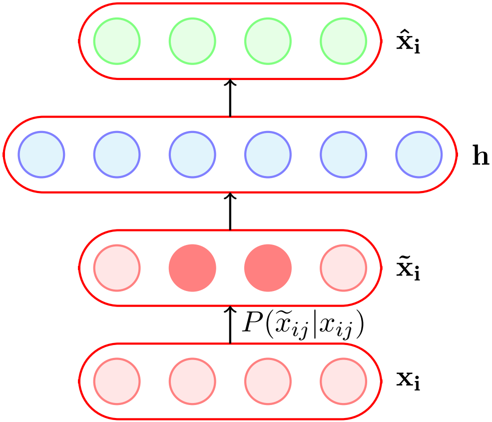
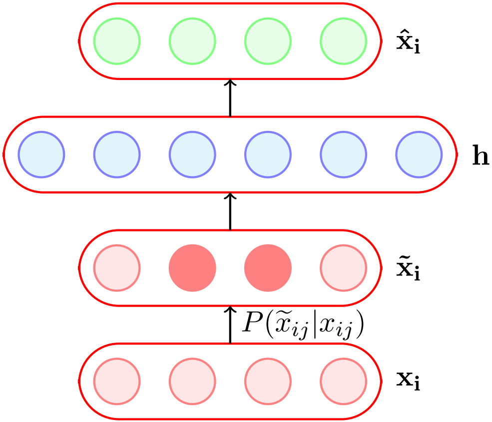
Module 7.4: Denoising Autoencoders



- A denoising encoder simply corrupts the input data using a probabilistic process ($P(\widetilde{x}_{ij}|x_{ij})$) before feeding it to the network



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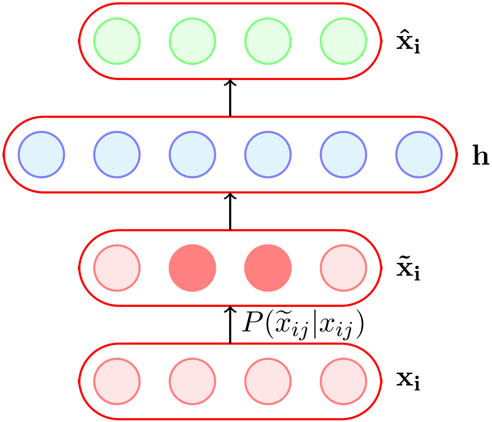

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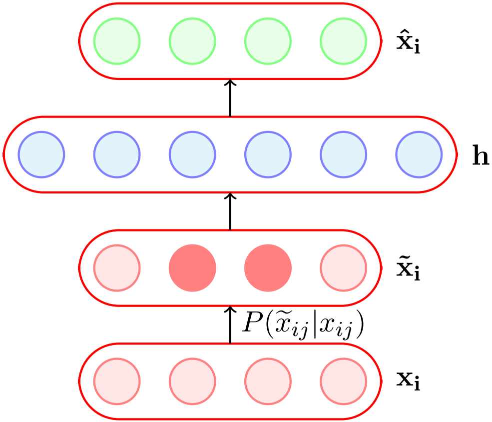

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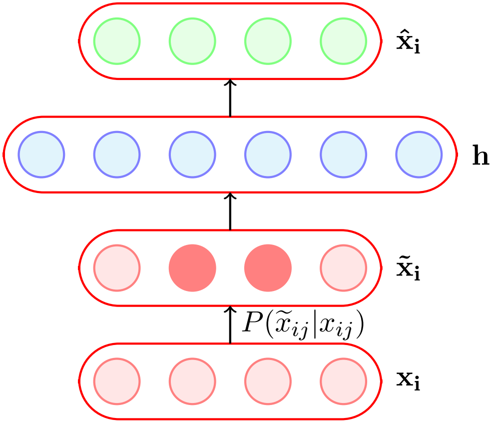

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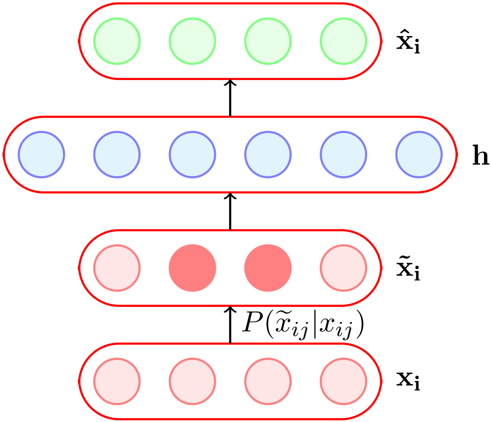
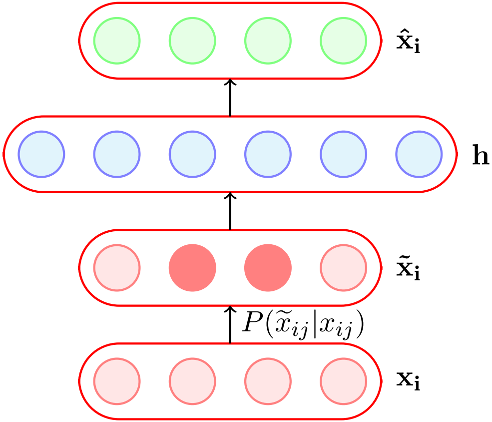
- In other words, with probability q the input is flipped to 0 and with probability $(1 - q)$ it is retained as it is

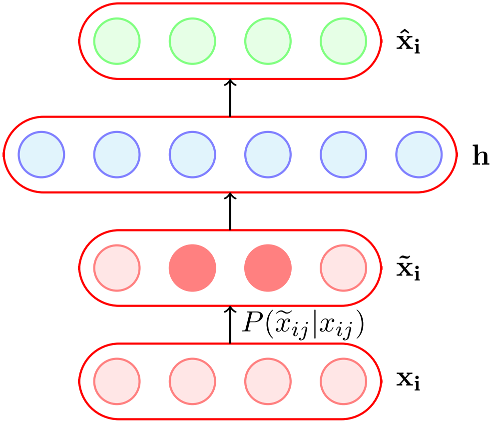


- How does this help ?



- How does this help ?
- This helps because the objective is still to reconstruct the original (un-corrupted) $\mathbf{x}_i$

$$\arg\min_{\theta} \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} (\hat{x}_{ij} - x_{ij})^2$$



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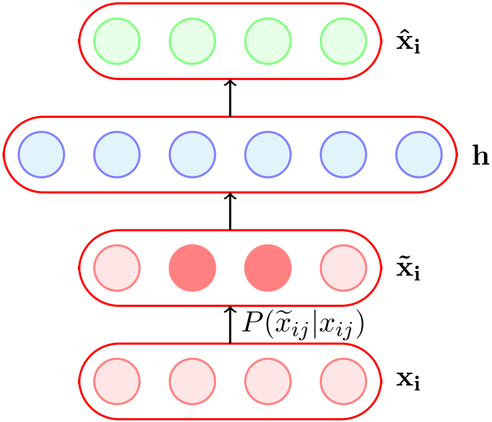

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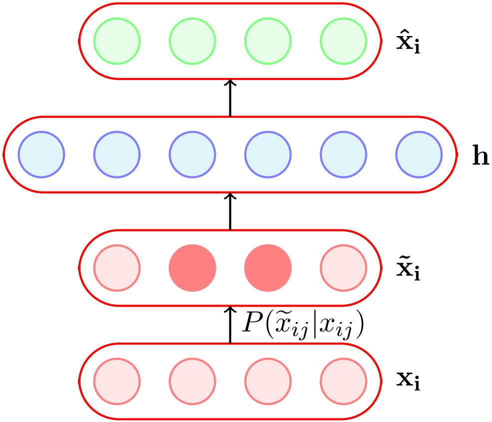

For example, it will have to learn to reconstruct a corrupted x_{ij} correctly by relying on its interactions with other elements of $\mathbf{x}_i$

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We will now see a practical application in which AEs are used and then compare Denoising Autoencoders with regular autoencoders

Task: Hand-written digit recognition

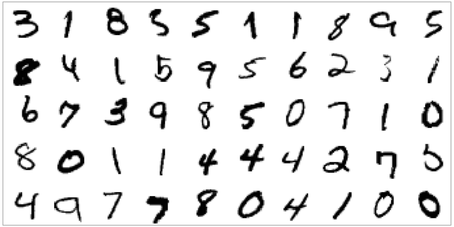
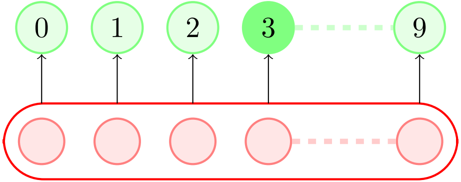


Figure: MNIST Data



$$|\mathbf{x}_i| = 784 = 28 \times 28$$

28*28

Figure: Basic approach(we use raw data as input features)

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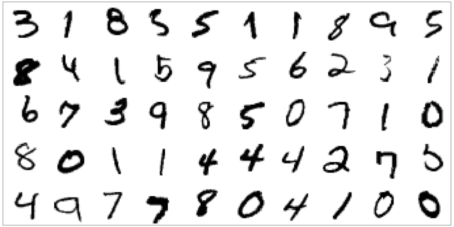

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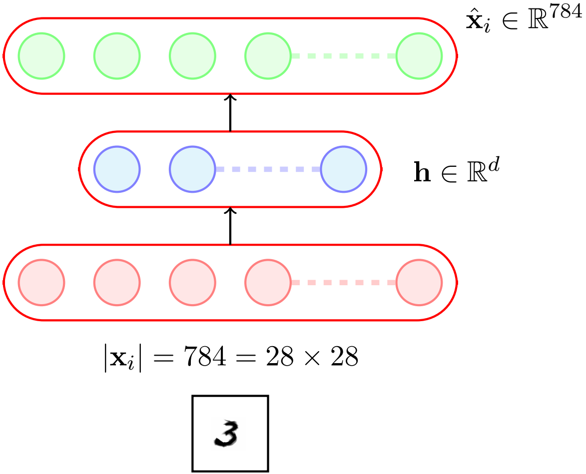

Figure: AE approach (first learn important characteristics of data)

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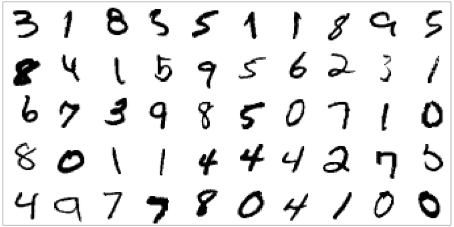

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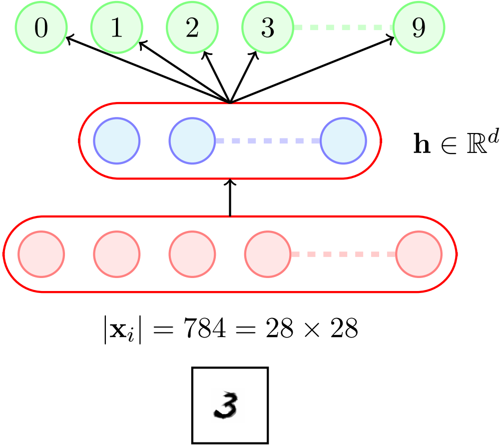
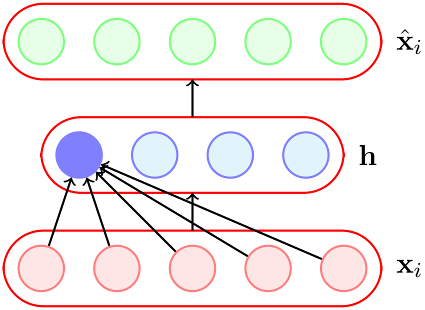


Figure: AE approach (and then train a classifier on top of this hidden representation)

We will now see a way of visualizing AEs and use this visualization to compare different AEs



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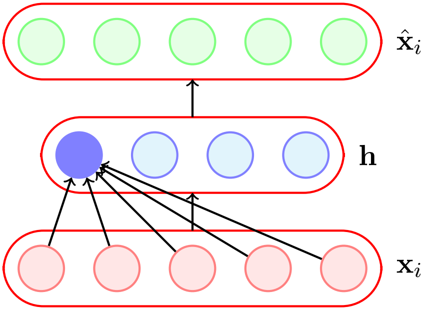

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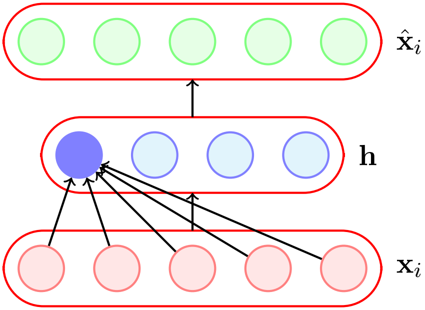
- For example,

$$\mathbf{h}_1 = \sigma(W_1^T \mathbf{x}_i) \; [\textit{ignoring bias } b]$$

Where W_1 is the trained vector of weights connecting the input to the first hidden neuron



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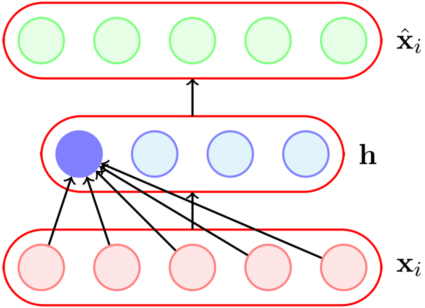

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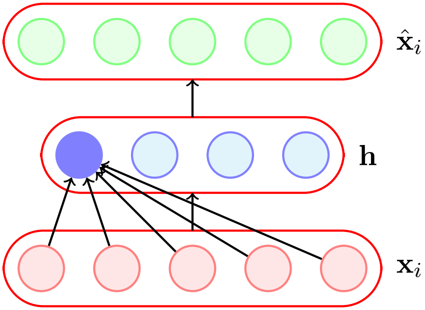
- Suppose we assume that our inputs are normalized so that $\|\mathbf{x}_i\| = 1$



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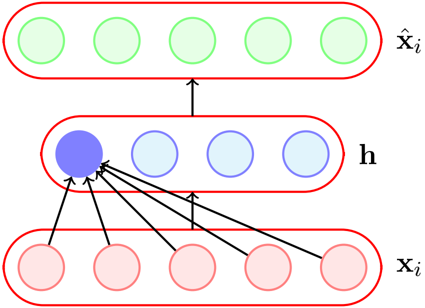

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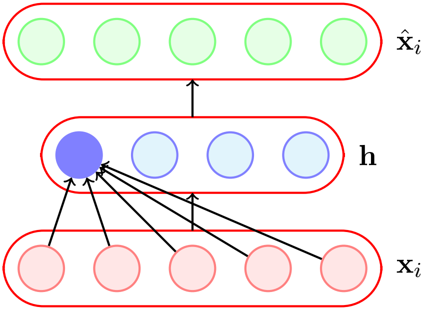

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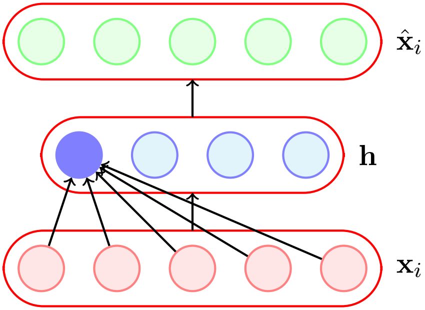

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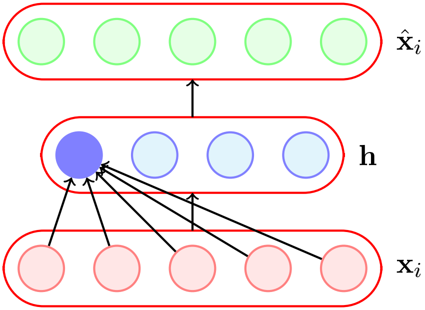

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- These $\mathbf{x}_i$'s are computed by the above formula using the weights $(W_1, W_2 \ldots W_k)$ learned by the respective autoencoders

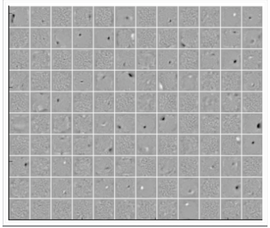


Figure: Vanilla AE (No noise)

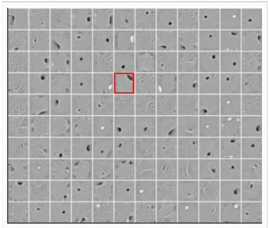


Figure: 25% Denoising AE (q=0.25)

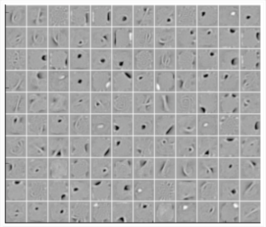
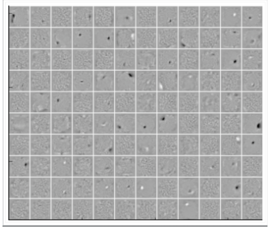


Figure: 50% Denoising AE (q=0.5)

- The vanilla AE does not learn many meaningful patterns



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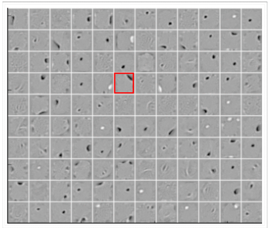

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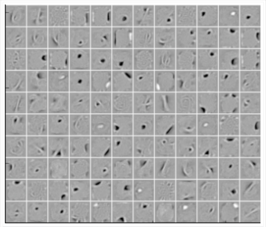

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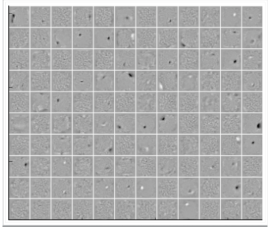

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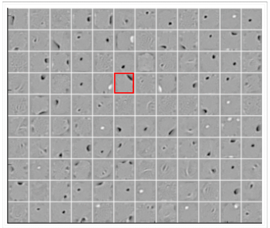

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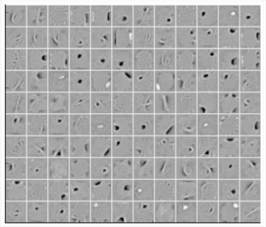

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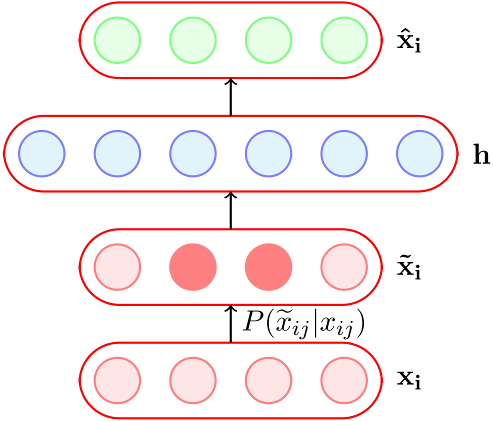
- As the noise increases the filters become more wide because the neuron has to rely on more adjacent pixels to feel confident about a stroke



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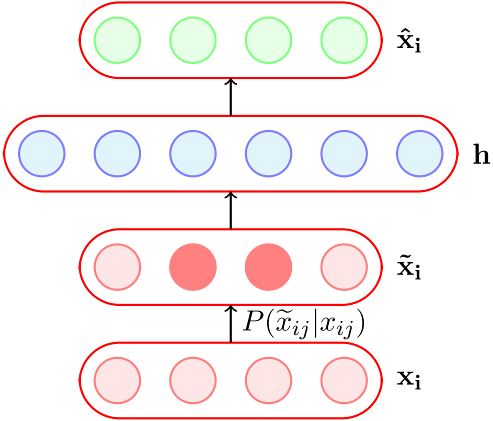

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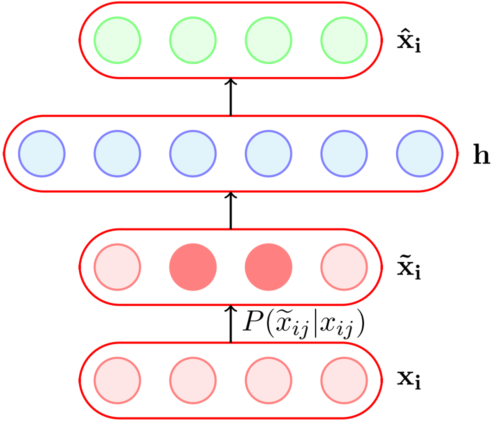
- Another way of corrupting the inputs is to add a Gaussian noise to the input

$$\widetilde{x}_{ij} = x_{ij} + \mathcal{N}(0, 1)$$



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- We will now use such a denoising AE on a different dataset and see their performance

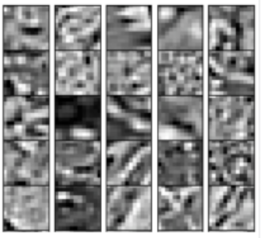


Figure: Data

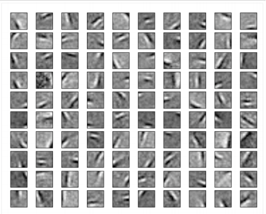


Figure: AE filters

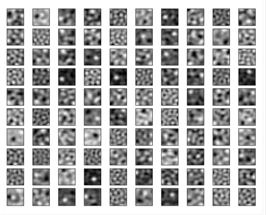
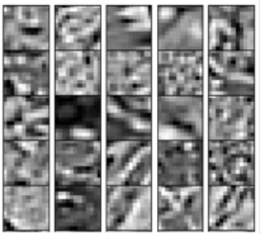


Figure: Weight decay filters

- The hidden neurons essentially behave like edge detectors



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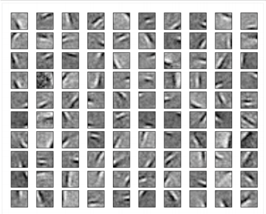

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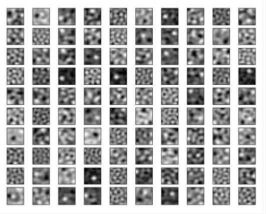

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- PCA does not give such edge detectors