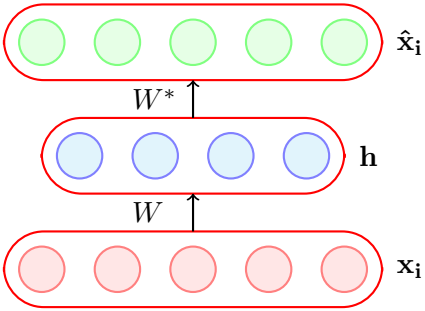
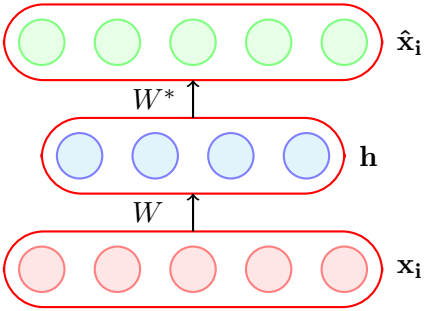
Module 7.1: Introduction to Autoencoders


$\hat{\mathbf{x}}_i$
W^*
$\mathbf{h}$
W
$\mathbf{x}_i$



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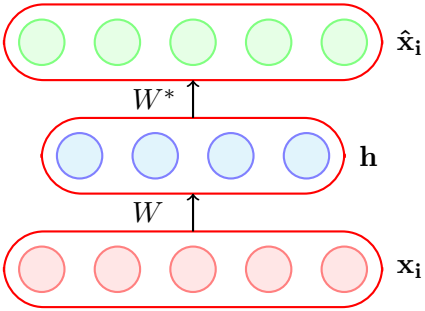

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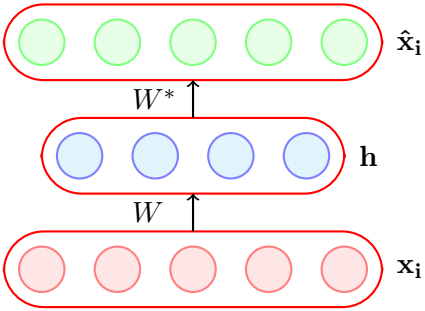
- Encodes its input $\mathbf{x_i}$ into a hidden representation $\mathbf{h}$



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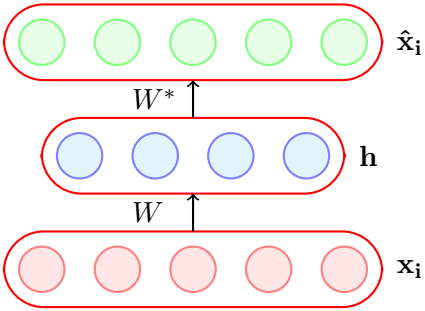

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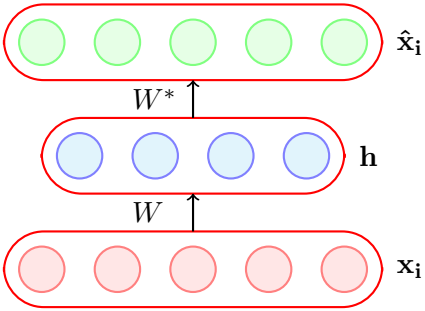

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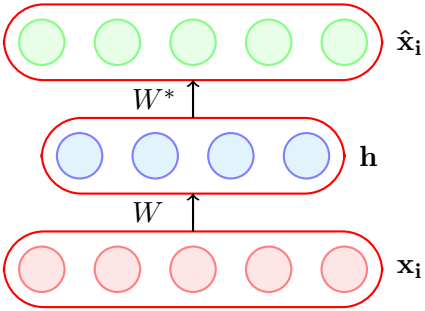

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- Encodes its input $\mathbf{x_i}$ into a hidden representation $\mathbf{h}$
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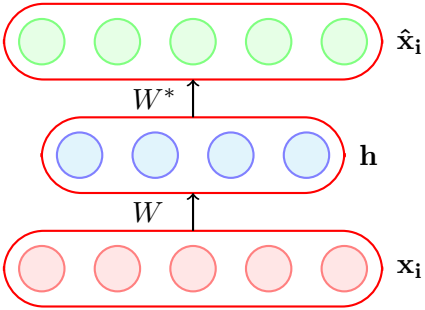
- The model is trained to minimize a certain loss function which will ensure that $\hat{\mathbf{x}}_\mathbf{i}$ is close to $\mathbf{x_i}$ (we will see some such loss functions soon)



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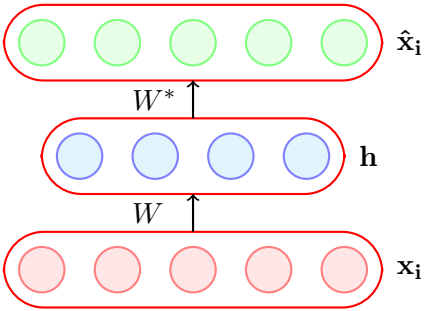

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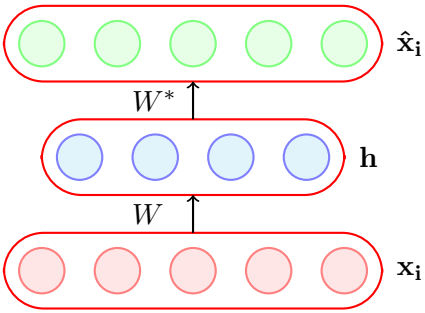

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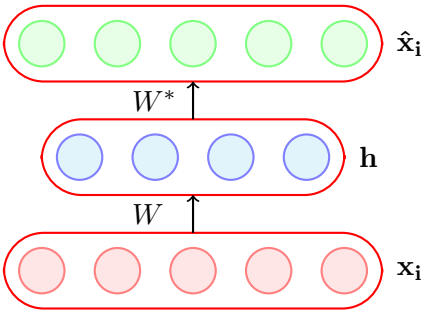

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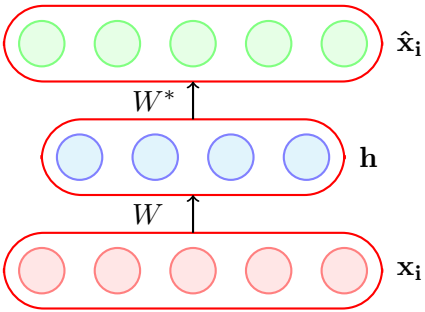
- $\mathbf{h}$ is a loss-free encoding of $\mathbf{x_i}$. It captures all the important characteristics of $\mathbf{x_i}$



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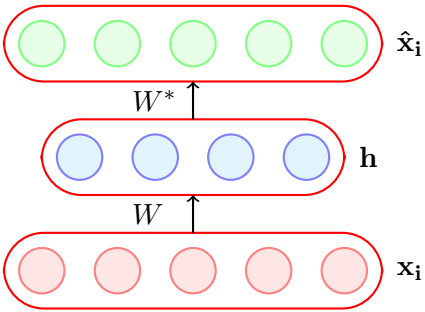
- Do you see an analogy with PCA?



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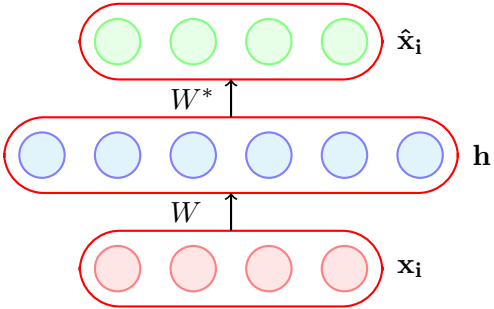

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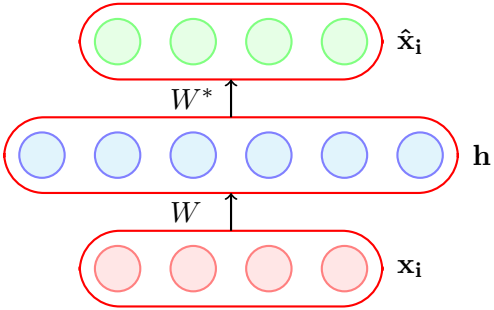

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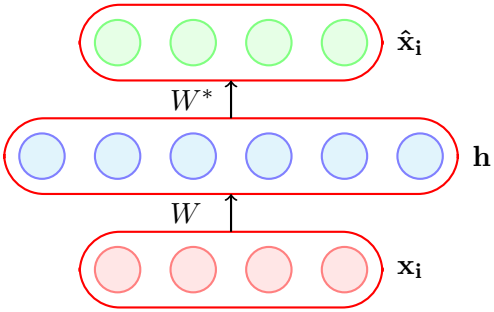

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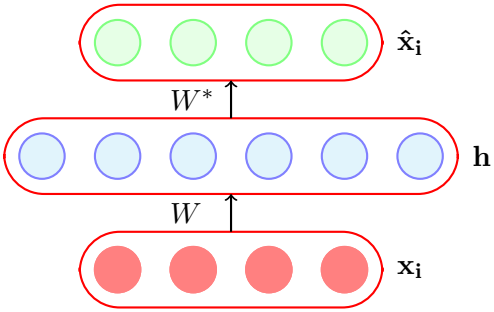

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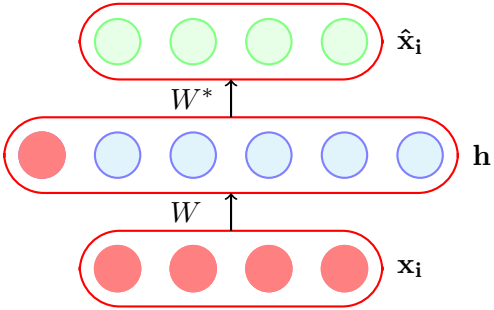

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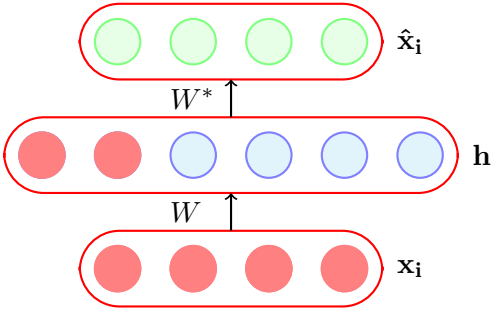

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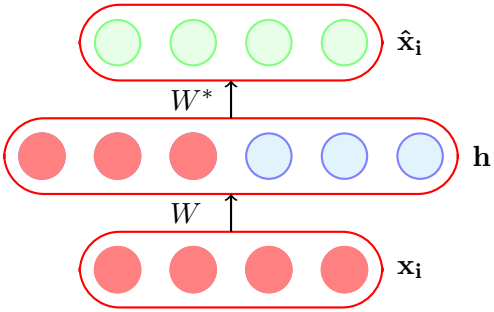

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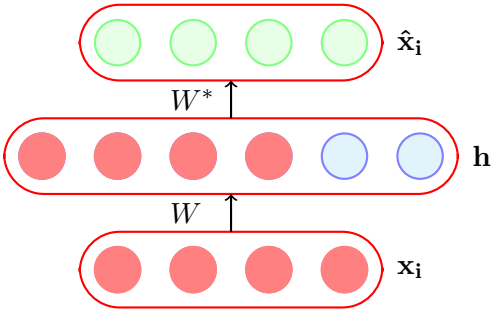

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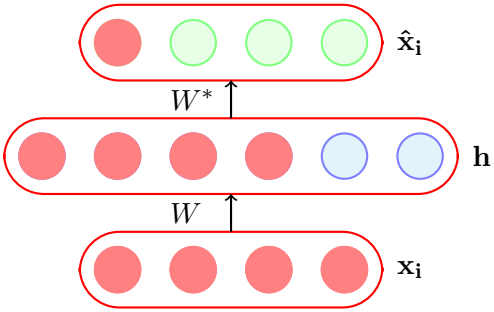

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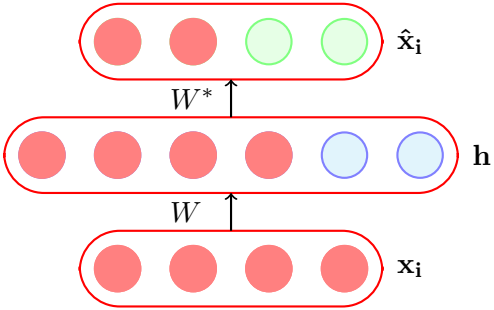

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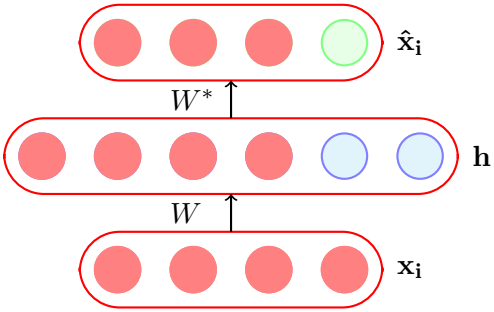

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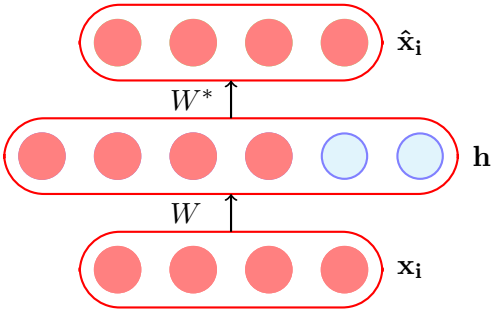

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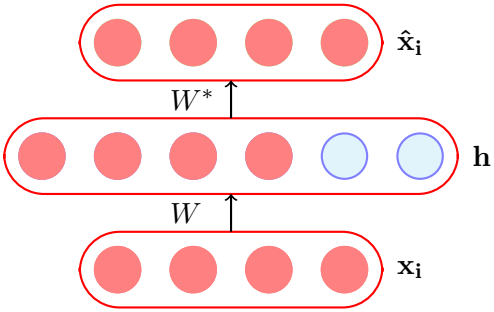

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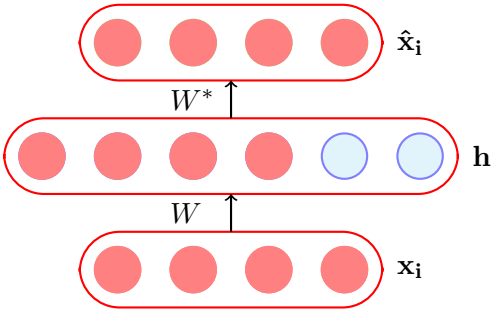
- Such an identity encoding is useless in practice as it does not really tell us anything about the important characteristics of the data



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The Road Ahead

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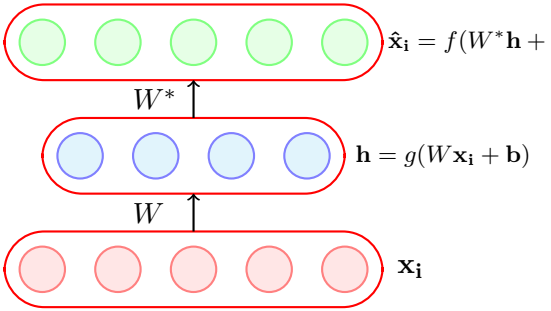
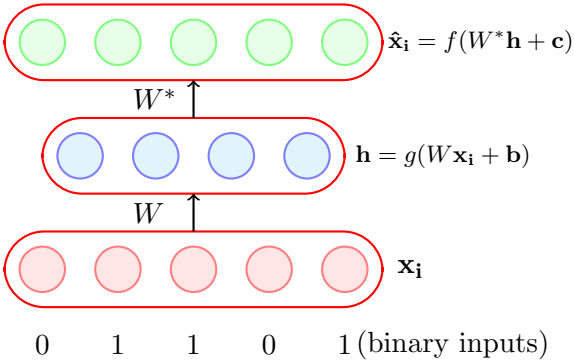
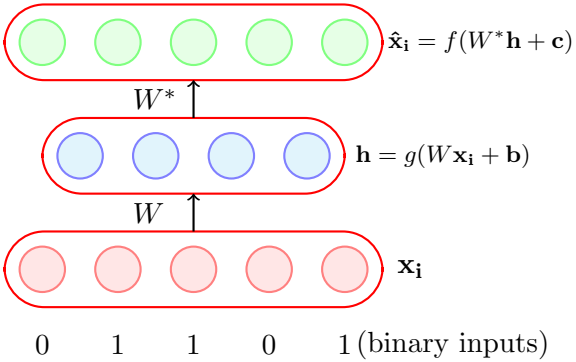

$\hat{\mathbf{x}}_\mathbf{i} = f(W^*\mathbf{h} + \mathbf{c})$
W^*
$\mathbf{h} = g(W\mathbf{x}_\mathbf{i} + \mathbf{b})$
W
$\mathbf{x}_\mathbf{i}$
0 1 1 0 1 (binary inputs)



- Suppose all our inputs are binary (each $x_{ij} \in \{0, 1\}$)



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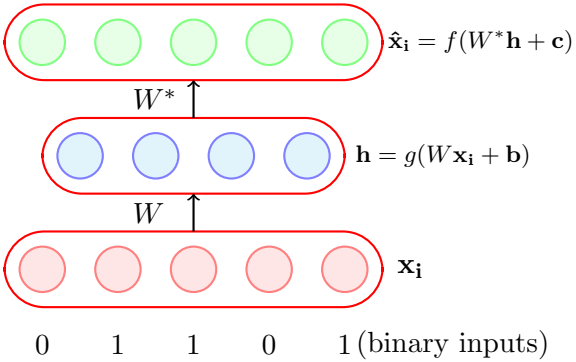

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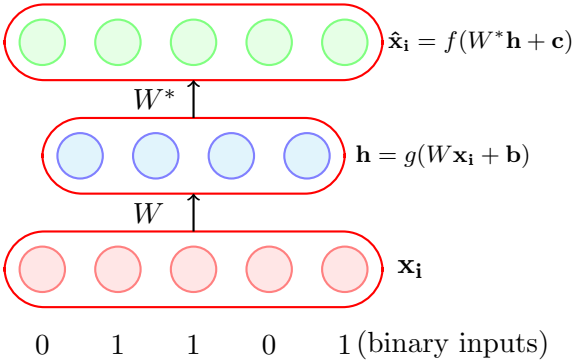

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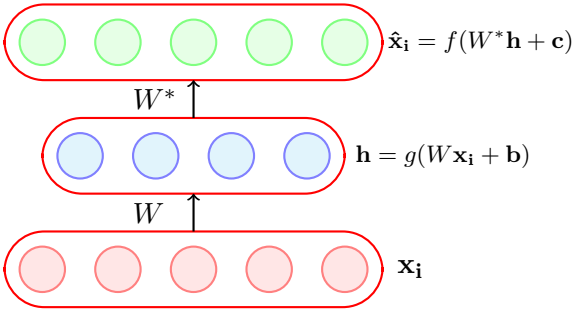

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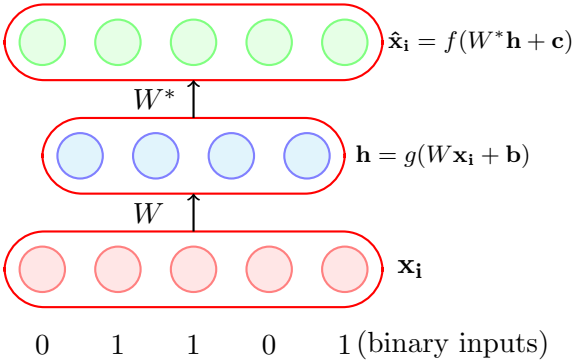
$$\hat{\mathbf{x}}_\mathbf{i} = \mathit{logistic}(W^*\mathbf{h} + \mathbf{c})$$



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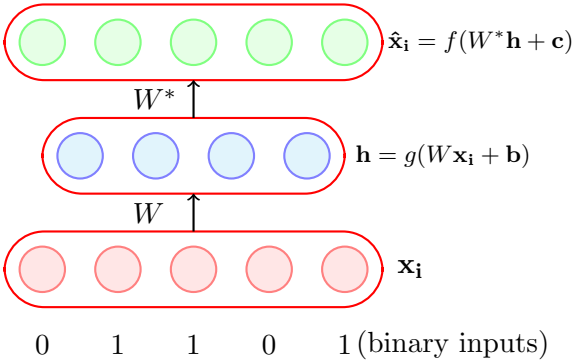
$$\hat{\mathbf{x}}_\mathbf{i} = W^*\mathbf{h} + \mathbf{c}$$
$$\hat{\mathbf{x}}_\mathbf{i} = \mathit{logistic}(W^*\mathbf{h} + \mathbf{c})$$

- Logistic as it naturally restricts all outputs to be between 0 and 1



g is typically chosen as the sigmoid function

- Suppose all our inputs are binary (each $x_{ij} \in \{0, 1\}$)
- Which of the following functions would be most apt for the decoder?

$$\hat{\mathbf{x}}_\mathbf{i} = \tanh(W^*\mathbf{h} + \mathbf{c})$$
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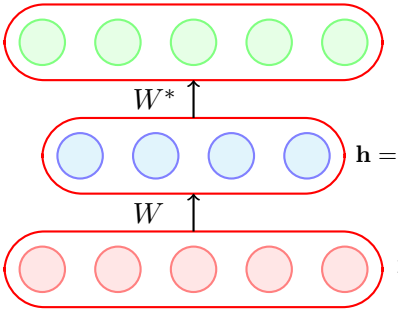
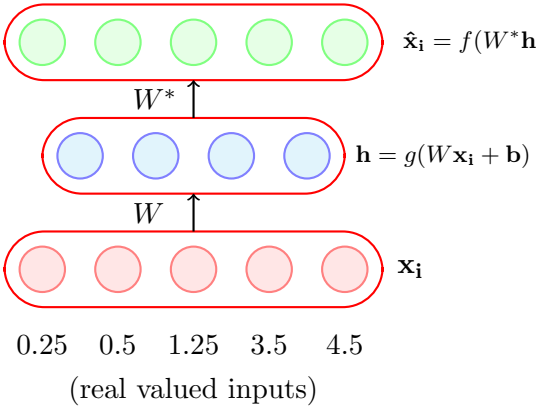
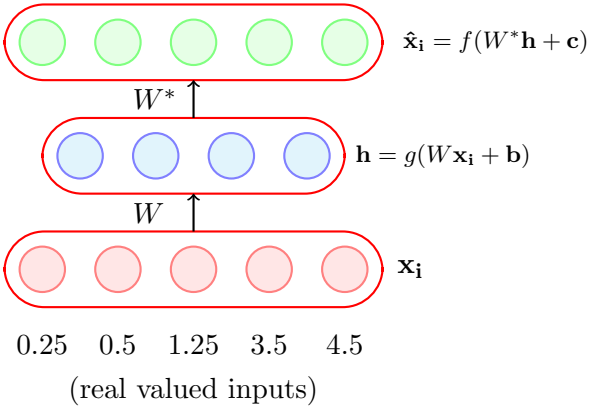

$\hat{\mathbf{x}}_\mathbf{i} = f(W^*\mathbf{h} + \mathbf{c})$
W^*
$\mathbf{h} = g(W\mathbf{x}_\mathbf{i} + \mathbf{b})$
W
$\mathbf{x}_\mathbf{i}$
0.25 0.5 1.25 3.5 4.5
(real valued inputs)



- Suppose all our inputs are real (each $x_{ij} \in \mathbb{R}$)



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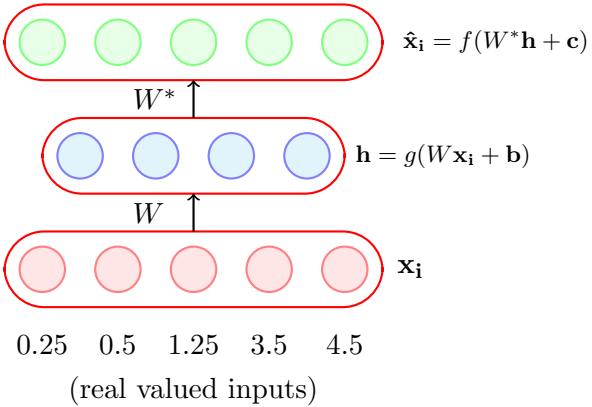

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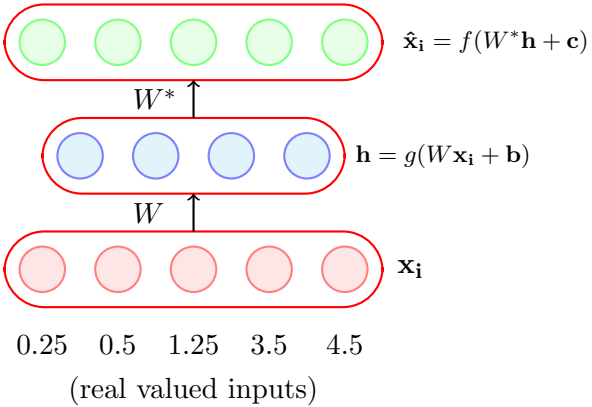

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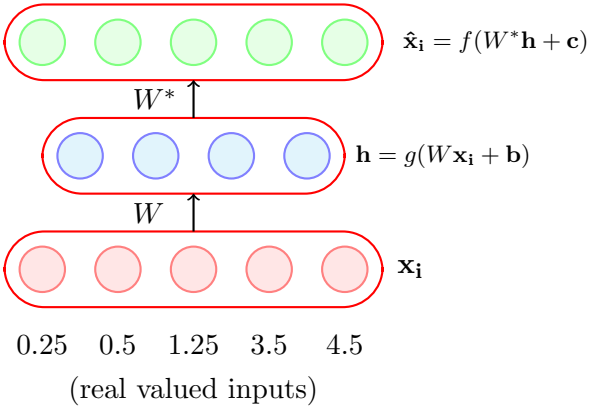

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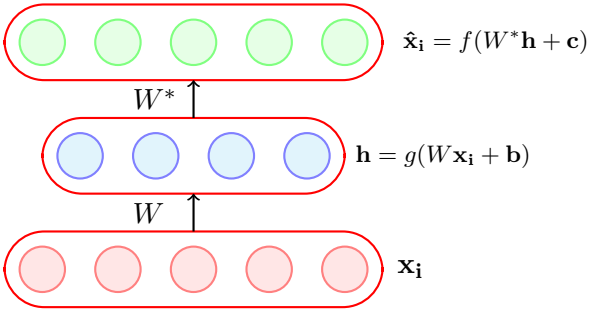

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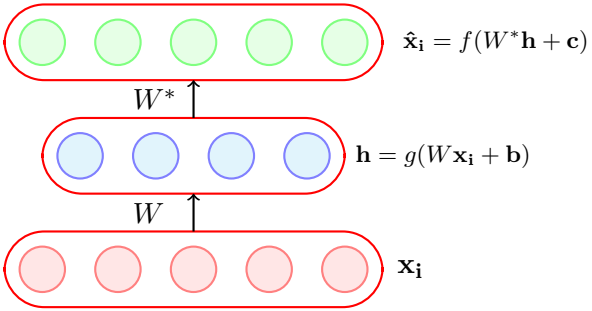
- What will logistic and tanh do?



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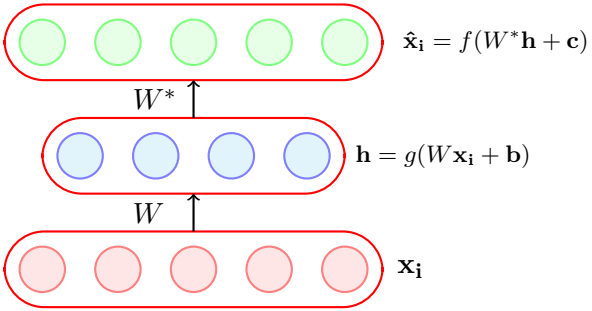

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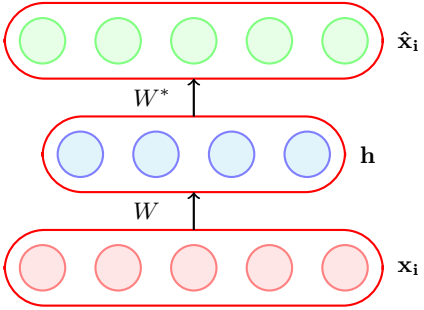
The Road Ahead

- Choice of $f(\mathbf{x_i})$ and $g(\mathbf{x_i})$
- Choice of loss function



$$\mathbf{h} = g(W\mathbf{x_i} + \mathbf{b})$$
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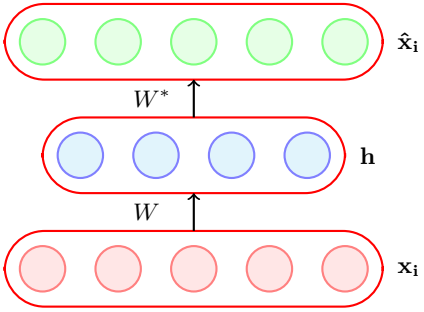

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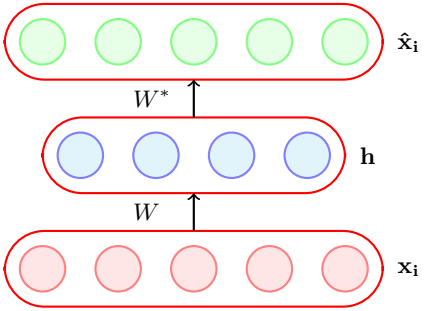
- Consider the case when the inputs are real valued



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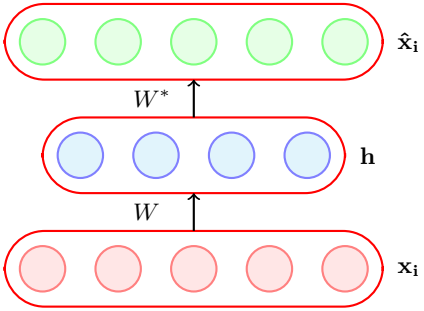
- Consider the case when the inputs are real valued
- The objective of the autoencoder is to reconstruct $\hat{\mathbf{x}}_\mathbf{i}$ to be as close to $\mathbf{x_i}$ as possible



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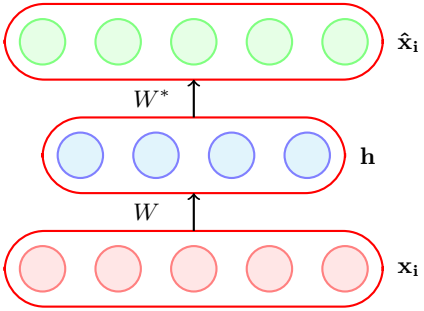
$$\min_{W,W^*,\mathbf{c},\mathbf{b}} \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} (\hat{x}_{ij} - x_{ij})^2$$



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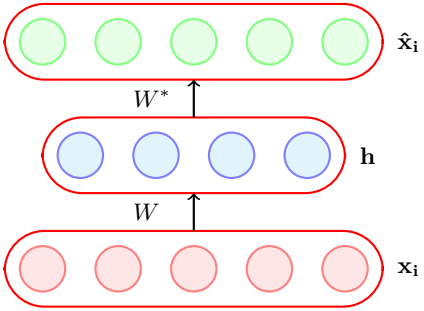

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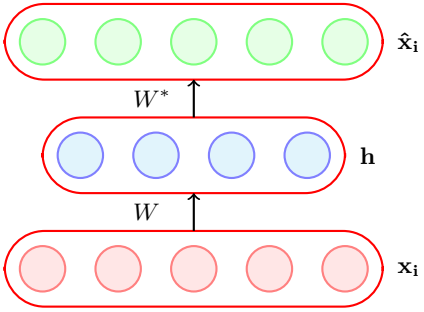
- We can then train the autoencoder just like a regular feedforward network using backpropagation



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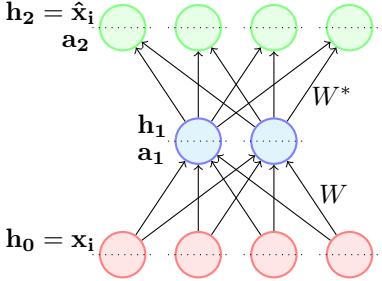
- We can then train the autoencoder just like a regular feedforward network using backpropagation
- All we need is a formula for $\frac{\partial \mathscr{L}(\theta)}{\partial W^*}$ and $\frac{\partial \mathscr{L}(\theta)}{\partial W}$ which we will see now

$$\mathscr{L}(\theta) = (\hat{\mathbf{x}}_\mathbf{i} - \mathbf{x}_\mathbf{i})^T(\hat{\mathbf{x}}_\mathbf{i} - \mathbf{x}_\mathbf{i})$$



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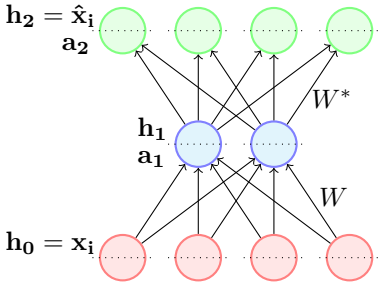

- Note that the loss function is shown for only one training example.

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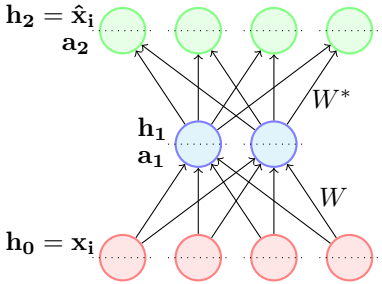

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- $\dfrac{\partial \mathscr{L}(\theta)}{\partial W^*} = \dfrac{\partial \mathscr{L}(\theta)}{\partial \mathbf{h_2}} \boxed{\dfrac{\partial \mathbf{h_2}}{\partial \mathbf{a_2}} \dfrac{\partial \mathbf{a_2}}{\partial W^*}}$

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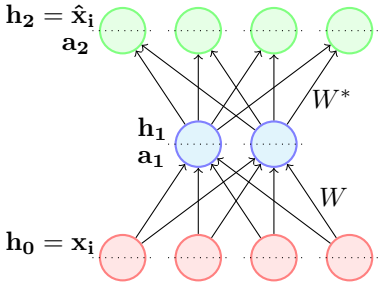

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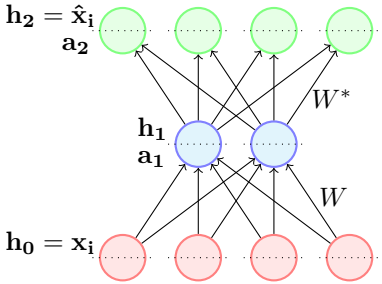

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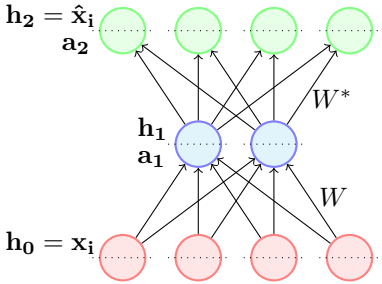
- We have already seen how to calculate the expression in the boxes when we learnt backpropagation

$$\mathscr{L}(\theta) = (\hat{\mathbf{x}}_\mathbf{i} - \mathbf{x}_\mathbf{i})^T(\hat{\mathbf{x}}_\mathbf{i} - \mathbf{x}_\mathbf{i})$$



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$$\frac{\partial \mathscr{L}(\theta)}{\partial \mathbf{h_2}} = \frac{\partial \mathscr{L}(\theta)}{\partial \hat{\mathbf{x}}_\mathbf{i}}$$

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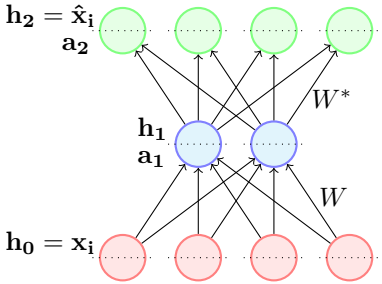

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$$\begin{aligned}\frac{\partial \mathscr{L}(\theta)}{\partial \mathbf{h_2}} &= \frac{\partial \mathscr{L}(\theta)}{\partial \hat{\mathbf{x}}_\mathbf{i}} \\ &= \nabla_{\hat{\mathbf{x}}_\mathbf{i}} \{(\hat{\mathbf{x}}_\mathbf{i} - \mathbf{x}_\mathbf{i})^T(\hat{\mathbf{x}}_\mathbf{i} - \mathbf{x}_\mathbf{i})\}\end{aligned}$$

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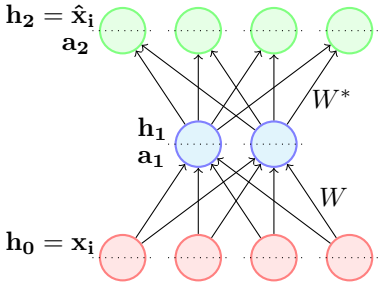

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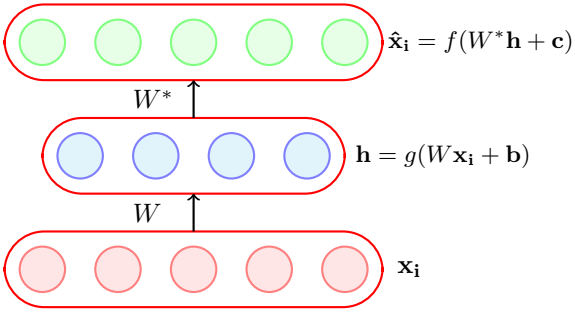
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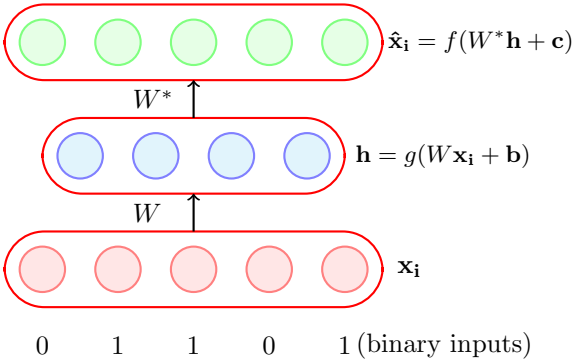
W^*
$\mathbf{h} = g(W\mathbf{x}_\mathbf{i} + \mathbf{b})$
W
$\mathbf{x}_\mathbf{i}$
0 1 1 0 1 (binary inputs)



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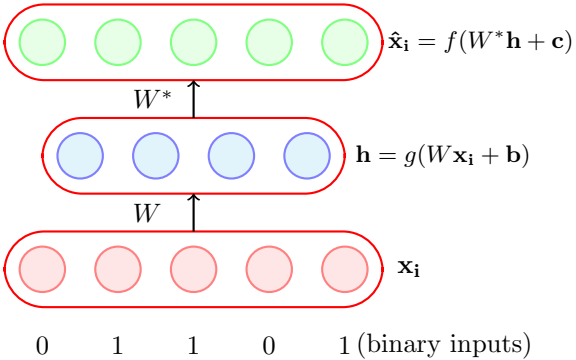
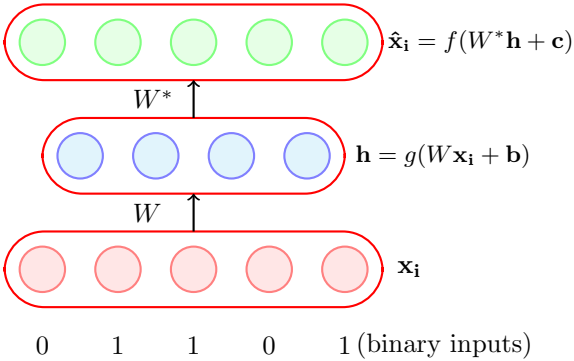


- Consider the case when the inputs are binary
- We use a sigmoid decoder which will produce outputs between 0 and 1, and can be interpreted as probabilities.



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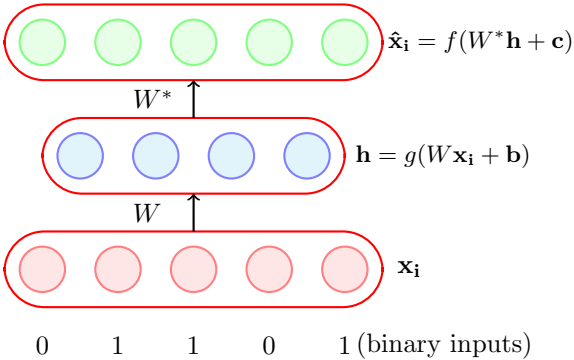

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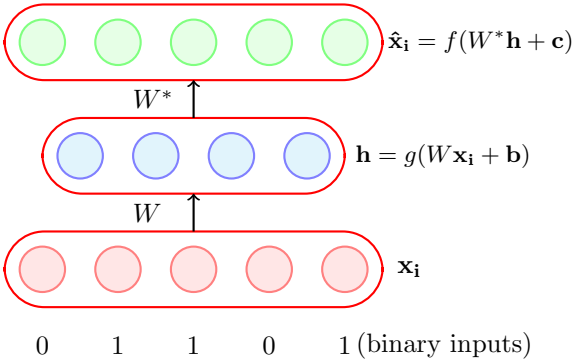

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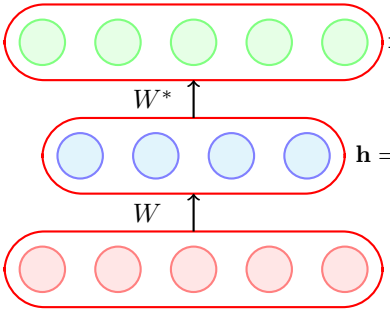

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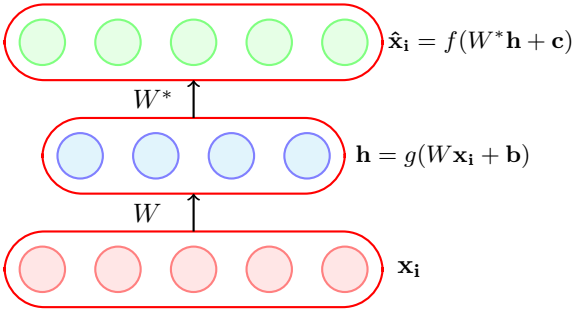

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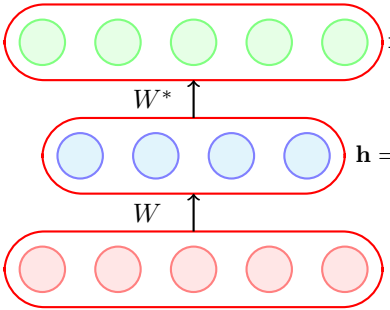

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Indeed the above function will be minimized when $\hat{x}_{ij} = x_{ij}$!

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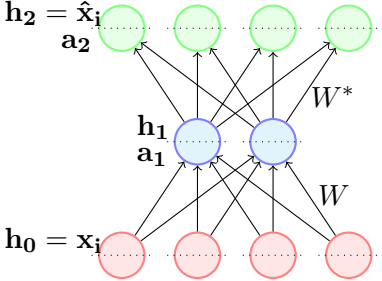

$L20pt$

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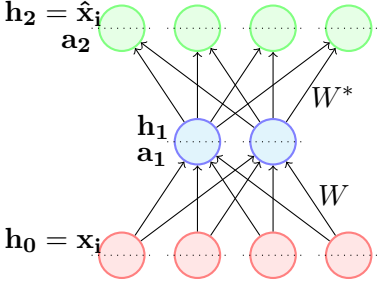

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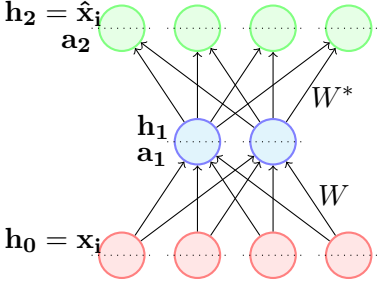

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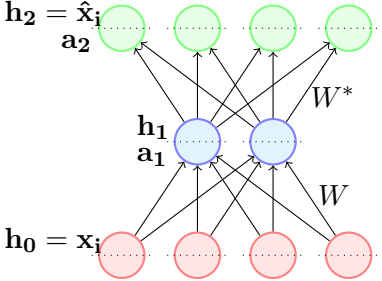

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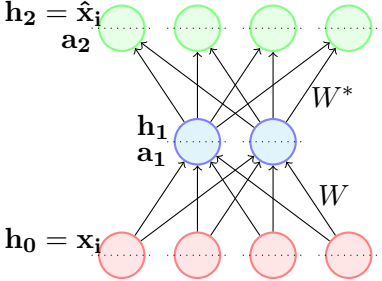

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$$\frac{\partial \mathscr{L}(\theta)}{\partial h_{2j}} = -\frac{x_{ij}}{\hat{x}_{ij}} + \frac{1 - x_{ij}}{1 - \hat{x}_{ij}}$$

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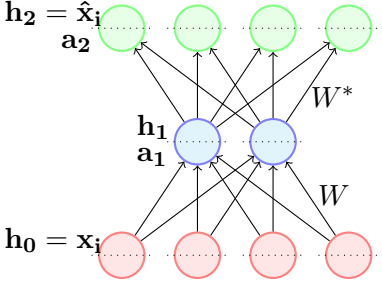

$$L20pt\frac{\partial \mathscr{L}(\theta)}{\partial \mathbf{h_2}} = \frac{\partial \mathscr{L}(\theta)}{\partial h_{21}} \frac{\partial \mathscr{L}(\theta)}{\partial h_{22}} : \frac{\partial \mathscr{L}(\theta)}{\partial h_{2n}}$$

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