Module 6.7 : PCA : Practical Example

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- What we have plotted here are the first 16 eigen vectors of $X^T X$ (basically, treating each 10K dimensional eigen vector as a 100×100 dimensional image)

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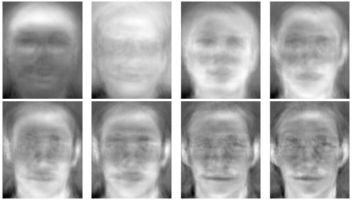
$$\sum_{i=1}^{4} \alpha_{1i} p_i$$



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$$\sum_{i=1}^{8} \alpha_{1i} p_i$$

$$\sum_{i=1}^{12} \alpha_{1i} p_i$$

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- This significantly reduces the storage cost without much loss in image quality