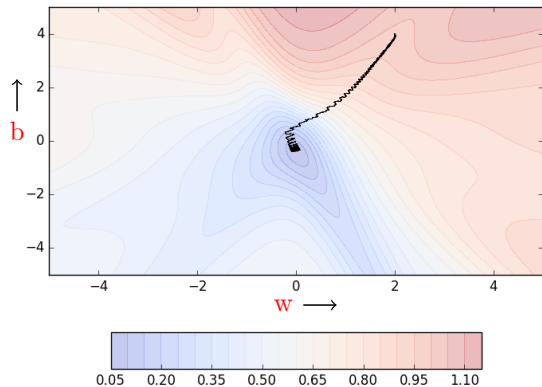




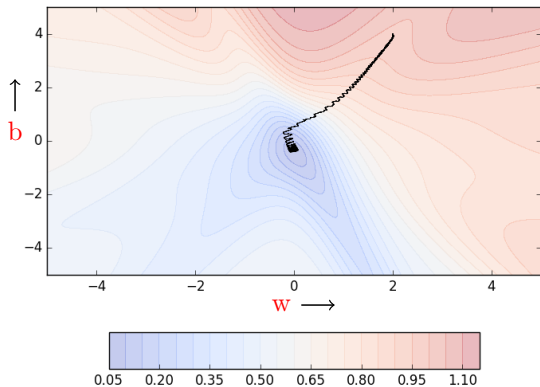




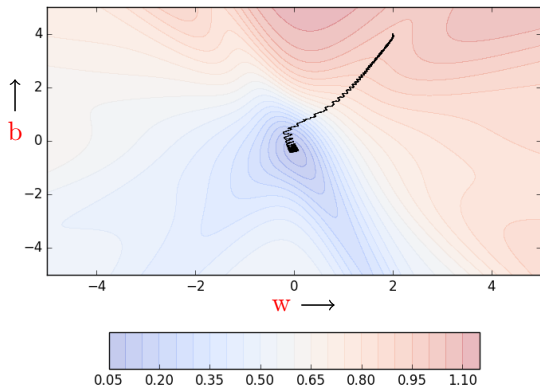
- We see many oscillations. Why ?



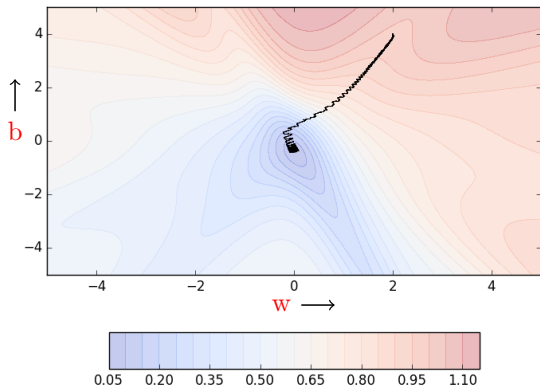
- We see many oscillations. Why? Because we are making greedy decisions.



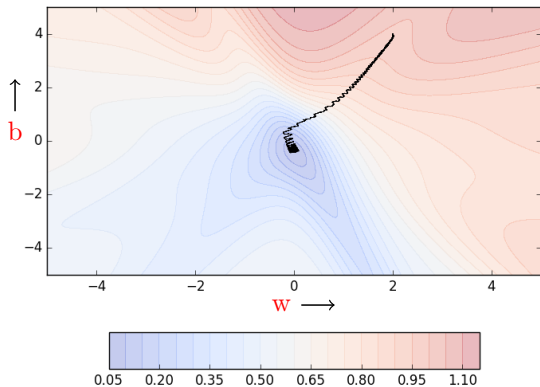
- We see many oscillations. Why? Because we are making greedy decisions.
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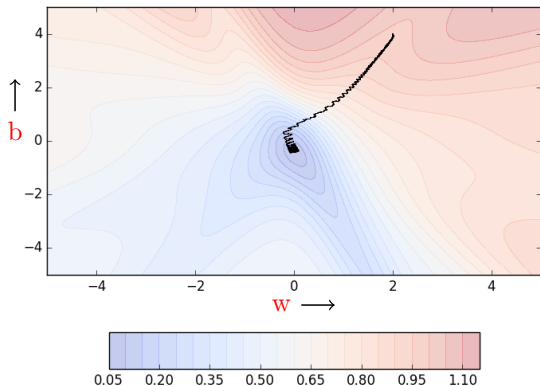
- We see many oscillations. Why ? Because we are making greedy decisions.
- Each point is trying to push the parameters in a direction most favorable to it (without being aware of how this affects other points)



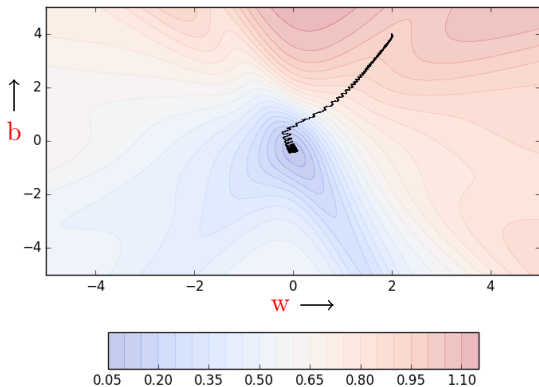
- We see many oscillations. Why ? Because we are making greedy decisions.
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- A parameter update which is locally favorable to one point may harm other points



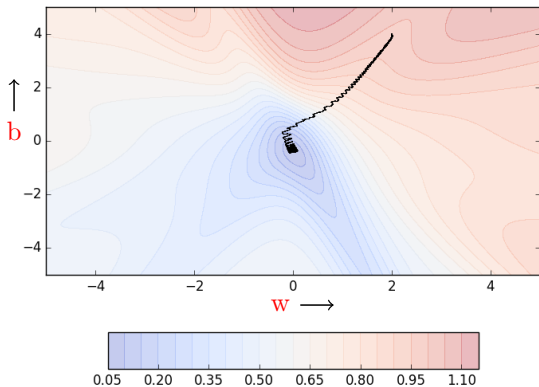
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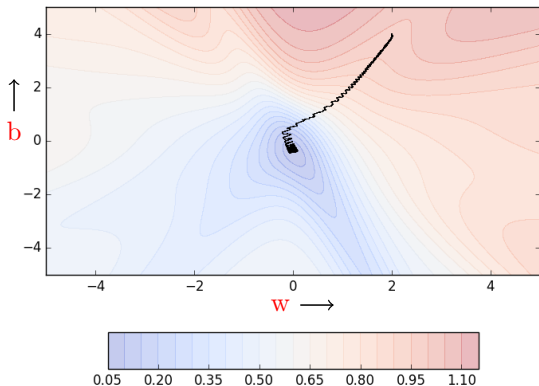
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- Can we reduce the oscillations by improving our stochastic estimates of the gradient



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- Can we reduce the oscillations by improving our stochastic estimates of the gradient (currently estimated from just 1 data point at a time)



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- A parameter update which is locally favorable to one point may harm other points (its almost as if the data points are competing with each other)
- Can we reduce the oscillations by improving our stochastic estimates of the gradient (currently estimated from just 1 data point at a time)
- Yes, let's look at mini-batch gradient descent



```
def do_mini_batch_gradient_descent() :
    w, b, eta = -2, -2, 1.0
    mini_batch_size, num_points_seen = 2, 0
    for i in range(max_epochs) :
        dw, db, num_points = 0, 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)
            num_points_seen +=1

        if num_points_seen % mini_batch_size == 0 :
            # seen one mini_batch
            w = w - eta * dw
            b = b - eta * db
            dw, db = 0, 0 #reset gradients
```

- Notice that the algorithm updates the parameters after it sees *mini_batch_size* number of data points

```
def do_stochastic_gradient_descent():
    w, b, eta, max_epochs = -2, -2, 1.0, 1000
    for i in range(max_epochs):
        dw, db = 0, 0
        for x, y in zip(X, Y):
            dw = grad_w(w, b, x, y)
            db = grad_b(w, b, x, y)
            w = w - eta * dw
            b = b - eta * db
```

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- Notice that the algorithm updates the parameters after it sees *mini_batch_size* number of data points
- The stochastic estimates are now slightly better

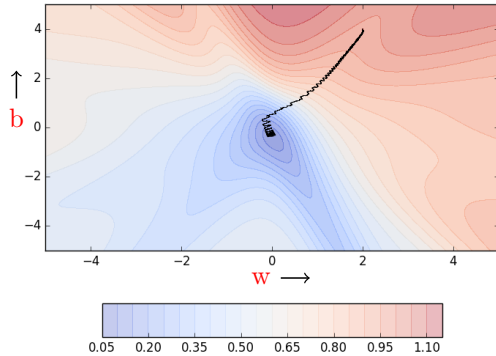
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    w, b, eta, max_epochs = -2, -2, 1.0, 1000
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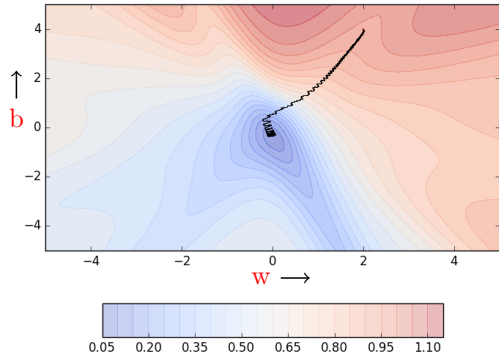
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def do_mini_batch_gradient_descent() :
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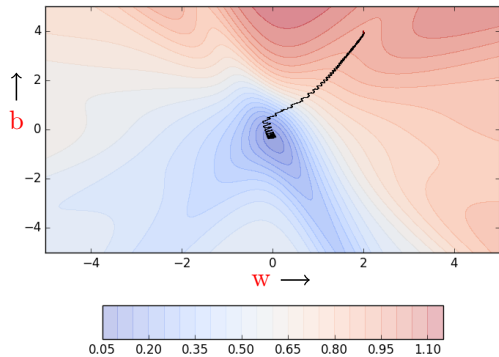
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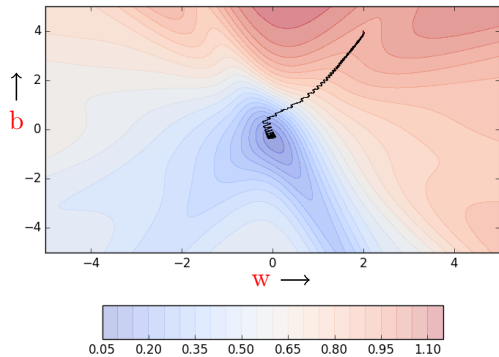
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def do_stochastic_gradient_descent():
    w, b, eta, max_epochs = -2, -2, 1.0, 1000
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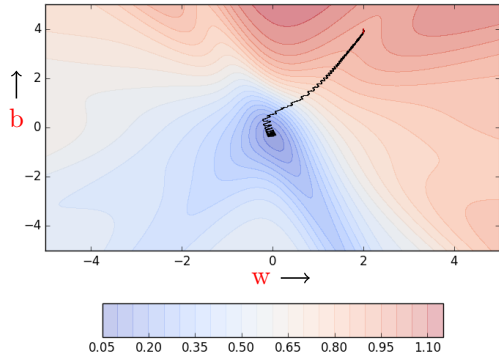
- Notice that the algorithm updates the parameters after it sees *mini_batch_size* number of data points
- The stochastic estimates are now slightly better
- Let's see this algorithm in action when we have $k = 2$

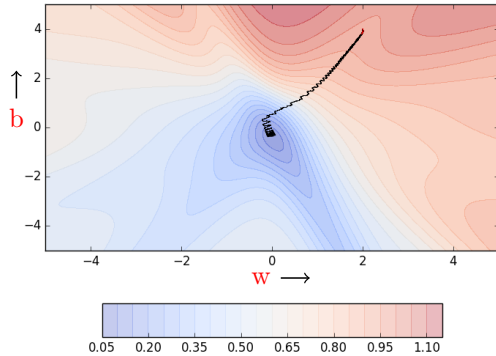


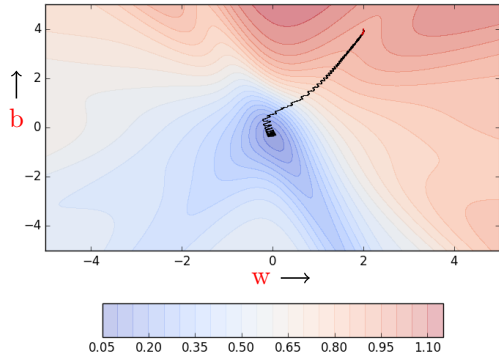


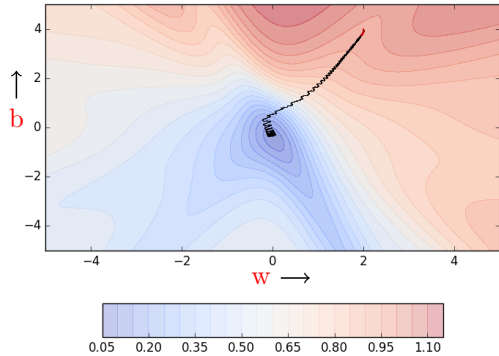


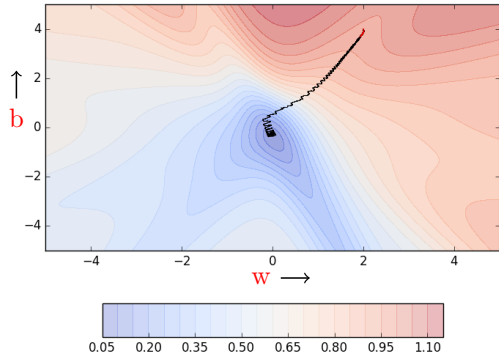


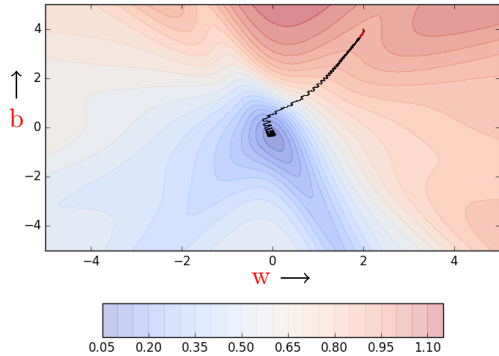


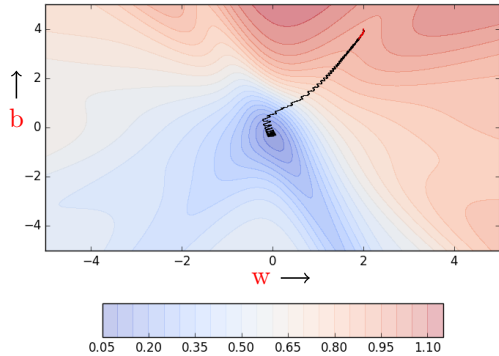


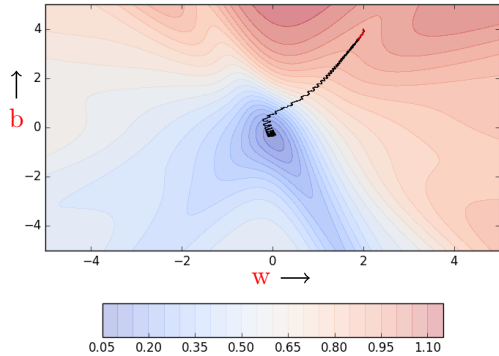


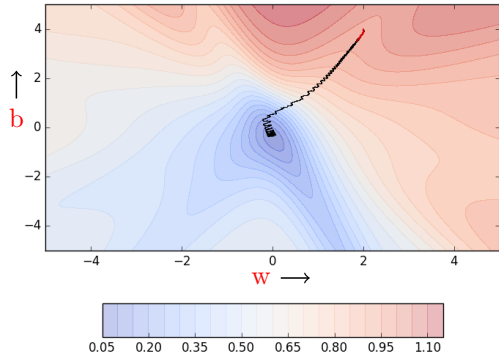


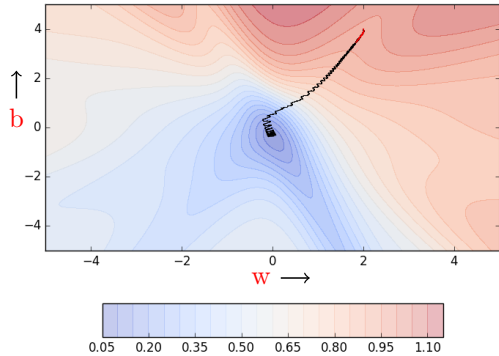


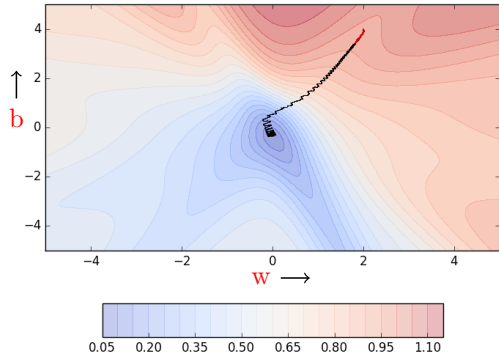


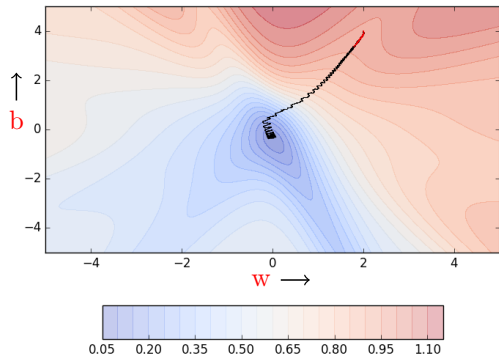


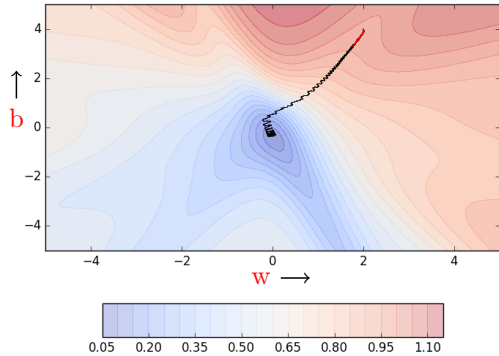


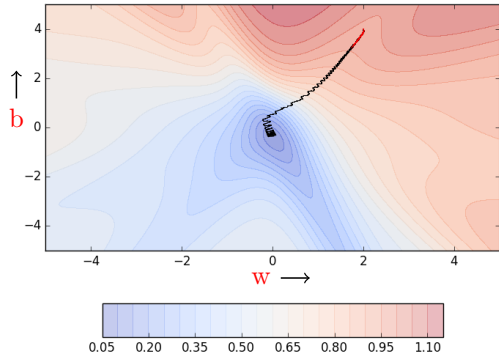


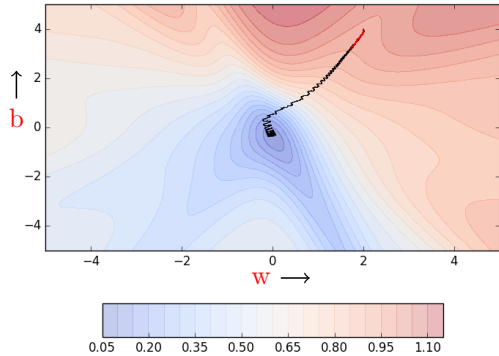


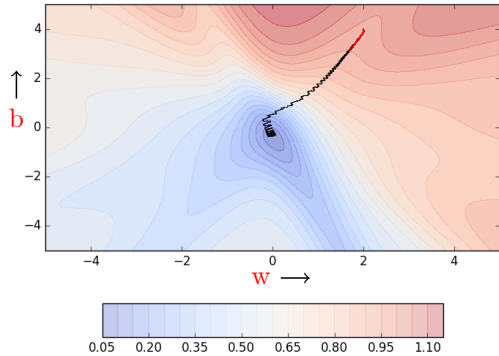


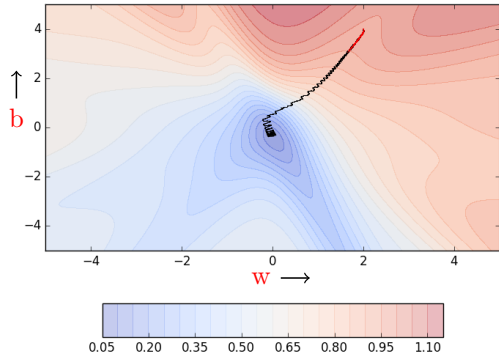


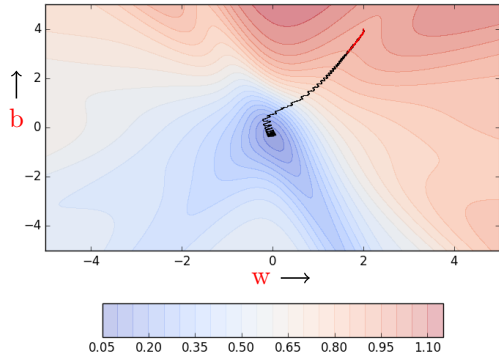


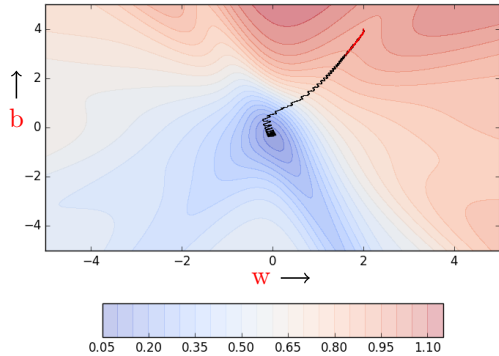


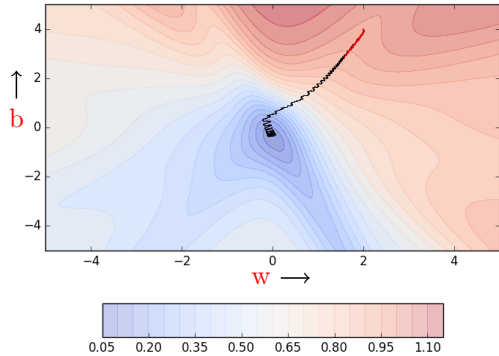


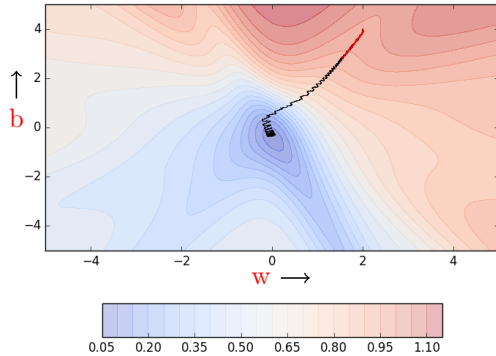


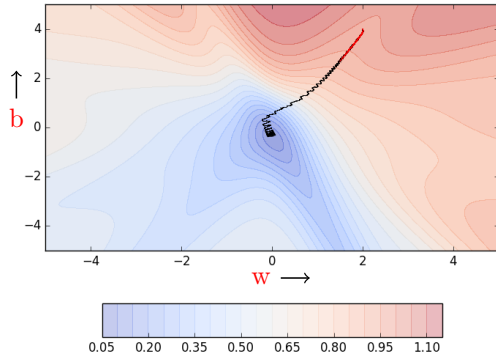


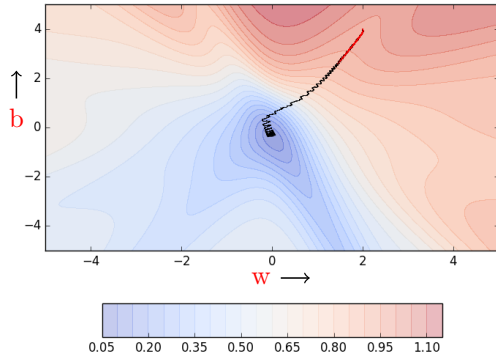


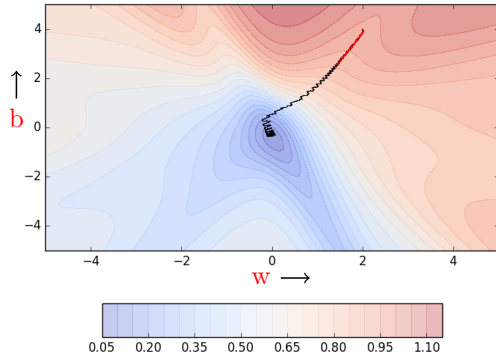


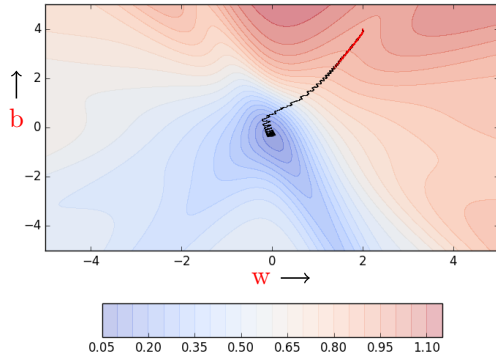


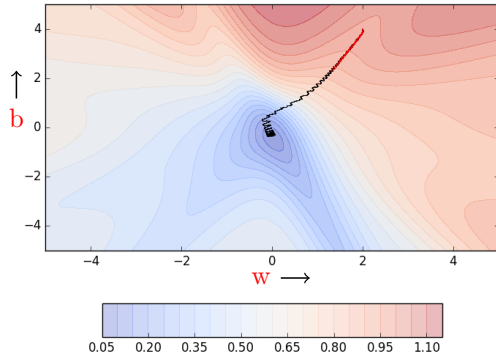


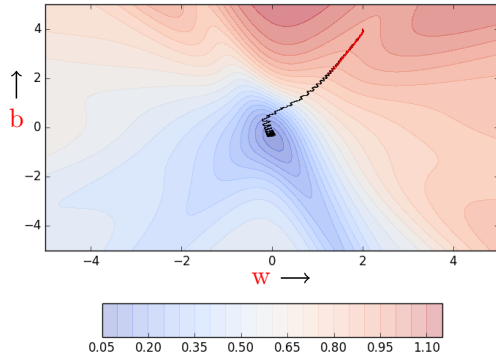


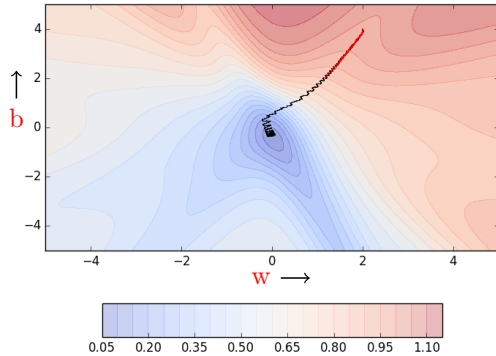


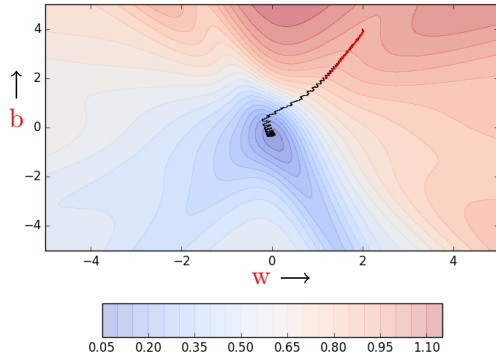


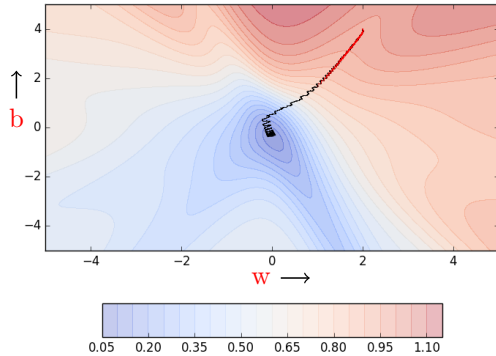


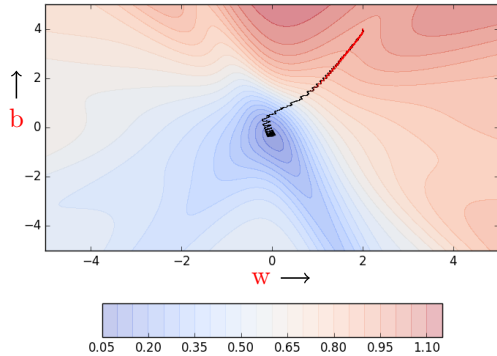


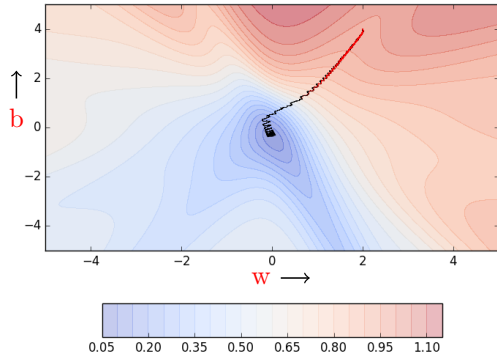


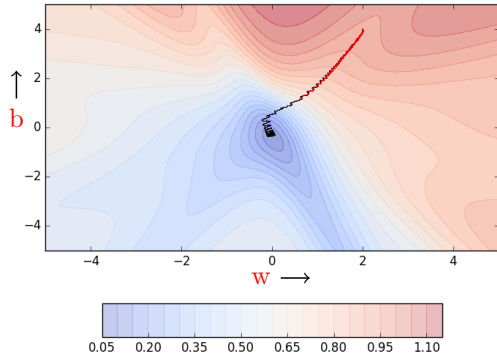


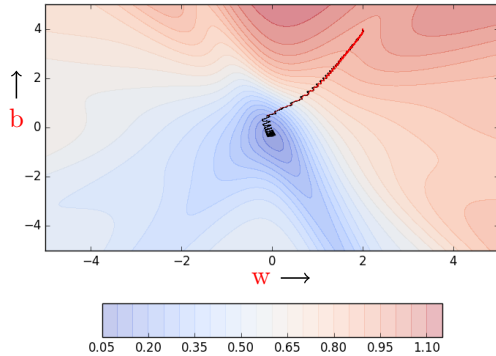


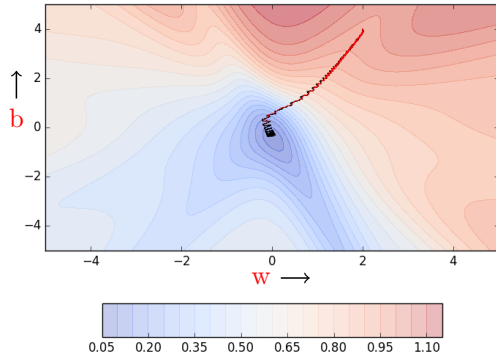


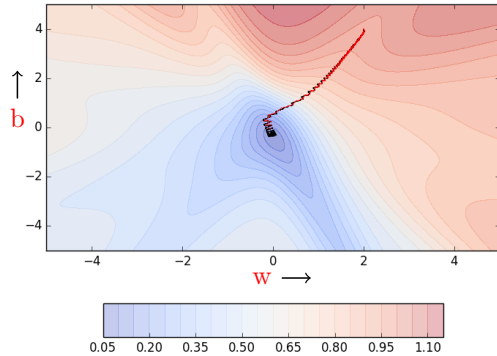


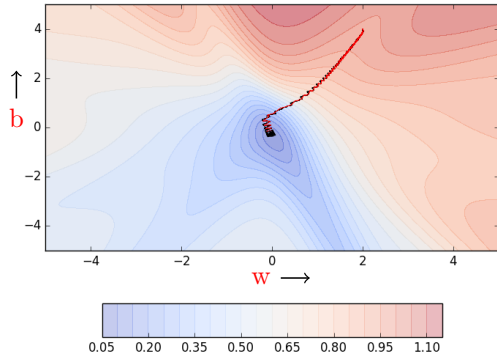


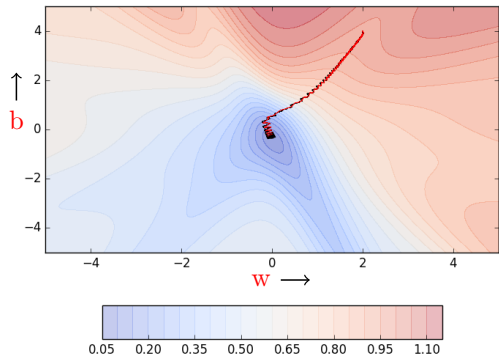


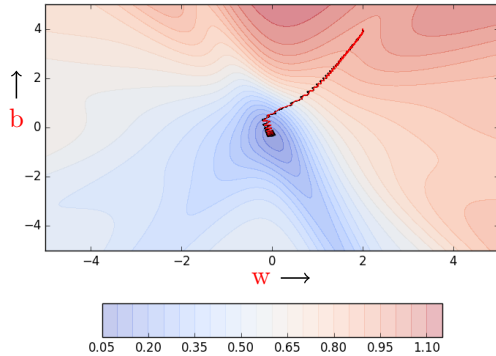


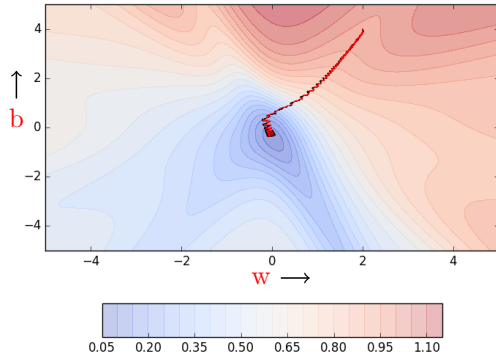




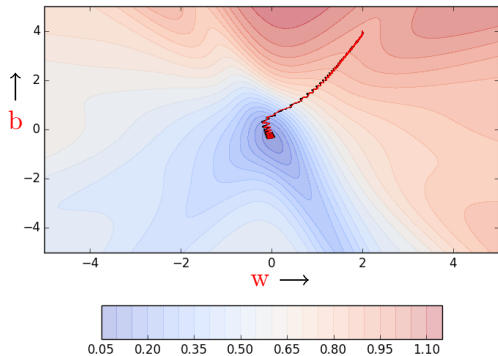




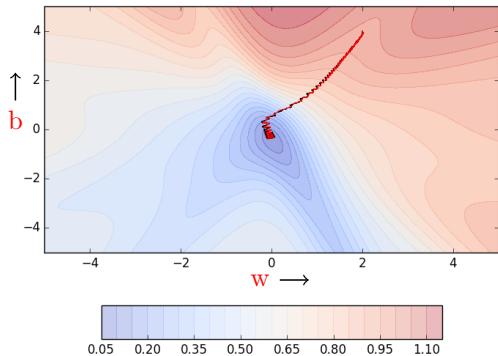




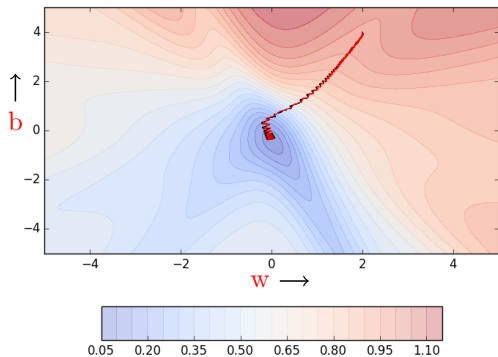
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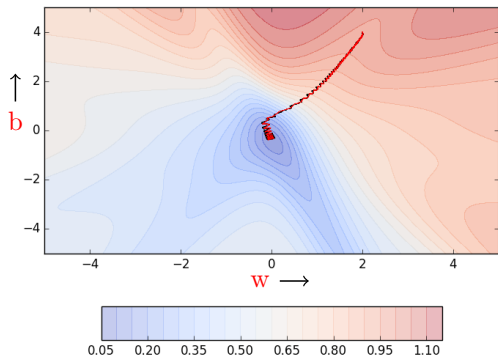
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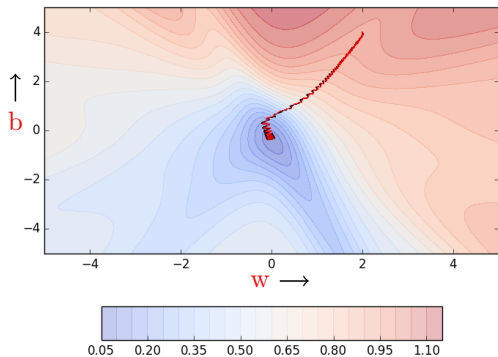
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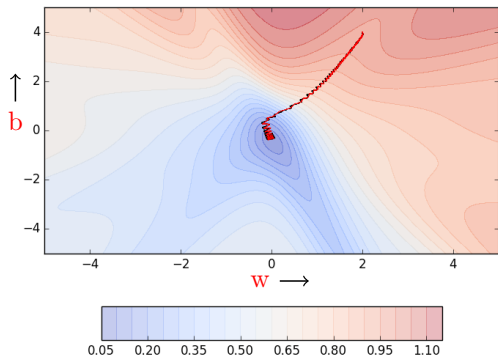
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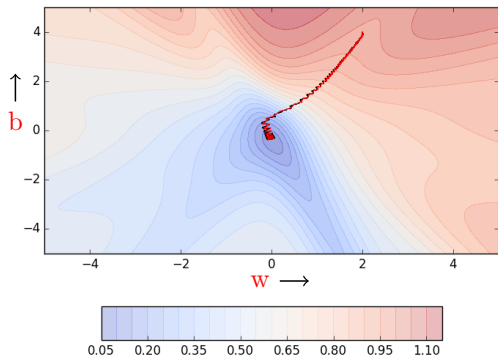
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- The higher the value of k the more accurate are the estimates
- In practice, typical values of k are 16, 32, 64
- Of course, there are still oscillations and they will always be there as long as we are using an approximate gradient as opposed to the true gradient



Some things to remember

- 1 epoch = one pass over the entire data
- 1 step = one update of the parameters
- N = number of data points
- B = Mini batch size

Algorithm	# of steps in 1 epoch
Vanilla (Batch) Gradient Descent	
Stochastic Gradient Descent	
Mini-Batch Gradient Descent	

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Mini-Batch Gradient Descent	$\frac{N}{B}$

Similarly, we can have stochastic versions of Momentum based gradient descent and Nesterov accelerated based gradient descent

```
def do_momentum_gradient_descent() :
    w, b, eta = init_w, init_b, 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw += grad_w(w, b, x, y)
            db += grad_b(w, b, x, y)

        v_w = gamma * prev_v_w + eta* dw
        v_b = gamma * prev_v_b + eta* db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
        prev_v_b = v_b
```

```
def do_stochastic_momentum_gradient_descent() :
    w, b, eta = init_w, init_b, 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            dw = grad_w(w, b, x, y)
            db = grad_b(w, b, x, y)

        v_w = gamma * prev_v_w + eta* dw
        v_b = gamma * prev_v_b + eta* db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
        prev_v_b = v_b
```

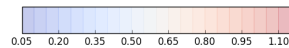
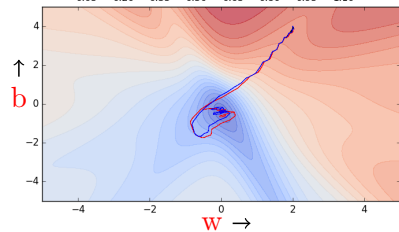
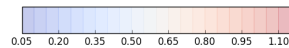
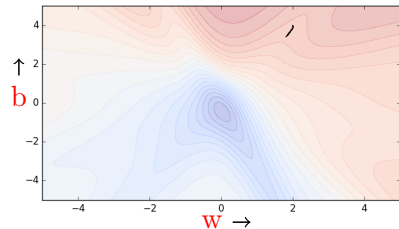


```
def do_nesterov_accelerated_gradient_descent() :
    w, b, eta = init_w, init_b , 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        #do partial updates
        v_w = gamma * prev_v_w
        v_b = gamma * prev_v_b
        for x,y in zip(X, Y) :
            #calculate gradients after partial update
            dw += grad_w(w - v_w, b - v_b, x, y)
            db += grad_b(w - v_w, b - v_b, x, y)

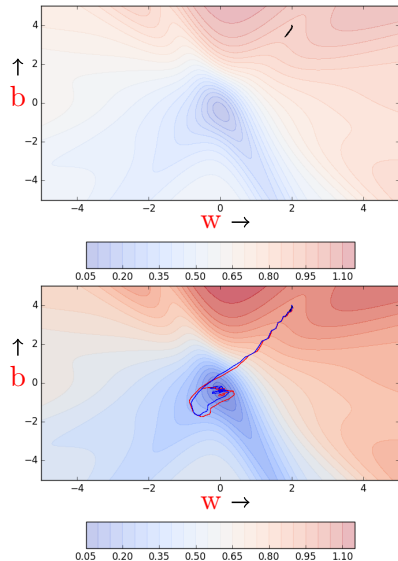
        #now do the full update
        v_w = gamma * prev_v_w + eta * dw
        v_b = gamma * prev_v_b + eta * db
        w = w - v_w
        b = b - v_b
        prev_v_w = v_w
        prev_v_b = v_b
```

```
def do_nesterov_accelerated_gradient_descent() :
    w, b, eta = init_w, init_b, 1.0
    prev_v_w, prev_v_b, gamma = 0, 0, 0.9
    for i in range(max_epochs) :
        dw, db = 0, 0
        for x,y in zip(X, Y) :
            #do partial updates
            v_w = gamma * prev_v_w
            v_b = gamma * prev_v_b
            #calculate gradients after partial update
            dw = grad_w(w - v_w, b - v_b, x, y)
            db = grad_b(w - v_w, b - v_b, x, y)

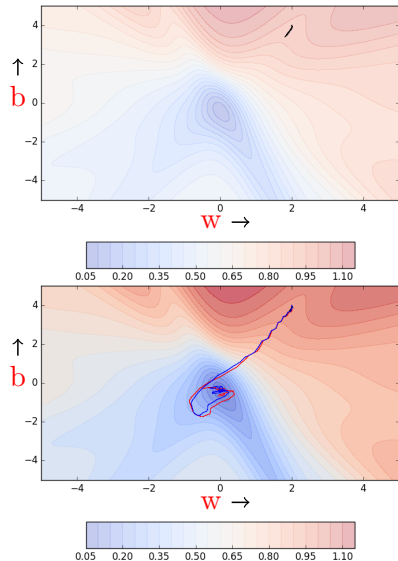
            v_w = gamma * prev_v_w + eta * dw
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            w = w - v_w
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            prev_v_w = v_w
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```



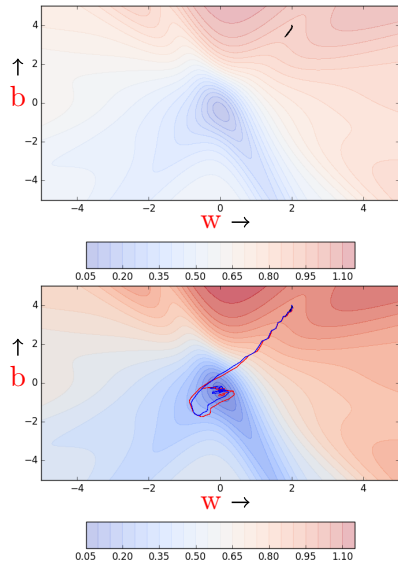
- While the stochastic versions of both Momentum [red] and NAG [blue] exhibit oscillations the relative advantage of NAG over Momentum still holds



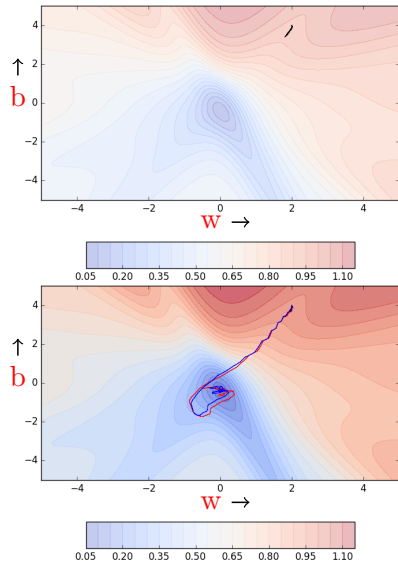
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- While the stochastic versions of both Momentum [red] and NAG [blue] exhibit oscillations the relative advantage of NAG over Momentum still holds (i.e., NAG takes relatively shorter u-turns)
- Further both of them are faster than stochastic gradient descent (after 60 steps, stochastic gradient descent [black - top figure] still exhibits a very high error whereas NAG and Momentum are close to convergence)



And, of course, you can also have the mini batch version of Momentum and NAG...

And, of course, you can also have the mini batch version of Momentum and NAG...I leave that as an exercise :-)