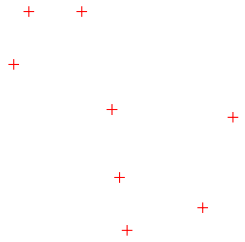
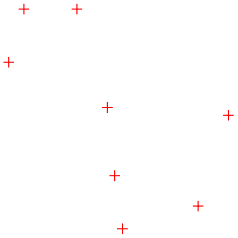
Module 8.1 : Bias and Variance

We will begin with a quick overview of bias, variance and the trade-off between them.



The points were drawn from a sinusoidal function (the true $f(x)$)

- Let us consider the problem of fitting a curve through a given set of points



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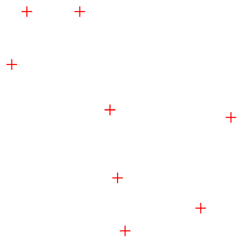
- Let us consider the problem of fitting a curve through a given set of points
- We consider two models :



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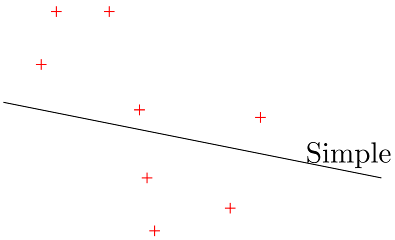
- Let us consider the problem of fitting a curve through a given set of points
- We consider two models :

$$\underset{(degree:1)}{Simple} \quad y = \hat{f}(x) = w_1 x + w_0$$



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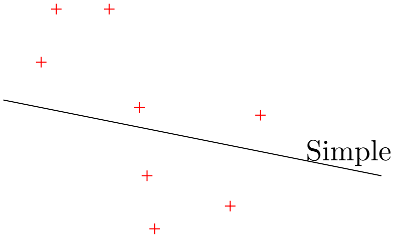

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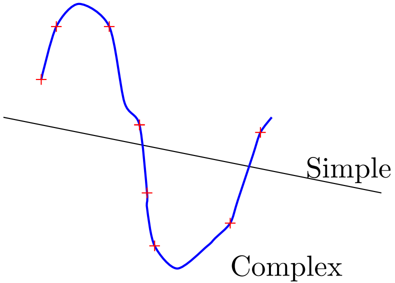
- Let us consider the problem of fitting a curve through a given set of points
- We consider two models :

$$\underset{(degree:1)}{Simple} \quad y = \hat{f}(x) = w_1 x + w_0$$

$$\underset{(degree:25)}{Complex} \quad y = \hat{f}(x) = \sum_{i=1}^{25} w_i x^i + w_0$$



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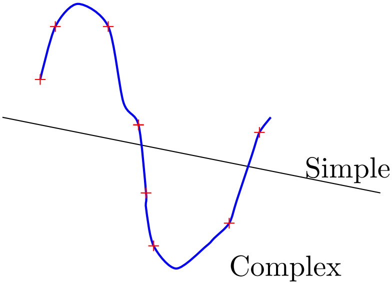

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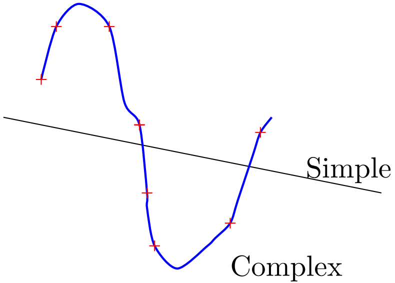
- Note that in both cases we are making an assumption about how y is related to x. We have no idea about the true relation $f(x)$



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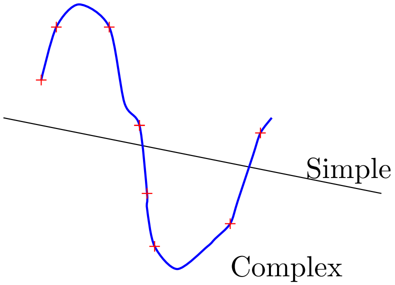
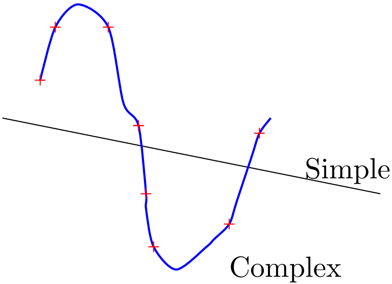
- Note that in both cases we are making an assumption about how y is related to x. We have no idea about the true relation $f(x)$
- The training data consists of 100 points



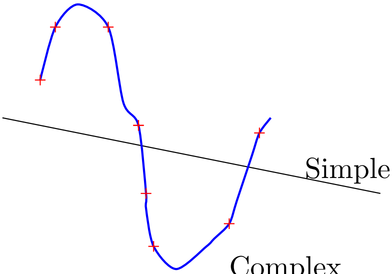
The points were drawn from a sinusoidal function (the true $f(x)$)

- We sample 25 points from the training data and train a simple and a complex model



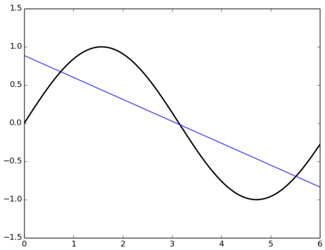
The points were drawn from a sinusoidal function (the true $f(x)$)

- We sample 25 points from the training data and train a simple and a complex model
- We repeat the process 'k' times to train multiple models (each model sees a different sample of the training data)



The points were drawn from a sinusoidal function (the true $f(x)$)

- We sample 25 points from the training data and train a simple and a complex model
- We repeat the process 'k' times to train multiple models (each model sees a different sample of the training data)
- We make a few observations from these plots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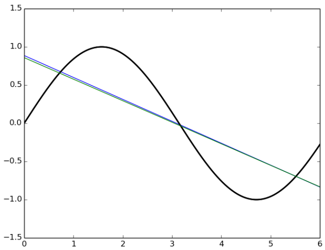
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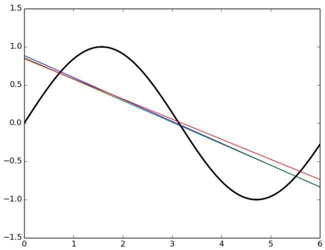

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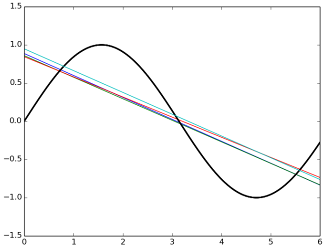
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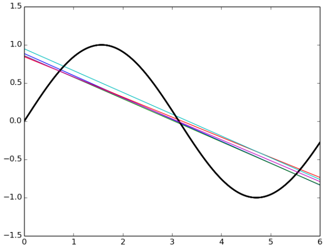
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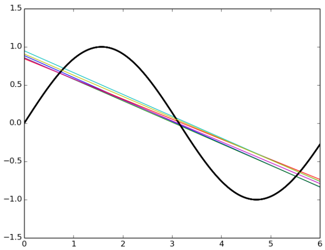
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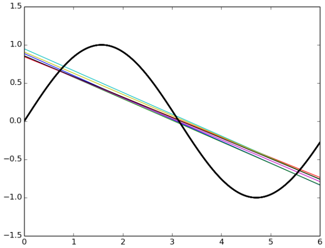
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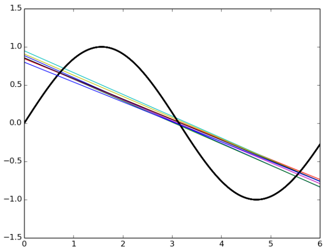
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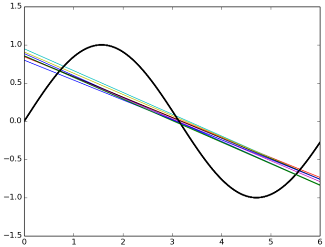
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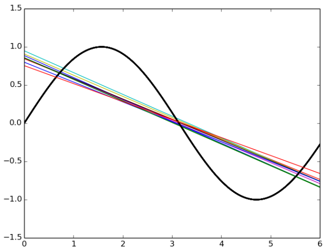
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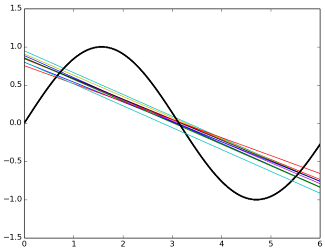
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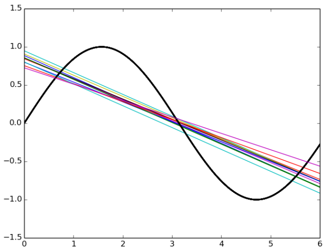
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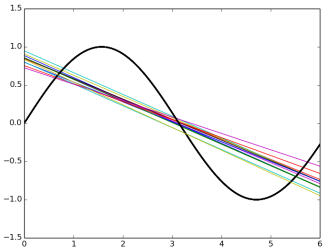
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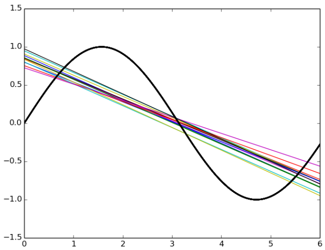
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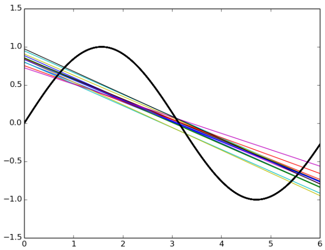
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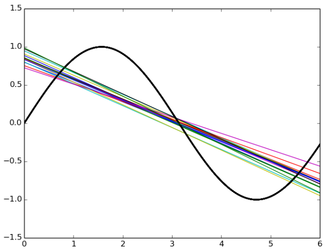
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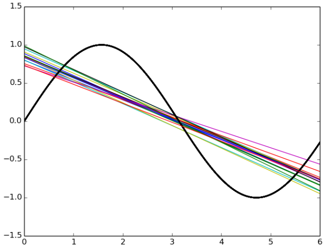
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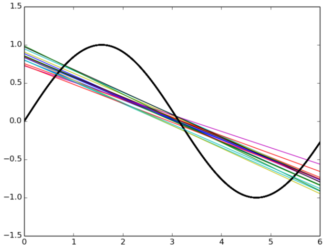

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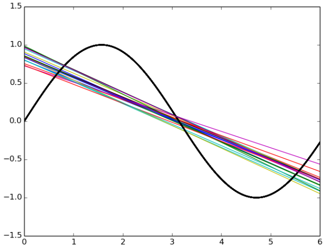
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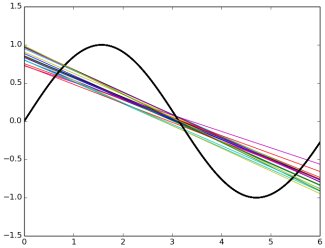
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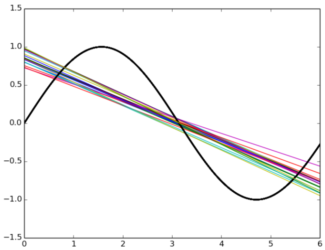

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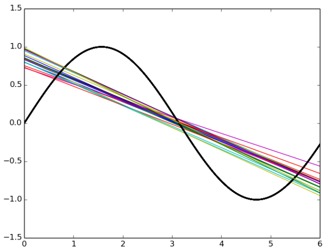

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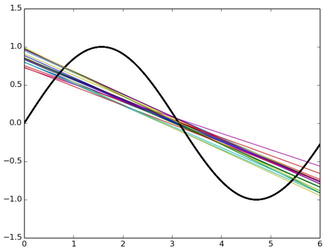
- However they are very far from the true sinusoidal curve (under fitting)



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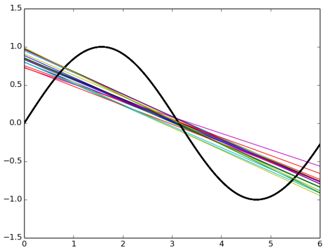

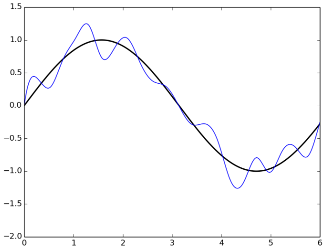

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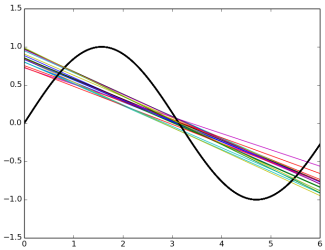

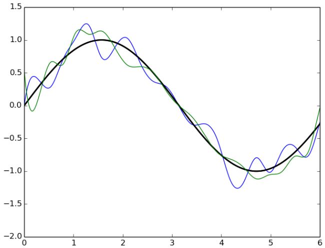

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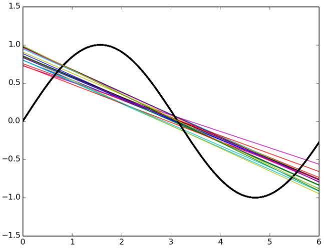

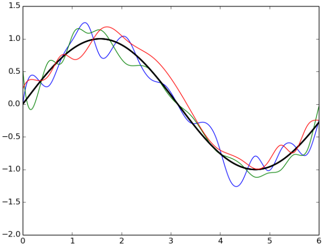

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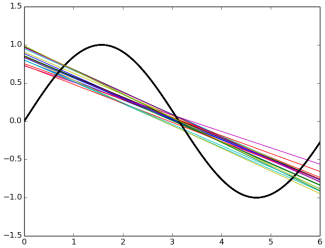

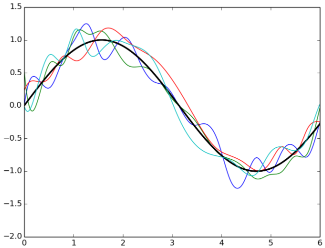

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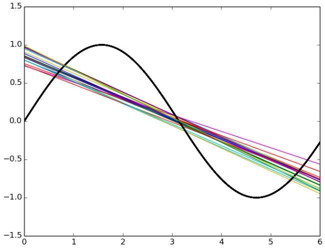

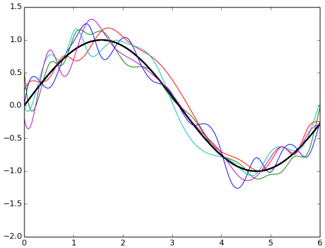

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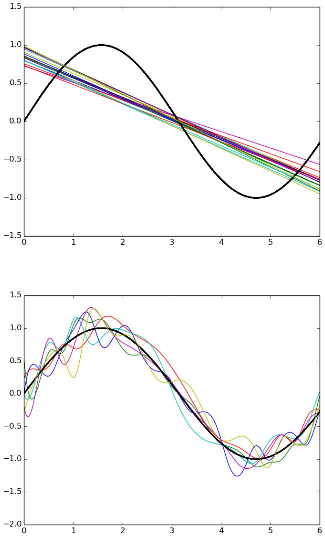

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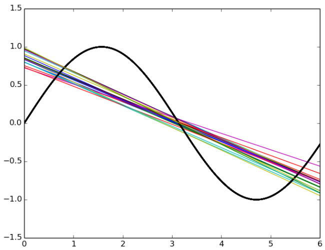

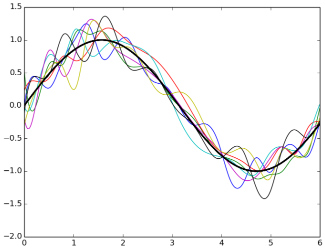

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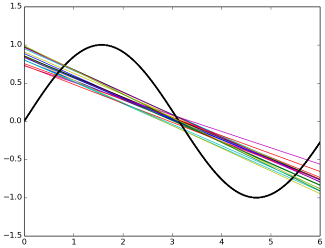

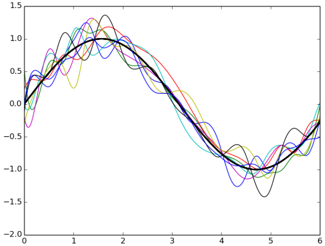

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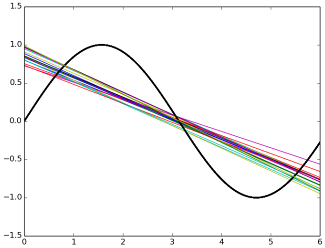

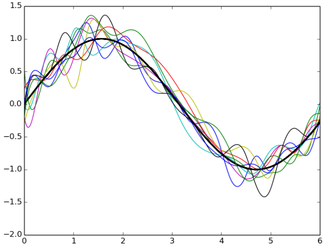

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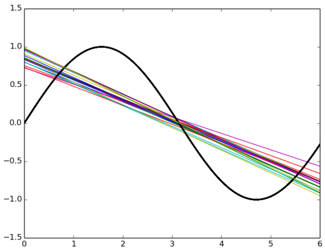

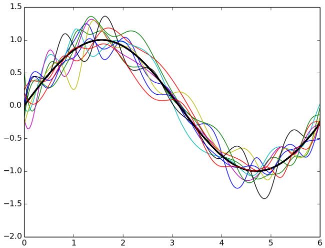

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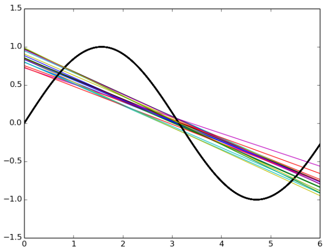

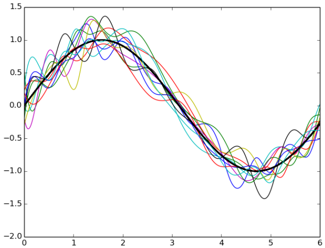

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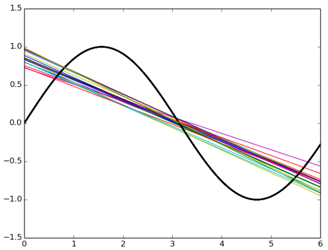

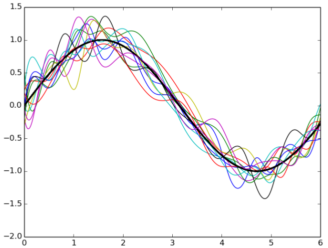

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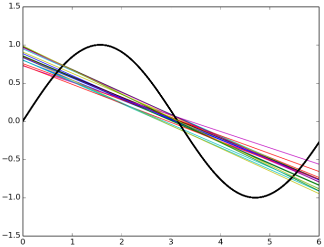

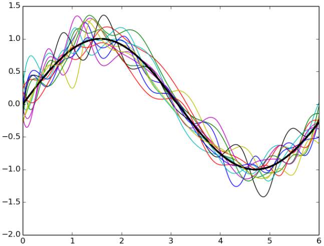

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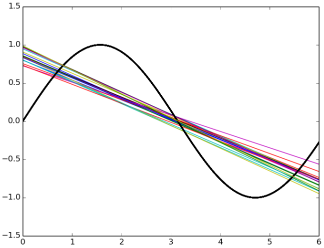

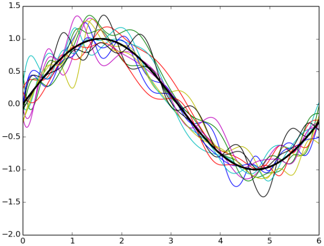

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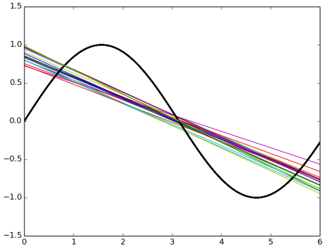

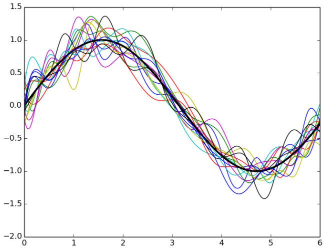

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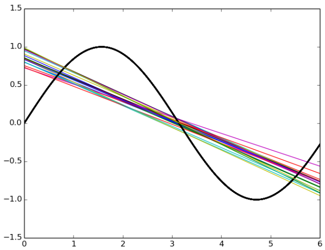

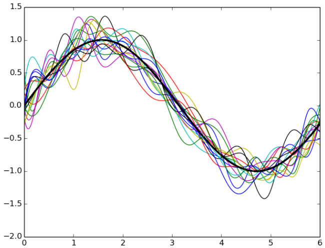

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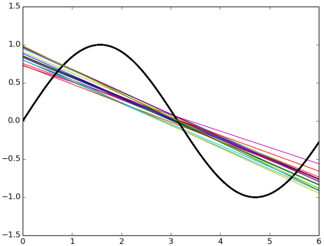

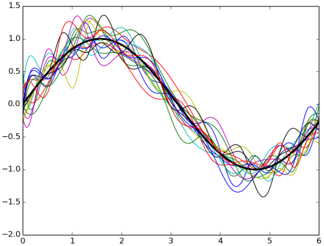

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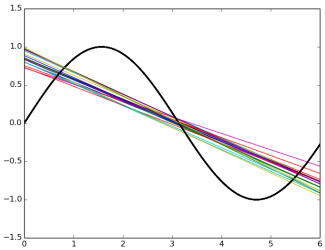

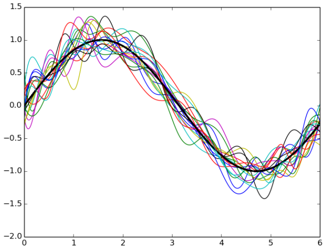

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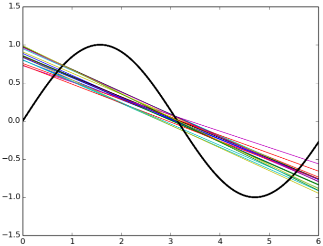

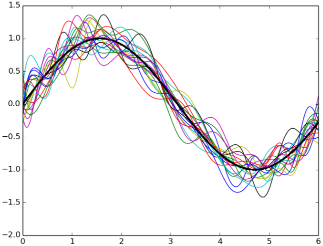

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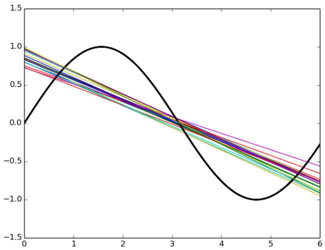

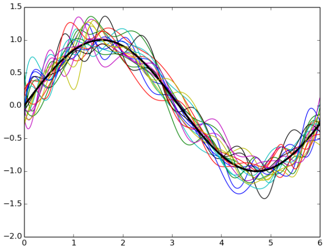

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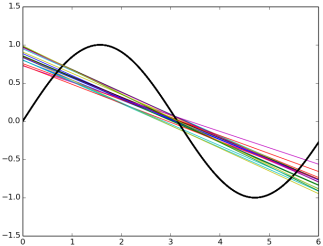
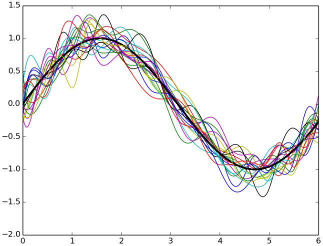
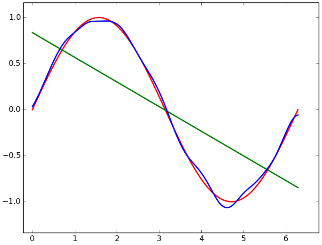




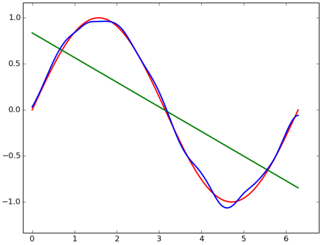
- Simple models trained on different samples of the data do not differ much from each other
- However they are very far from the true sinusoidal curve (under fitting)
- On the other hand, complex models trained on different samples of the data are very different from each other (high variance)



Green Line: Average value of $\hat{f}(x)$ for the simple model
Blue Curve: Average value of $\hat{f}(x)$ for the complex model
Red Curve: True model (f(x))

- Let $f(x)$ be the true model (sinusoidal in this case) and $\hat{f}(x)$ be our estimate of the model (simple or complex, in this case) then,

$$\text{Bias } (\hat{f}(x)) = E[\hat{f}(x)] - f(x)$$



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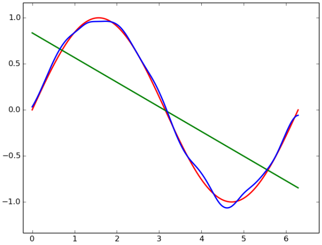
- $E[\hat{f}(x)]$ is the average (or expected) value of the model



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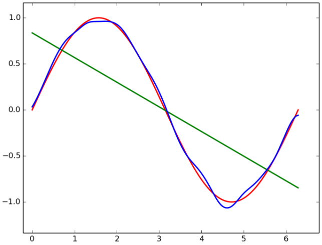
- We can see that for the simple model the average value (green line) is very far from the true value $f(x)$ (sinusoidal function)



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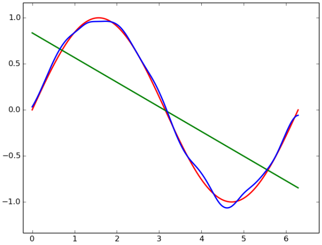
- $E[\hat{f}(x)]$ is the average (or expected) value of the model
- We can see that for the simple model the average value (green line) is very far from the true value $f(x)$ (sinusoidal function)
- Mathematically, this means that the simple model has a high bias



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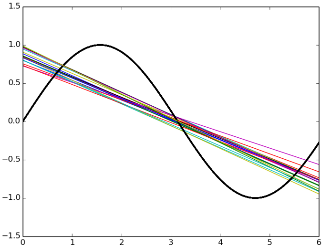
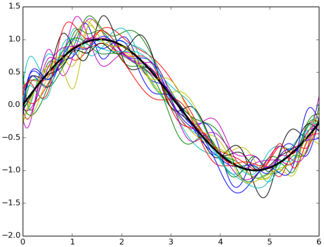
$$\text{Bias } (\hat{f}(x)) = E[\hat{f}(x)] - f(x)$$

- $E[\hat{f}(x)]$ is the average (or expected) value of the model
- We can see that for the simple model the average value (green line) is very far from the true value $f(x)$ (sinusoidal function)
- Mathematically, this means that the simple model has a high bias
- On the other hand, the complex model has a low bias

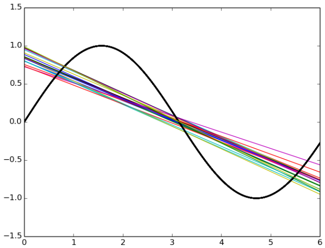


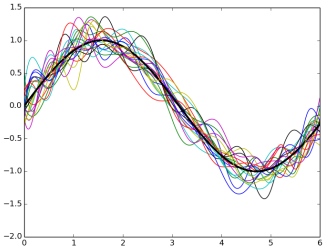


- We now define,

$$\text{Variance } (\hat{f}(x)) = E[(\hat{f}(x) - E[\hat{f}(x)])^2]$$

(Standard definition from statistics)





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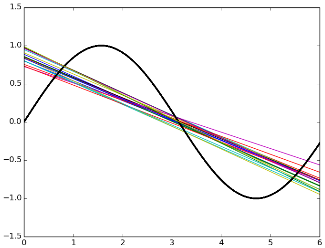
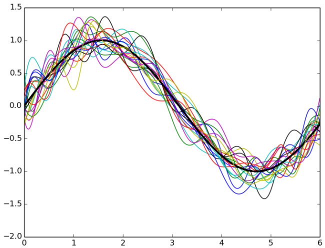
- Roughly speaking it tells us how much the different $\hat{f}(x)$'s (trained on different samples of the data) differ from each other





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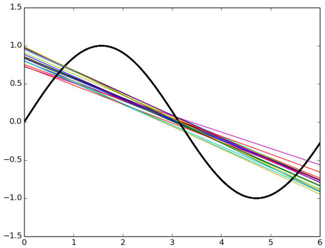
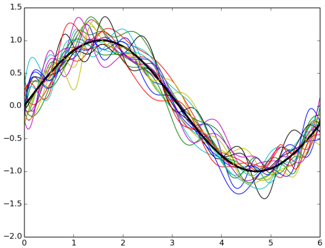
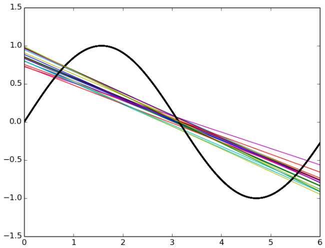
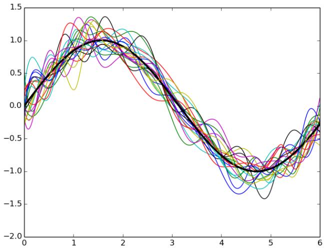
- Roughly speaking it tells us how much the different $\hat{f}(x)$'s (trained on different samples of the data) differ from each other
- It is clear that the simple model has a low variance whereas the complex model has a high variance



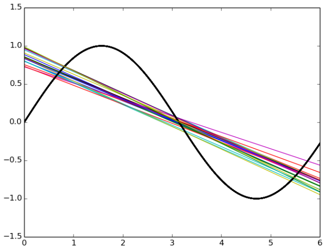


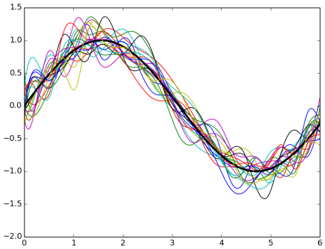
- In summary (informally)



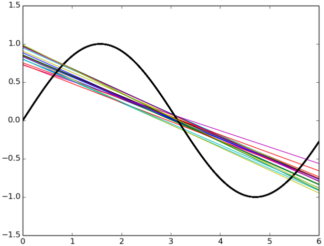


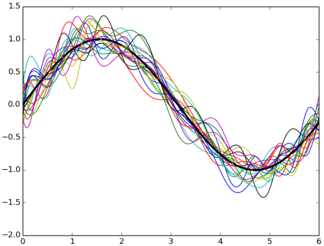
- In summary (informally)
- Simple model: high bias, low variance



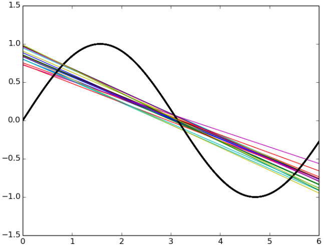


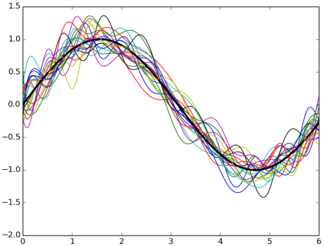
- In summary (informally)
- Simple model: high bias, low variance
- Complex model: low bias, high variance





- In summary (informally)
- Simple model: high bias, low variance
- Complex model: low bias, high variance
- There is always a trade-off between the bias and variance





- In summary (informally)
- Simple model: high bias, low variance
- Complex model: low bias, high variance
- There is always a trade-off between the bias and variance
- Both bias and variance contribute to the mean square error. Let us see how